Advanced Statistical Signal Processing for Next Generation Trajectory Prediction

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Outline

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- **Methodological Challenges and Perspectives**
  - **Towards Robust TP Solutions**
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  - **From Centralized to Cooperative Processing**
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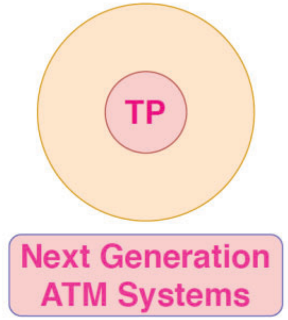
- **Results**
- **Conclusion**

Introduction and Background

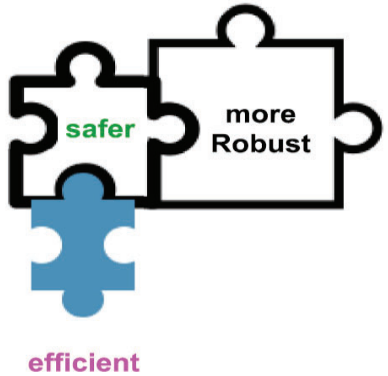
Accurate and reliable TP:

- Next generation of **on-board** and **ground-board** DSTs:
  - Traffic synchronization and separation management
- Enhanced safety net:
  - (Partially) automated environment, on-ground, airborne
  - Distributed system



The aim of this PhD:

New SSP approach
to improve TP
in new generation of ATM systems



TP for the flight execution phase.

*** Trajectory Prediction (TP), Decision Support Tools (DST), * Statistical Signal Processing (SSP), * Air Traffic Management (ATM).**

Introduction and Background

ATM operations are evolving towards a trajectory-centric paradigm

- Airports
- Airspace Users
- ANSPs
- NM

SESAR and NexGen proposed a new concept of operations aiming to build an ATM system based on the notion of TBO.

TBO $\neq$ current airspace-centric paradigm:

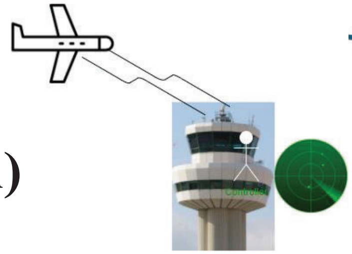
- Dynamically managing flights on an end-to-end time basis,
- Enabling AUs to fly their preferred flight trajectories.

On-board TP: in FMS for trajectory planning and to compute the estimates of the fuel on-board, times of arrival, the location of the top of descent, …

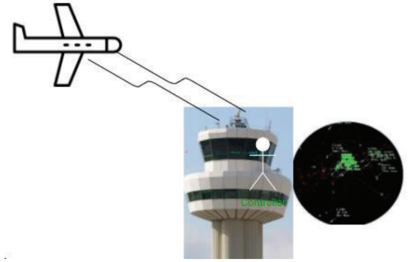
Ground-based TP: CDR algorithm, estimate ATC sector loads, air-ground synchronization tools, …

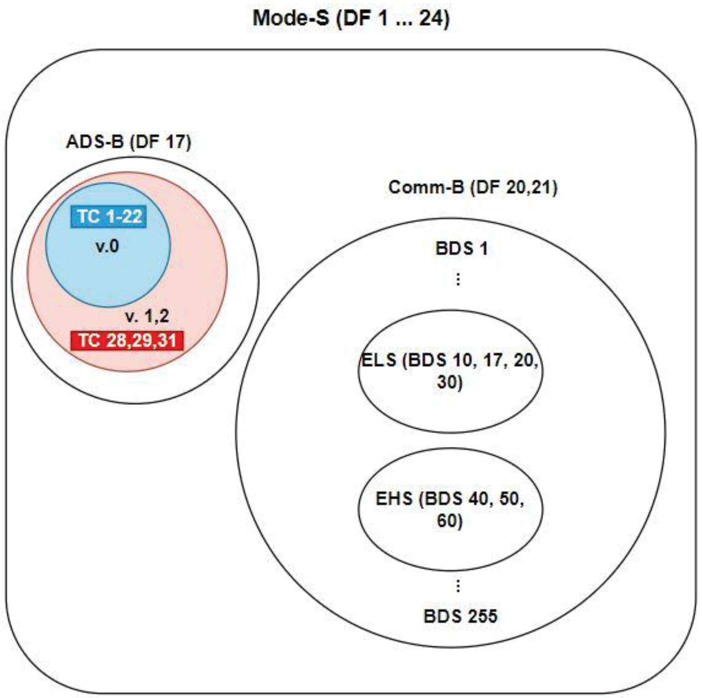
** Air Traffic Management (ATM), * Trajectory Based Operations (TBO), Airspace Users (AUs), * Air Navigation Service Provider (ANSP), * Network Manager (NM), * Single European Sky ATM Research (SESAR), * Next Generation Air Transportation System (NexGen), Service Provider (SP), * flight management systems (FMS), * Conflict Detection and Resolution (CDR),*

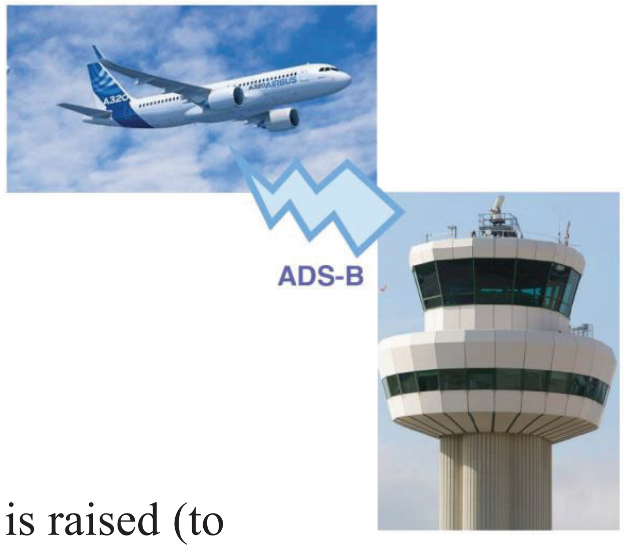
Introduction and Background



- **Primary surveillance radar (the aircraft azimuth)**
- **Secondary surveillance radar**
  - Mode A (an **aircraft identity** by a 4-digit octal code)
  - Mode C (the **barometric altitude**)
  - Mode S







- **ADS-C-EPP**
  - new automation and **shared data** within the **TBO** paradigm is raised (to **predict** and **share** very **accurate trajectory data** via **ADS–C EPP** reports).

***** *Enhanced Mode-S Surveillance (EHS), * True AirSpeed (TAS), * Indicated AirSpeed (IAS) , * Automatic Dependent Surveillance Broadcast (ADS-B) , * Vertical Speed (VS), * Trajectory Based Operations (TBO), * Automatic Dependent Surveillance - Contract Extended Projected Profile (ADS-C EPP),*

Methodological Challenges and Perspectives

- **Physical modelling of the system**

- **Model-based (SSP) approaches for TP problem in the execution phase of the flight:**
  - Dynamic aircraft model, (i.e., **point-mass model**)
  - Available data, (i.e., **ADS-B** that can be used for both **air-ground** and **air-air** applications)
  - Characterization of the system **uncertainty**,
  - Time in TBO (to model **4D TP**)

TP is an Estimation problem

- **The dynamic complex system**
  - time-varying, nonlinear, non-Gaussian, with a certain model uncertainty and model mismatch.

** Statistical Signal Processing (SSP), * Automatic Dependent Surveillance Broadcast (ADS-B) , * Trajectory Prediction (TP),*

Methodological Challenges and Perspectives

- **From methodological standpoint:**
  - Optimal estimation in such complex dynamic system?
  - How to deal with deviations from model assumptions (uncertainties, model mismatch, attacks)?
  - Being a safety-critical application, which is the trade-off between optimality and robustness?
  - Do the methodologies scale properly with the number of aircraft present in the airspace of interest?
  - Optimal detection metrics to avoid heuristic rules?
  - In order to allow self-separation, how to move from centralized to cooperative/distributed processing?
  - From a practical perspective, are the available and new methodologies to come certifiable?

**** Trajectory Prediction (TP),***

Methodological Challenges and Perspectives

- Towards **Robust TP** Solutions.
  - standard **KF**s rely on the **complete knowledge** of the system.
  - diverge in **highly** nonlinear systems. (**sampling-based** strategies)
  - poor performance in **non**-Gaussian problems. (**Monte Carlo** methods)
  - **Robust** filtering techniques for **real-life** in order to cope with **mismatched system models**:
    - **linear constraints** (the possible impact of mismatched process and measurement matrices can be mitigated)
    - **Robust statistics** techniques (outliers in the system can be mitigated)
    - **Variational Bayesian-Based** filtering solutions (unknown noise statistics' parameters can be included in the filter formulation)
    - **Nonparametric Bayesian** estimation (if the complete system densities are unknown)
- From **Single** to **Multiple Aircraft TP**.
  - **MTT** provides a **statistical** framework to cope with **unknown time-varying number of targets**, **false alarms**, **missed detections**, and **clutter.**
- From **Centralized** to **Cooperative** Processing.
  - **Graph-based** techniques.

**** Trajectory Prediction (TP), * Kalman Filter (KF), * Multiple Target Tracking (MTT),***

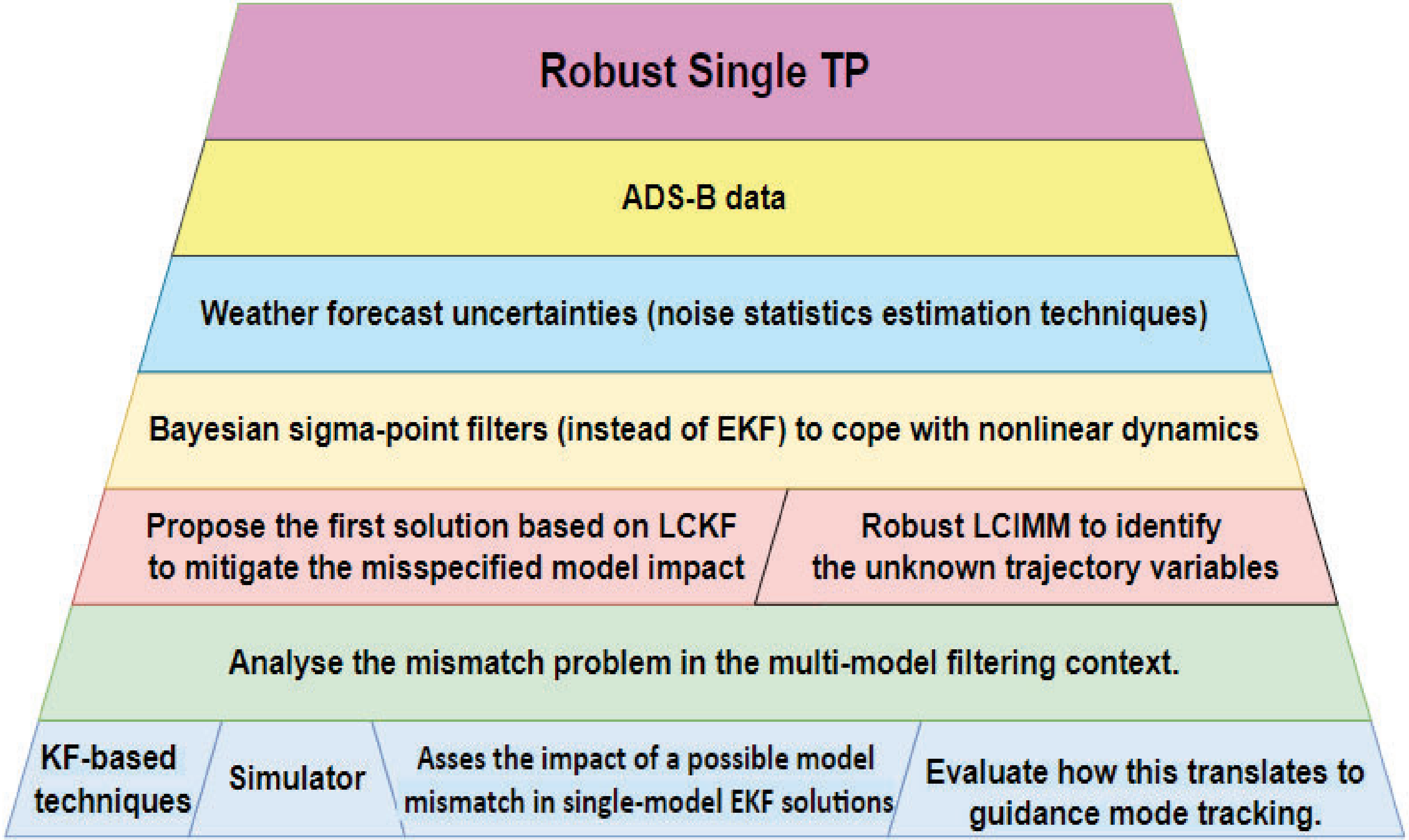
PhD Research Objectives

- Towards **Robust TP** Solutions.
  - O1: **Probabilistic characterization** of the **TP** problem at hand, and formal analysis on the limitations of standard filtering techniques for TP (i.e., impact of a **misspecified** system).
  - O2: **Robustification** of the current filtering techniques and development of **new robust approaches for TP** (i.e., relying on **linearly constrained** filtering, using **covariance estimation** techniques or advanced **Bayesian filtering** solutions).
- From **Single** to **Multiple Aircraft TP**.
  - O3: Extension of the **robust filtering** approaches developed in O1 to **multiple aircraft TP**.
- From **Centralized** to **Cooperative** Processing.
  - O4: Development of **distributed or cooperative robust filtering** techniques, as an extension of the methodologies developed in O1 to enable **self-separation**.

**** Trajectory Prediction (TP), * Objective (O),***

Methodology to Achieve the Research Objectives

- Towards **Robust TP** Solutions (O1 and O2):



- Objects O3 and O4

** Trajectory Prediction (TP), * Extended Kalman Filter(EKF), * Automatic Dependent Surveillance Broadcast (ADS-B), * Linear Constraints Kalman Filter (LCKF), * Linear Constraints Interacting Multiple-model (LCIMM),*

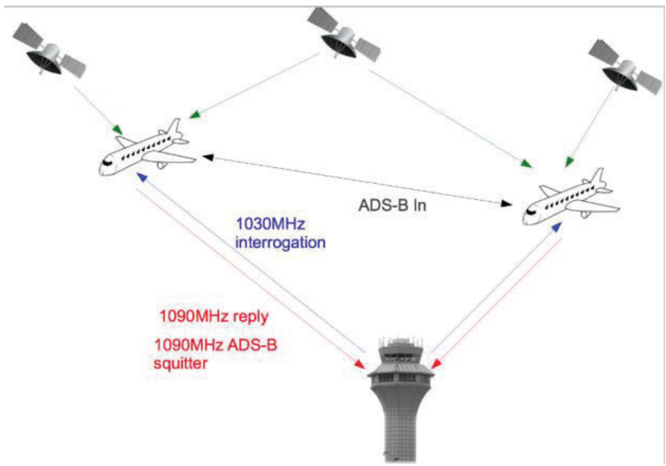
Methodology to Achieve the Research Objectives

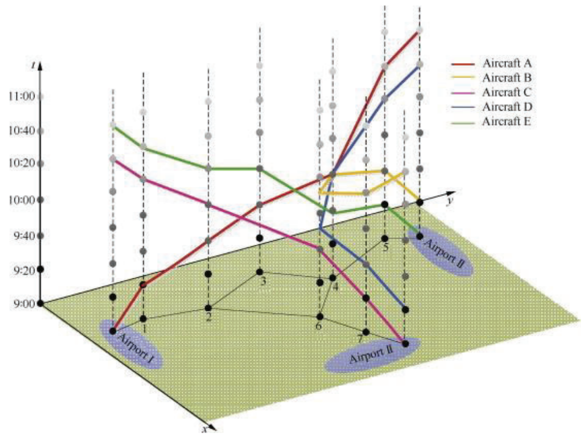
Case studies to appraise the impact of the proposed methods in applications which demand TP:

- Improving the **MTCD-like** systems or more general **CDR** algorithms.

- Improving **self-separation** algorithms.

- **Multi aircraft** CD.





** Trajectory Prediction (TP), * Medium Term Conflict Detection (MTCD), * Conflict Detection and Resolution (CDR),*

Results

Real-time identification of high-lift devices (flaps/slats) deployment based on surveillance input data (Radar/ADS-B):

- this estimation aims to enhance ground-based TPs.
- aiming at detecting atypical trajectories and/or preventing unstabilised approaches.

The scope of this paper:

- the execution phase of the flight.
- a model-based methodology to identify, in real-time, the moment that flaps/slats are deployed on descending trajectories.
- Detection of the deployment moment of the first flaps/slats configuration.
- capable of being implemented in real-time applications.

** Trajectory Prediction (TP), * Air Traffic Control (ATC), * Decision Support Tools (DSTs), * Aircraft Dependent Surveillance-Broadcast (ADS-B),*

"**Real-time Identification of High-Lift Devices Deployment in Aircraft Descents**" Homeyra Khaledian et al

Results

- **High-lift devices are designed to be used in the take-off and initial climb; and final stages of the approach and landing.**
- **Commercial airliners are typically equipped with flaps/slats, which have different positions or configurations that are progressively deployed during the approach (and progressively retracted during the climb):**
  - Clean configuration: no flaps/slats deployed.
  - Airbus example: for most models there are 5 different configurations CONF 1, CONF 1+F, CONF 2, CONF 3, and FULL.
- **Each flaps/slats configuration:**
  - has a minimum and maximum speed.
  - different drag coefficient parameters (usage of flaps/slats increases Drag)
- **Environment variables affect the exact deployment moment:**
  - Weather (especially in gusting and/or strong wind conditions)
  - Obstacles below the flight path
  - How *busy* is the crew in performing other tasks

"**Real-time Identification of High-Lift Devices Deployment in Aircraft Descents**" Homeyra Khaledian et al

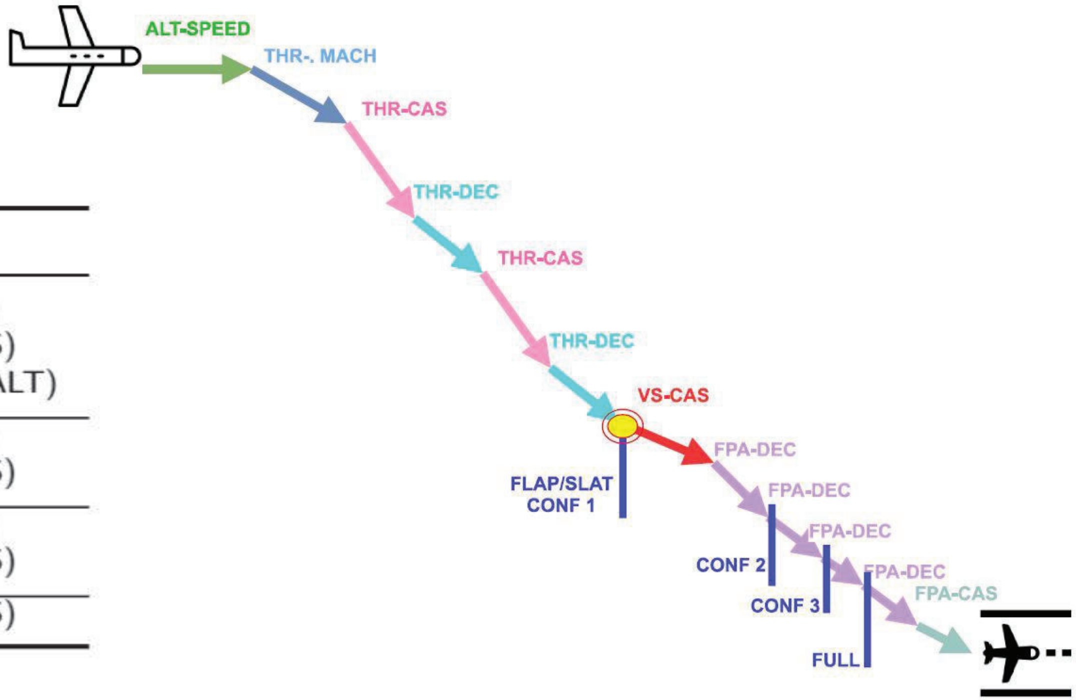
Results

- **Two aircraft intents are needed to close the two degrees of freedom of the dynamics of the aircraft in the vertical domain**

- **Different combinations are possible.**

- **Intents considered in this work:**

Intent 1	Intent 2
Fixed Throttle (THR)	Deceleration (DEC) Constant CAS (CAS) Constant altitude (ALT)
Constant vertical speed (VS)	Deceleration (DEC) Constant CAS (CAS)
Constant flight path angle (FPA)	Deceleration (DEC) Constant CAS (CAS)
Constant altitude (ALT)	Constant CAS (CAS)

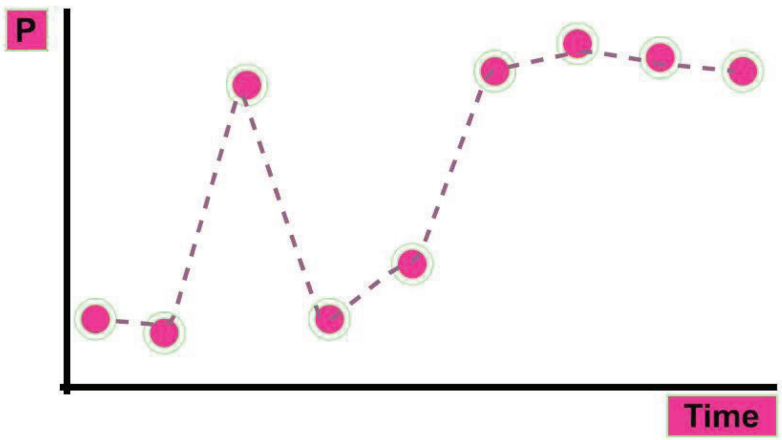


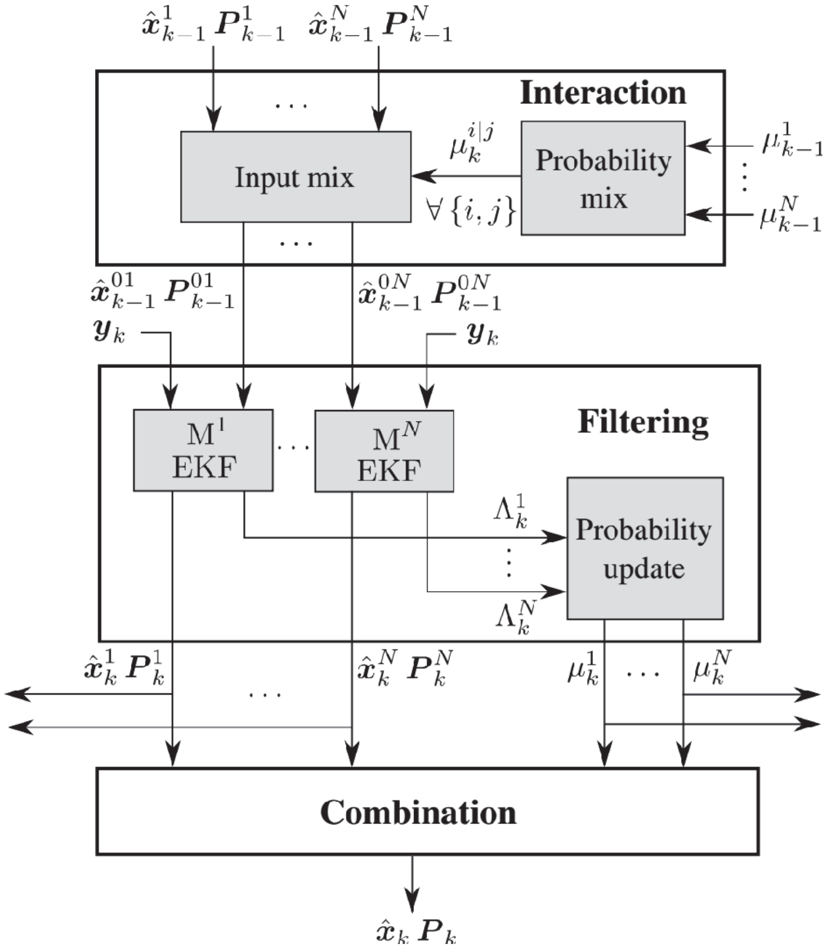
**** Throttle (THR), * Vertical Speed (VS), * Flight Path Angle (FPA), * Altitude (ALT), * Deceleration (DEC), * Calibrated AirSpeed (CAS),***

"Real-time Identification of High-Lift Devices Deployment in Aircraft Descents" Homeyra Khaledian et al

Results

- KF in single linear dynamics SSM.
- EKF in single non-linear dynamic SSMs.
- EKF-IMM.
- Moving Average technique:





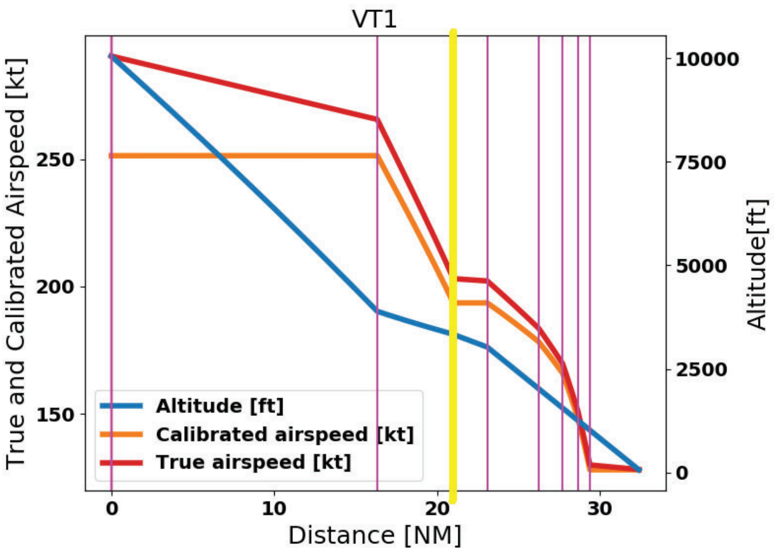
** Kalman Filter (KF), * State-Space Model (SSM), * Extended KF (EKF), * EKF-Interacting Multiple Model (EKF-IMM), * Probability (P),*

"**Real-time Identification of High-Lift Devices Deployment in Aircraft Descents**" Homeyra Khaledian et al

Results

- The trajectory simulator generates flight data (emulates the same information obtained from ADS-B and a Mode S receiver).

- Vertical profile specification of the Validation Trajectory (VT1), simulating a typical Airbus A320 approach:



Phase	Aircraft Intent	Intent1	Intent2	End Condition	Configuration	Landing Gear
1	THR-CAS	Idle	250 kt	$h_p = 4000$ ft	CLEAN	UP
2	THR-DEC	Idle	0.3	$CAS = 193$ kt	CLEAN	UP
3	VS-CAS	-500 ft/min	193 kt	$\Delta s = 2$ NM	CONF 1	UP
4	FPA-DEC	$-3°$	0.76	$h_p = 2000$ ft	CONF 1	UP
5	FPA-DEC	$-3°$	0.6835	$h_p = 1500$ ft	CONF 2	UP
6	FPA-DEC	$-3°$	0.53	$CAS = 147$ kt	CONF 3	DOWN
7	FPA-DEC	$-3°$	0.472	$CAS = 128$ kt	FULL	DOWN
8	FPA-CAS	$-3°$	128 kt	$h_p = 50$ ft	FULL	DOWN

**** Aircraft Dependent Surveillance-Broadcast (ADS-B), * Enhanced Mode S surveillance (EHS),***

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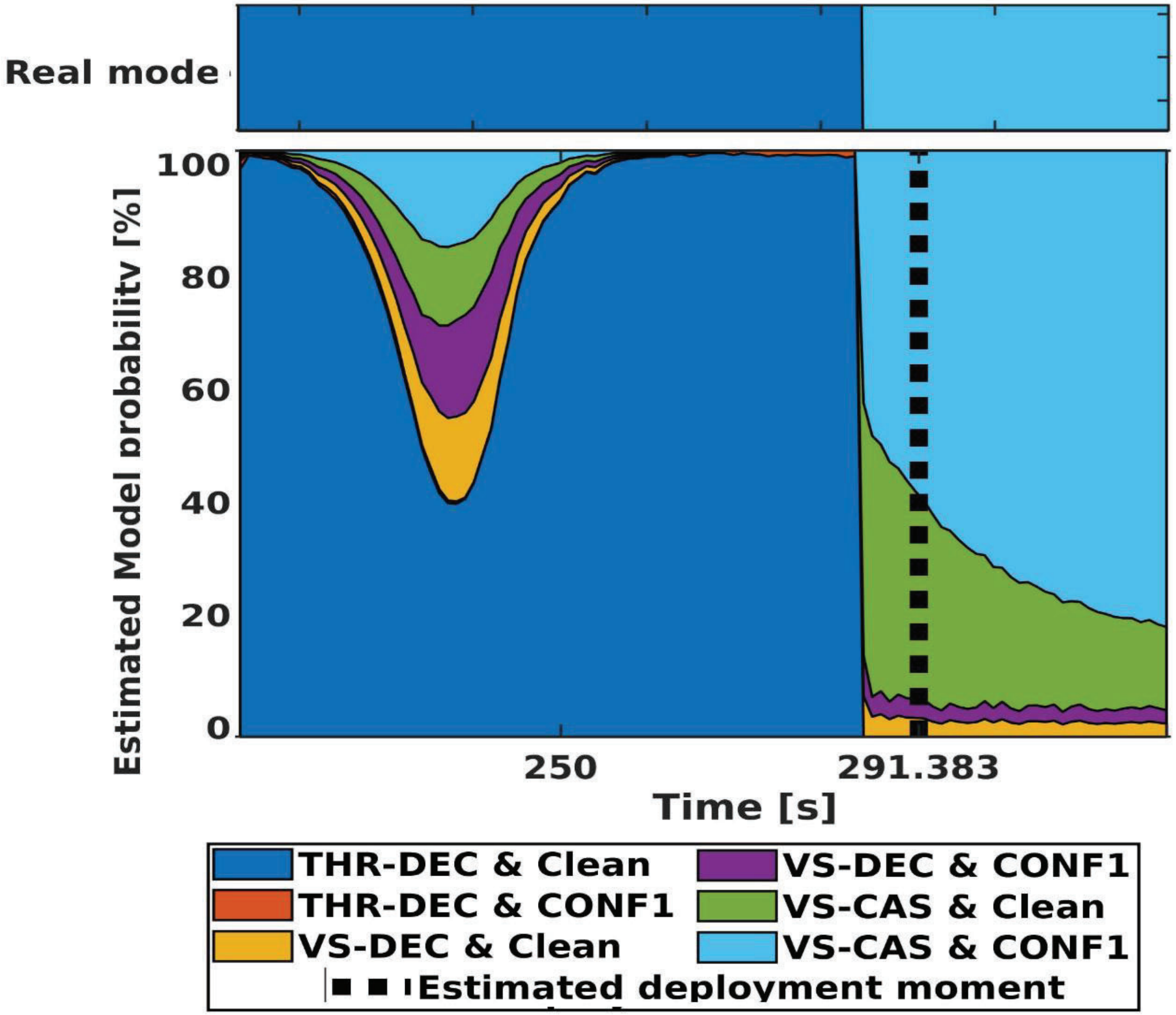
Results

Results for Validation Trajectory 1 (VT1)



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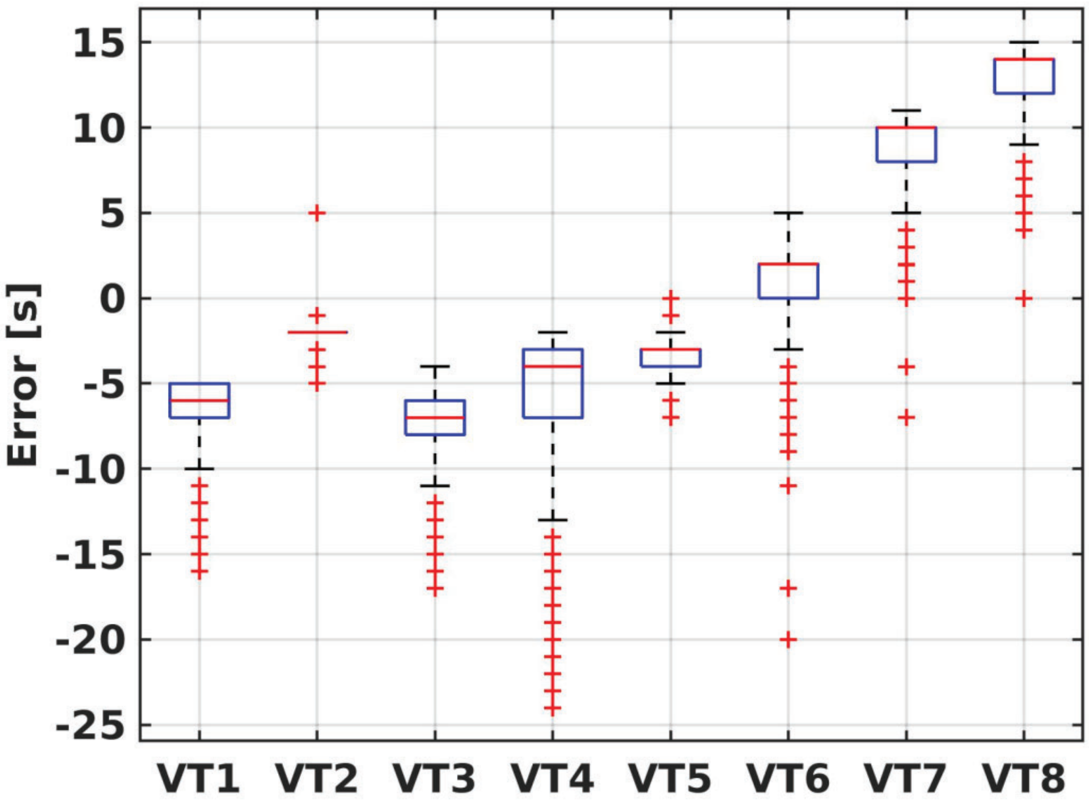
Results



**** Validation Trajectory (VT),***

"Real-time Identification of High-Lift Devices Deployment in Aircraft Descents" Homeyra Khaledian et al

Results

We consider a nonlinear discrete SSM,

$$\begin{aligned}\mathbf{x}_k &= \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{w}_{k-1} \\ &= \mathbf{f}_{k-1}(\mathbf{x}_{k-1}) + \mathbf{m}_{\mathbf{w}_{k-1}} + d\mathbf{w}_{k-1}, \\ \mathbf{y}_k &= \mathbf{h}_k(\mathbf{x}_k) + \mathbf{v}_k = \mathbf{h}_k(\mathbf{x}_k) + \mathbf{m}_{\mathbf{v}_k} + d\mathbf{v}_k,\end{aligned}$$

with $\mathbf{f}_{k-1}(\cdot)$ and $\mathbf{h}_k(\cdot)$ known system model (process and measurement) functions, and $\mathbb{E}[d\mathbf{w}_{k-1}] = \mathbf{0}$, $\mathbb{E}[d\mathbf{v}_k] = \mathbf{0}$. Standard approach to derive a nonlinear filter of $\mathbf{x}_k$: linearization at the vicinity of a nominal trajectory.

The standard EKF recursion is given by

$$\hat{\mathbf{x}}^b_{k|k} \simeq \hat{\mathbf{x}}^b_{k|k-1} + \mathbf{K}^b_k \left(\mathbf{y}_k - \hat{\mathbf{y}}^b_{k|k-1}\right),$$

$$\hat{\mathbf{x}}^b_{k|k-1} \simeq \mathbf{f}_{k-1}\left(\hat{\mathbf{x}}^b_{k-1|k-1}\right) + \mathbf{m}_{\mathbf{w}_{k-1}}, \hat{\mathbf{y}}^b_{k|k-1} \simeq \mathbf{h}_k\left(\hat{\mathbf{x}}^b_{k|k-1}\right) + \mathbf{m}_{\mathbf{v}_k},$$

where the Kalman gain is recursively computed as

$$\mathbf{P}^b_{k|k-1} \simeq \mathbf{F}_{k-1}\mathbf{P}^b_{k-1|k-1}\mathbf{F}^H_{k-1} + \mathbf{C}_{\mathbf{w}_{k-1}}, \mathbf{P}^b_{k|k} \simeq \left(\mathbf{I} - \mathbf{K}^b_k\mathbf{H}_k\right)\mathbf{P}^b_{k|k-1},$$

$$\mathbf{K}^b_k \simeq \mathbf{P}^b_{k|k-1}\mathbf{H}^H_k\left(\mathbf{H}_k\mathbf{P}^b_{k|k-1}\mathbf{H}^H_k + \mathbf{C}_{\mathbf{v}_k}\right)^{-1},$$

with $\mathbf{F}_{k-1} \simeq \frac{\partial \mathbf{f}_{k-1}\left(\hat{\mathbf{x}}^b_{k-1|k-1}\right)}{\partial \mathbf{x}^T_{k-1}}$ and $\mathbf{H}_k \simeq \frac{\partial \mathbf{h}_k\left(\hat{\mathbf{x}}^b_{k|k-1}\right)}{\partial \mathbf{x}^T_k}$.

"**On Parametric Model Mismatch in Nonlinear EKF Approximations**" Homeyra Khaledian et al

Results

Main assumptions on the EKF derivation: known $\mathbf{f}_k(\cdot)$ and $\mathbf{h}_k(\cdot)$, $\mathbf{m}_{\mathbf{w}_k}$, $\mathbf{m}_{\mathbf{v}_k}$, $\mathbf{C}_{\mathbf{w}_k}$, $\mathbf{C}_{\mathbf{v}_k}$, and perfect initialisation, $\mathbf{P}^b_{0|0} = \mathbf{C}_{\mathbf{x}_0}$ and $\hat{\mathbf{x}}^b_{0|0} = \mathbb{E}\{\mathbf{x}_0\}$.

> The EKF performance strongly depends on this knowledge.
> Key question: which is the impact on filtering methods where modelling errors may appear?

Model mismatch?

- Parametric model: $\mathbf{f}_{k-1}(\cdot) \triangleq \mathbf{f}_{k-1}(\cdot, \omega)$ and $\mathbf{h}_k(\cdot) \triangleq \mathbf{h}_k(\cdot, \theta)$, where ω and θ are supposed to be deterministic vectors.
- The existence of uncertainty on the nonlinear SSM can be taken into account as $\widehat{\omega} = \omega + d\widehat{\omega}$ and $\widehat{\theta} = \theta + d\widehat{\theta}$.

We want to cope with: a true (T) and a mismatched (M) nonlinear SSM,

$$\mathrm{M}: \begin{cases} \mathbf{x}'_k = \mathbf{f}_{k-1}(\mathbf{x}'_{k-1}, \widehat{\omega}) + \mathbf{m}_{\mathbf{w}_{k-1}} + d\mathbf{w}_{k-1} \\ \mathbf{y}_k = \mathbf{h}_k(\mathbf{x}'_k, \widehat{\theta}) + \mathbf{m}_{\mathbf{v}_k} + d\mathbf{v}_k \end{cases}$$

$$\mathrm{T}: \begin{cases} \mathbf{x}_k = \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \omega) + \mathbf{m}_{\mathbf{w}_{k-1}} + d\mathbf{w}_{k-1} \\ \mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k, \theta) + \mathbf{m}_{\mathbf{v}_k} + d\mathbf{v}_k \end{cases}$$

Goal: characterizing estimator bias and error covariance under mismatch!

"**On Parametric Model Mismatch in Nonlinear EKF Approximations**" Homeyra Khaledian et al

Results

The Kalman gain $\mathbf{K}_k$ is the solution of $\mathbf{K}_k^b = \arg\min_{\mathbf{K}_k} \left\{ \mathbf{P}_{k|k}^J (\mathbf{K}_k) \right\}$, with $\mathbf{P}_{k|k}^J (\mathbf{K}_k) = \mathbb{E}\left[(\widehat{\mathbf{x}}_{k|k} (\mathbf{K}_k) - \mathbf{x}_k)(\widehat{\mathbf{x}}_{k|k} (\mathbf{K}_k) - \mathbf{x}_k)^H \right]$, and the estimation error

$$\begin{aligned} \mathbf{e}_{k|k} &= \mathbf{f}_{k-1}\left(\widehat{\mathbf{x}}_{k-1|k-1}^b, \widehat{\omega}\right) - \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \widehat{\omega}) + \mathbf{K}_k \mathbf{h}_k \left(\mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \widehat{\omega}) + \mathbf{m}_{\mathbf{w}_{k-1}} + d\mathbf{w}_{k-1}, \widehat{\theta} \right) \\ &- \mathbf{K}_k \mathbf{h}_k \left(\mathbf{f}_{k-1}\left(\widehat{\mathbf{x}}_{k-1|k-1}^b, \widehat{\omega}\right) + \mathbf{m}_{\mathbf{w}_{k-1}}, \widehat{\theta} \right) + \mathbf{K}_k d\mathbf{v}_k - d\mathbf{w}_{k-1} + \varepsilon_k(\mathbf{K}_k) = \mathbf{e}_{k|k}^b + \varepsilon_k(\mathbf{K}_k), \end{aligned}$$

and the additional error term (and the 1st order EKF error approximation)

$$\begin{aligned} \varepsilon_k(\mathbf{K}_k) &= \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \widehat{\omega}) - \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \omega) + \mathbf{K}_k \mathbf{h}_k(\mathbf{x}_k, \theta) \\ &- \mathbf{K}_k \mathbf{h}_k \left(\mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \widehat{\omega}) + \mathbf{m}_{\mathbf{w}_{k-1}} + d\mathbf{w}_{k-1}, \widehat{\theta} \right) \\ &\simeq \left(\mathbf{I} - \mathbf{K}_k \widehat{\mathbf{H}}_k \right) \frac{\partial \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \widehat{\omega})}{\partial \omega^T} d\widehat{\omega} - \mathbf{K}_k \frac{\partial \mathbf{h}_k(\mathbf{x}_k, \widehat{\theta})}{\partial \theta^T} d\widehat{\theta}. \end{aligned}$$

with $\widehat{\mathbf{F}}_{k-1} = \left. \frac{\partial \mathbf{f}_{k-1}(\mathbf{x}_{k-1}, \widehat{\omega})}{\partial \mathbf{x}_{k-1}^T} \right|_{\mathbf{m}_{\mathbf{x}_{k-1}}}$, $\widehat{\mathbf{H}}_k = \left. \frac{\partial \mathbf{h}_k(\mathbf{x}_k, \widehat{\theta})}{\partial \mathbf{x}_k^T} \right|_{\widehat{\mathbf{m}}_{\mathbf{x}_k}}$.

"**On Parametric Model Mismatch in Nonlinear EKF Approximations**" Homeyra Khaledian et al

Results

Recursive bias and MSE expressions:

- The estimator bias is

$$\mathbb{E}[\mathbf{e}_{k|k}] = \mathbb{E}[\mathbf{e}^{b}_{k|k} + \varepsilon_k(\mathbf{K}_k)] = \mathbb{E}[\varepsilon_k(\mathbf{K}_k)]$$
$$\simeq \left(\mathbf{I} - \mathbf{K}_k\widehat{\mathbf{H}}_k\right)\mathbb{E}\left[\frac{\partial \mathbf{f}_{k-1}(\mathbf{x}_{k-1},\widehat{\omega})}{\partial \omega^T}\right] d\widehat{\omega} - \mathbf{K}_k\mathbb{E}\left[\frac{\partial \mathbf{h}_k(\mathbf{x}_k,\widehat{\theta})}{\partial \theta^T}\right] d\widehat{\theta},$$

$$\mathrm{Bias}_k \simeq \left(\mathbf{I} - \mathbf{K}_k\widehat{\mathbf{H}}_k\right)\widehat{\mathbf{F}}_{k-1}\mathrm{Bias}_{k-1} + \left(\mathbf{I} - \mathbf{K}_k\widehat{\mathbf{H}}_k\right)\frac{\partial \mathbf{f}_{k-1}\left(\mathbf{m}_{\mathbf{x}_{k-1}},\widehat{\omega}\right)}{\partial \omega^T} d\widehat{\omega} - \mathbf{K}_k\frac{\partial \mathbf{h}_k(\mathbf{m}_{\mathbf{x}_k},\widehat{\theta})}{\partial \theta^T} d\widehat{\theta}.$$

- The estimator error covariance is

$$\mathbf{P}_{k|k} = \mathbb{E}\left[\mathbf{e}_{k|k}\mathbf{e}^H_{k|k}\right] = \mathbf{P}^b_{k|k} + \mathbf{P}_{e,k}\ ;\ \mathbf{P}_{e,k} = 2\mathbb{E}\left[\mathbf{e}^b_{k|k}\varepsilon^H_k(\mathbf{K}_k)\right] + \mathbb{E}\left[\varepsilon_k(\mathbf{K}_k)\varepsilon^H_k(\mathbf{K}_k)\right],$$
$$\mathbf{P}_{e,k} \simeq \left(\mathbf{I} - \mathbf{K}_k\widehat{\mathbf{H}}_k\right)\widehat{\mathbf{F}}_{k-1}\mathbf{P}_{e,k-1}\widehat{\mathbf{F}}^H_{k-1}\left(\mathbf{I} - \mathbf{K}_k\widehat{\mathbf{H}}_k\right)^H + 2\mathbf{\Gamma}_k + \mathbf{\Delta}_k$$

(refer to the article for the $\mathbf{\Gamma}_k$ and $\mathbf{\Delta}_k$ expressions).

Note: if the SSM is linear, we recover the expressions in J. Vilà-Valls et al., "Modelling Mismatch and Noise Statistics Uncertainty in Linear MMSE Estimation," EUSIPCO 2019.

"**On Parametric Model Mismatch in Nonlinear EKF Approximations**" Homeyra Khaledian et al

Results

Mismatched-True SSM pair $(\widehat{\mathbf{u}}_{k-1} = \mathbf{u}_{k-1} + d\widehat{\mathbf{u}}_{k-1})$

$$\mathrm{M}: \begin{cases} \mathbf{x}'_k = \mathbf{f}_{k-1}\left(\mathbf{x}'_{k-1}\right) + \mathbf{g}_{k-1}\left(\widehat{\mathbf{u}}_{k-1}\right) + \mathbf{w}_{k-1} \\ \mathbf{y}_k = \mathbf{h}_k\left(\mathbf{x}'_k\right) + \mathbf{v}_k \end{cases}$$

$$\mathrm{T}: \begin{cases} \mathbf{x}_k = \mathbf{f}_{k-1}\left(\mathbf{x}_{k-1}\right) + \mathbf{g}_{k-1}\left(\mathbf{u}_{k-1}\right) + \mathbf{w}_{k-1} \\ \mathbf{y}_k = \mathbf{h}_k\left(\mathbf{x}_k\right) + \mathbf{v}_k \end{cases}$$

then the recursive bias and MSE expressions are,

$$(15d)\ \mathrm{Bias}_k \simeq (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{F}_{k-1}\mathrm{Bias}_{k-1} + (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\frac{\partial \mathbf{g}_{k-1}\left(\widehat{\mathbf{u}}_{k-1}\right)}{\partial \mathbf{u}_{k-1}^T}\,d\widehat{\mathbf{u}}_{k-1},$$

$$(15e)\ \mathbf{P}_{e,k} \simeq (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{F}_{k-1}\mathbf{P}_{e,k-1}\mathbf{F}_{k-1}^H\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^H$$
$$+ 2\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\mathbf{F}_{k-1}\mathrm{Bias}_{k-1}\partial\mathbf{u}^H\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^H + (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\,\partial\mathbf{u}\partial\mathbf{u}^H\,(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^H.$$

with

$$\frac{\partial \mathbf{f}_{k-1}\left(\mathbf{x}_{k-1}, \widehat{\mathbf{u}}_{k-1}\right)}{\partial \mathbf{u}_{k-1}^T}\,d\widehat{\mathbf{u}}_{k-1} = \frac{\partial \mathbf{g}_{k-1}\left(\widehat{\mathbf{u}}_{k-1}\right)}{\partial \mathbf{u}_{k-1}^T}\,d\widehat{\mathbf{u}}_{k-1} \triangleq \partial\mathbf{u},$$

“**On Parametric Model Mismatch in Nonlinear EKF Approximations**” Homeyra Khaledian et al

Results

- **System model mismatch (i.e., parametric errors in $\mathbf{f}_{k-1}(\cdot)$, $\mathbf{h}_k(\cdot)$ or inputs) induce an estimation bias and increase of the achievable MSE.**

- **We derived analytic expressions for the bias and MSE degradation under model mismatch.**

- **If we have a prior knowledge on the maximum expected error on the system model, we can evaluate the performance degradation.**

- **If the expected degradation is not acceptable, these results allow to have an insight for the derivation of new mitigation strategies.**

“**On Parametric Model Mismatch in Nonlinear EKF Approximations**” Homeyra Khaledian et al

Results

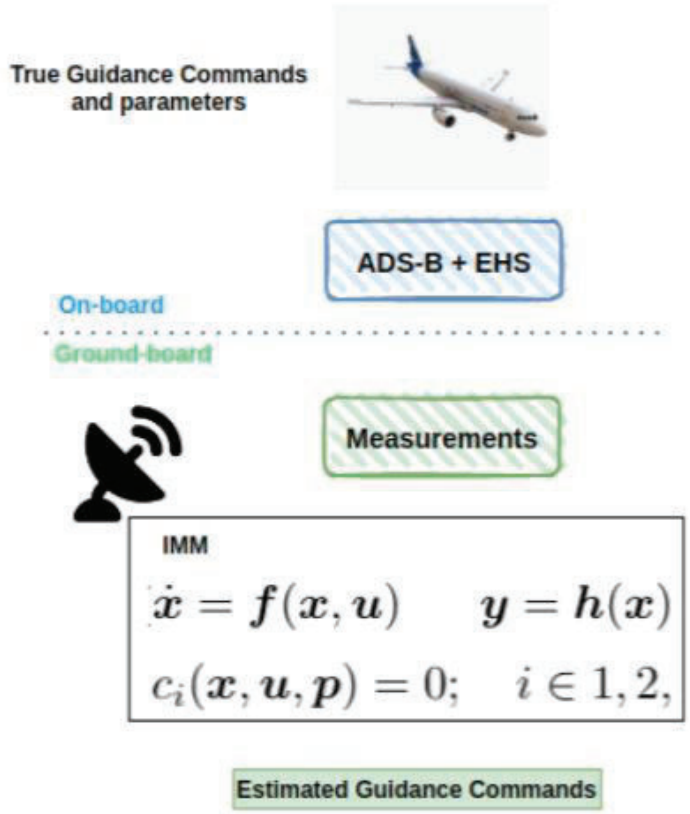


Fig. 1: General scheme of the overall problem formulation.

“On the Optimal Real-Time IMM-Based Guidance Modes Identification in Aircraft Climbs/Descents from Surveillance Data” Homeyra Khaledian et al

Results

TABLE I: Climb/Descent Guidance Modes considered in this article

Command 1 (Elevator)	Command 2 (Throttle)	Parameters vector	Control vector
MACH	THR	$\boldsymbol{p} = [\bar{M}, \bar{\pi}]$	$\pi(\boldsymbol{p}, \boldsymbol{x}) = \bar{\pi}$
CAS		$\boldsymbol{p} = [\bar{v}_{\text{CAS}}, \bar{\pi}]$	
ACC/DEC		$\boldsymbol{p} = [\bar{k}, \bar{\pi}]$	$\gamma(\boldsymbol{p}, \boldsymbol{x}) = \arcsin\left(\bar{k}(T_{\text{idle}} + \bar{\pi}(T_{\max} - T_{\text{idle}}) - D)(mg)^{-1}\right)(\dagger)$
VS	MACH	$\boldsymbol{p} = [\bar{v}_h, \bar{M}]$	$\gamma(\boldsymbol{p}, \boldsymbol{x}) = \arcsin(\bar{v}_h/v)$
	CAS	$\boldsymbol{p} = [\bar{v}_h, \bar{v}_{\text{CAS}}]$	
	ACC/DEC	$\boldsymbol{p} = [\bar{v}_h, \bar{k}]$	$\pi(\boldsymbol{p}, \boldsymbol{x}) = (D + \bar{k}^{-1} mg \sin\gamma - T_{\text{idle}})(T_{\max} - T_{\text{idle}})^{-1}(\dagger)$
FPA	MACH	$\boldsymbol{p} = [\bar{\gamma}_g, \bar{M}]$	$\gamma(\boldsymbol{p}, \boldsymbol{x}) = \arcsin\left(\sin\bar{\gamma}_g\left[\left(1 - \overline{W}_x^2 - \overline{W}_s^2 \sin^2\bar{\gamma}_g\right)^{1/2} + \overline{W}_s \cos\bar{\gamma}_g\right]\right)$
	CAS	$\boldsymbol{p} = [\bar{\gamma}_g, \bar{v}_{\text{CAS}}]$	
	ACC/DEC	$\boldsymbol{p} = [\bar{\gamma}_g, \bar{k}]$	$\pi(\boldsymbol{p}, \boldsymbol{x}) = (D + \bar{k}^{-1} mg \sin\gamma - T_{\text{idle}})(T_{\max} - T_{\text{idle}})^{-1}(\dagger)$
VS	THR	$\boldsymbol{p} = [\bar{v}_h, \bar{\pi}]$	$\pi(\boldsymbol{p}, \boldsymbol{x}) = \bar{\pi}$; $\gamma(\boldsymbol{p}, \boldsymbol{x}) = \arcsin(\bar{v}_h/v)$
FPA		$\boldsymbol{p} = [\bar{\gamma}_g, \bar{\pi}]$	$\pi(\boldsymbol{p}, \boldsymbol{x}) = \bar{\pi}$; $\gamma(\boldsymbol{p}, \boldsymbol{x}) = \arcsin\left(\sin\bar{\gamma}_g\left[\left(1 - \overline{W}_x^2 - \overline{W}_s^2 \sin^2\bar{\gamma}_g\right)^{1/2} + \overline{W}_s \cos\bar{\gamma}_g\right]\right)$
ALT		$\boldsymbol{p} = [\bar{v}_h = 0, \bar{\pi}]$	$\pi(\boldsymbol{p}, \boldsymbol{x}) = \bar{\pi}$; $\gamma(\boldsymbol{p}, \boldsymbol{x}) = 0$
ALT	SPD	$\boldsymbol{p} = [\bar{v}_h = 0, \bar{M}]$	$\gamma(\boldsymbol{p}, \boldsymbol{x}) = 0$; $\pi(\boldsymbol{p}, \boldsymbol{x}) = (D - T_{\text{idle}})(T_{\max} - T_{\text{idle}})^{-1}(\ddagger)$

$\overline{W}_s = W_s/v$ and $\overline{W}_x = W_x/v$ are the normalized components of the wind (head and cross wind respectively.

$^{(\dagger)}$ The energy share factor k is given as an input parameter for modes DEC or ACC ($\bar{k}$); or computed as a function of $\bar{M}$ or $\bar{v}_{\text{CAS}}$ for MACH and CAS modes, respectively. See appendix A for details.

$^{(\ddagger)}$ Note that the aerodynamic drag and maximum/idle thrust magnitudes all depend on $\bar{M}$, along with other state variables

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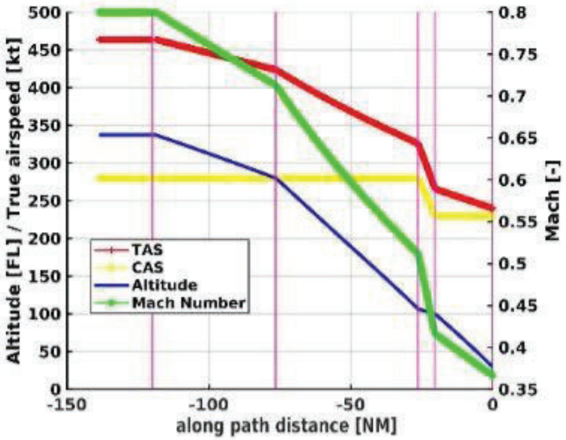
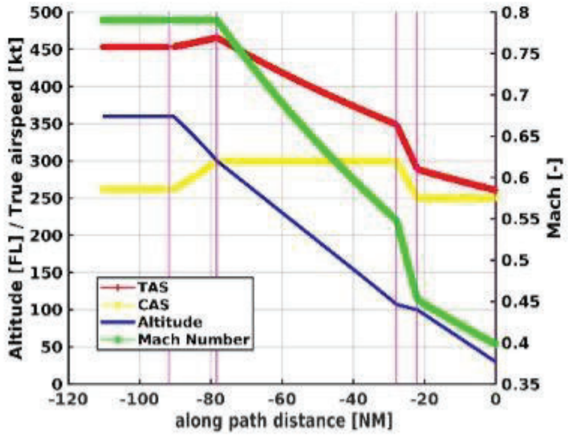
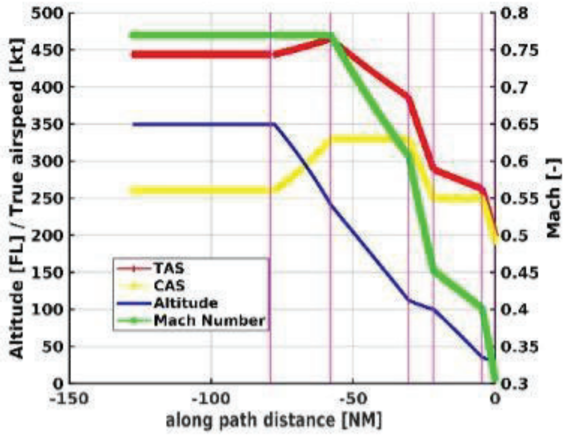
Results



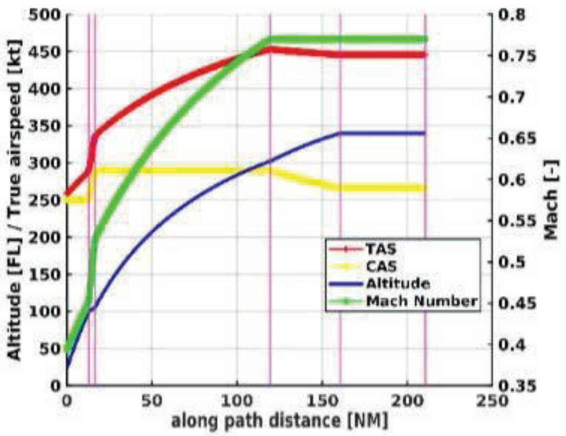
(a) VT1



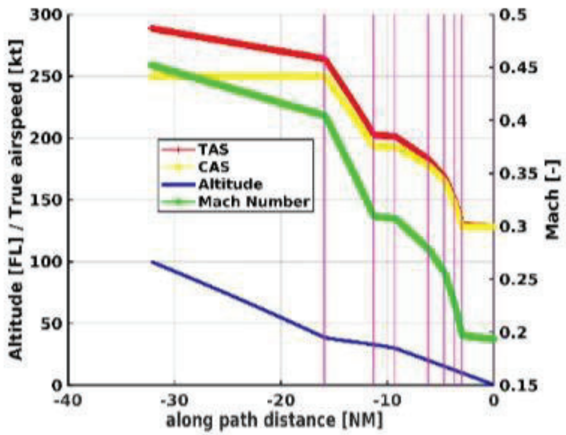
(b) VT2



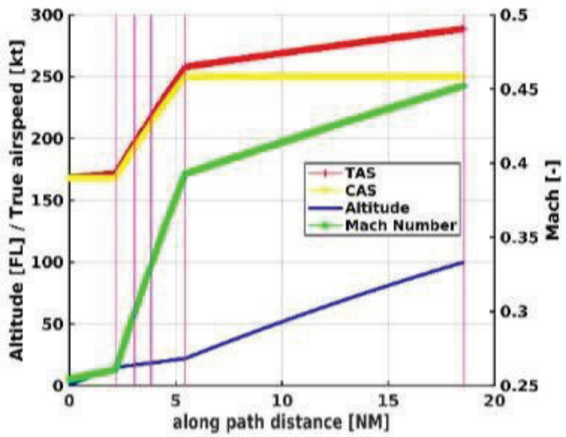
(c) VT3



(d) VT4



(e) VT5



(f) VT6

Results

TABLE VI: Vertical descent profile specification of VT5

IC: $h_p = 50\,\text{ft},\ s = 0\,\text{NM},\ v_{\text{CAS}} = 128\,\text{kt},\ m = 53000\,\text{kg}$

Phase	GM	Command1	Command2	End Condition	Configuration
1	FPA-CAS	$\bar{\gamma}_g = -3\,\text{deg}$	$\bar{v}_{\text{CAS}} = 128\,\text{kt}$	$h_p = 1000\,\text{ft}$	FULL-DOWN
2	FPA-DEC	$\bar{\gamma}_g = -3\,\text{deg}$	$\bar{k} = 0.472$	$v_{\text{CAS}} = 146.5\,\text{kt}$	FULL-DOWN
3	FPA-DEC	$\bar{\gamma}_g = -3\,\text{deg}$	$\bar{k} = 0.53$	$v_{\text{CAS}} = 165\,\text{kt}$	CONF3-DOWN
4	FPA-DEC	$\bar{\gamma}_g = -3\,\text{deg}$	$\bar{k} = 0.683$	$h_p = 2000\,\text{ft}$	CONF2-UP
5	FPA-DEC	$\bar{\gamma}_g = -3\,\text{deg}$	$\bar{k} = 0.76$	$h_p = 3000\,\text{ft}$	CONF1-UP
6	VS-CAS	$\bar{v}_h = -1000\,\text{ft/min}$	$\bar{v}_{\text{CAS}} = 193\,\text{kt}$	$\Delta s = 50\,\text{NM}$	CONF1-UP
7	DEC-THR	$\bar{k} = 0.3$	$\bar{\pi} = 0$	$v_{\text{CAS}} = 250\,\text{kt}$	CLEAN-UP
8	CAS-THR	$\bar{v}_{\text{CAS}} = 250\,\text{kt}$	$\bar{\pi} = 0$	$h_p = \text{FL}100$	CLEAN-UP

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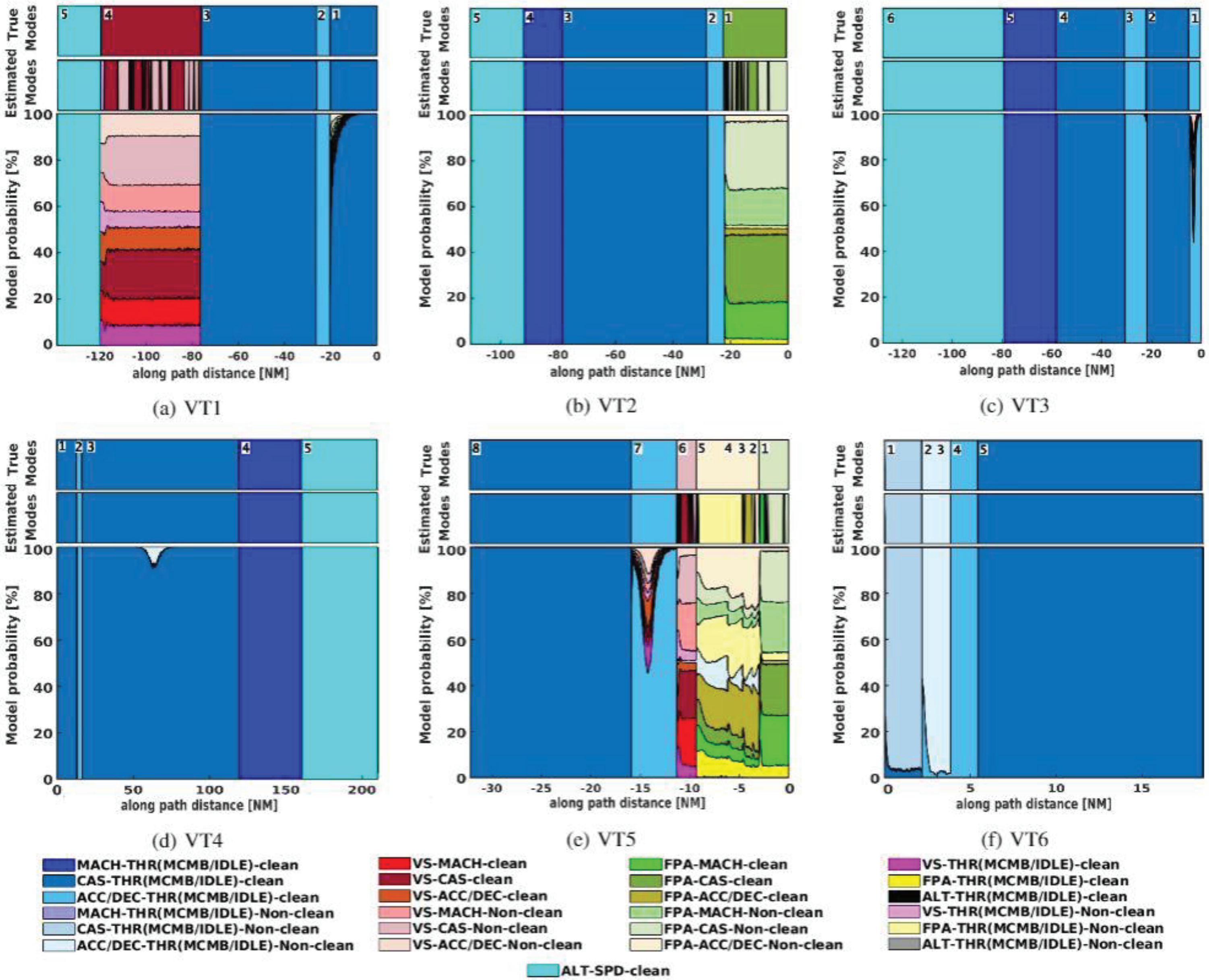

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Results

TABLE VIII: Average RMSE and guidance mode identification IMM-based results for the six representative VTs.

	VT1	VT2	VT3	VT4	VT5	VT6
e_{ident}	2.81 %	3.00 %	0.08 %	0.05 %	7.26 %	0.35 %
mean-RMSE						
h	10.47 ft	11.16 ft	13.00 ft	2.78 ft	6.91 ft	1.96 ft
s	0.06 NM	0.07 NM	0.07 NM	0.02 NM	0.01 NM	0.001 NM
v	0.56 kt	0.58 kt	0.51 kt	0.49 kt	0.53 kt	0.08 kt
m	70.41 kg	51.19 kg	7.68 kg	0.87 kg	5.51 kg	0.26 kg
τ	0.24 K	0.24 K	0.28 K	0.21 K	0.39 K	0.14 K
p	33.42 Pa	42.03 Pa	49.28 Pa	23.33 Pa	34.25 Pa	13.84 Pa

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Current Research

- **Specifying the measurement noise based on the tolerance of instrumental errors.**
- **Obtaining the guidance commands parameters from the noisy measurements.**
- **Estimator bias and error covariance under model mismatch (closed-form eq).**
- **The impact on the IMM filter performance induced by a possible model mismatch.**

Conclusion

I. TP problem in the execution phase of the flight.

II. Introduce our research path within the new concept of TBOs by SSP in order to increase the optimality and robustness of the solution.

III. The results illustrated the IMM-based guidance mode identification, and the impact of model mismatch, both with the proposed trajectory simulator.

IV. In future works, different SSP methods will be explored for robust TP.

***** *Trajectory Prediction (TP), * Trajectory Based Operation (TBO), * Statistical Signal Processing (SSP), *Interacting Multiple Model (IMM)*

Thank you for your attention

3rd Engage-KTN Summer School 2021 (Virtual)

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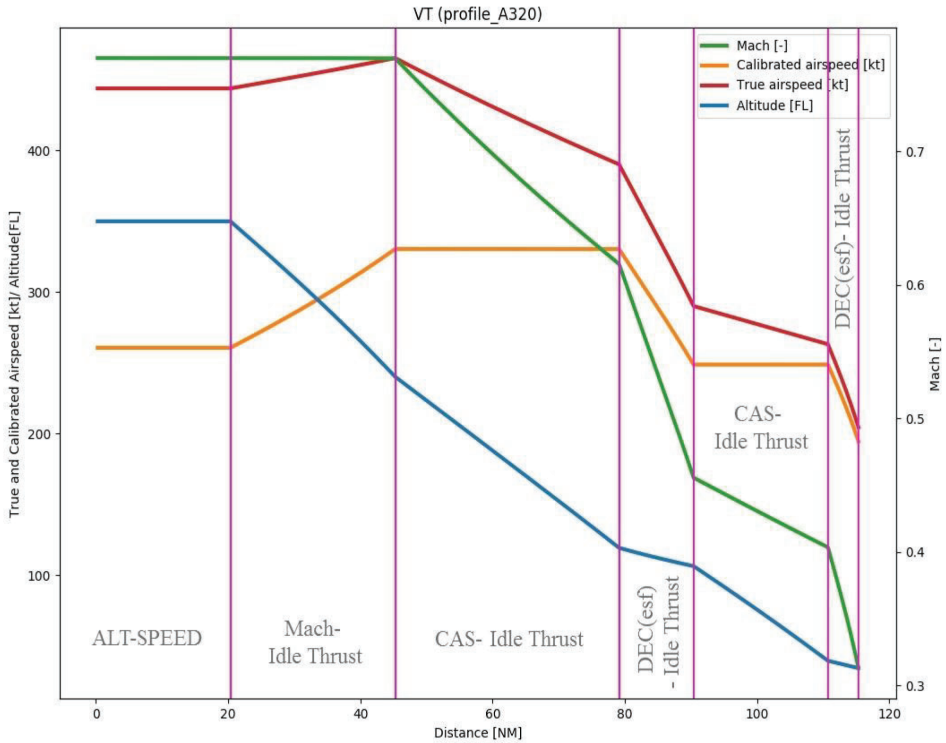
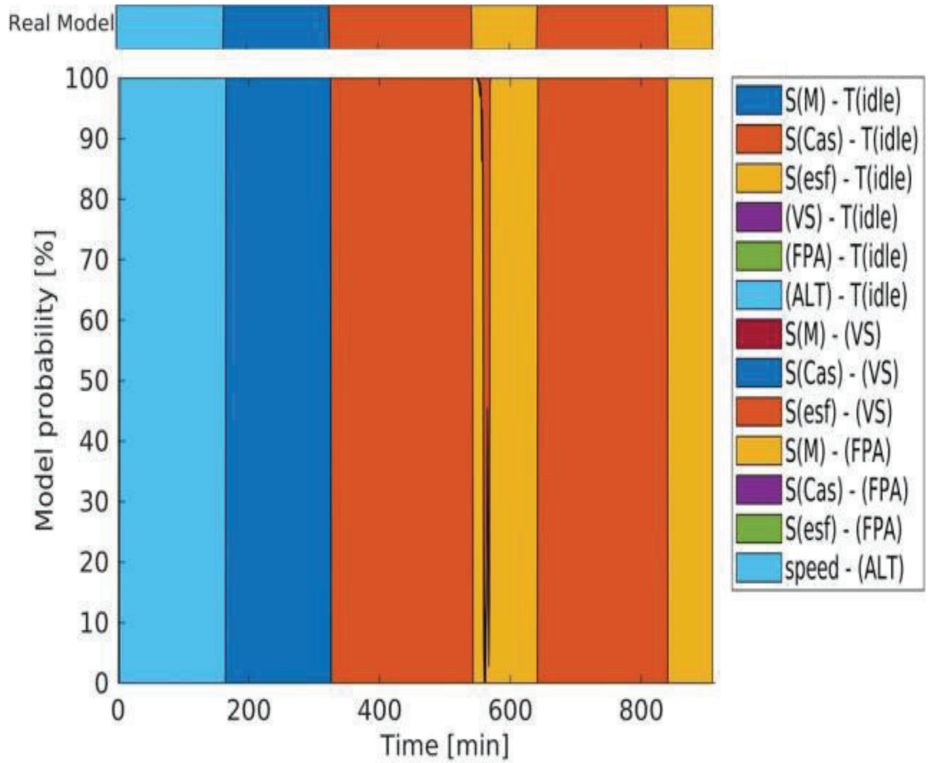
Preliminary Results: The IMM-based Guidance Mode Identification and The Impact of Pilot Input Mismatch

Phase	Mode	End condition
1	ALT-SPEED	Distance = 20 [NM]
2	Mach- Idle Thrust	v_CAS = 330 [kt]
3	CAS- Idle Thrust	h = FL120
4	DEC(esf)- Idle Thrust	v_CAS = 250 [kt]
5	CAS- Idle Thrust	Mach = 0.4 [-]
6	DEC(esf)- Idle Thrust	h = 3000 [ft]

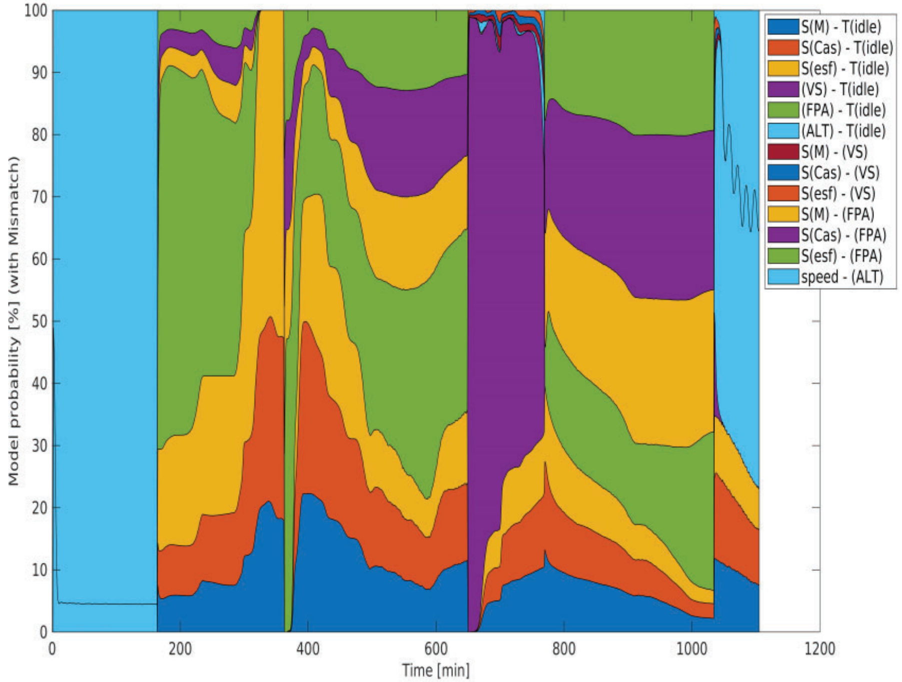


Trajectory Simulator

Preliminary Results: The IMM-based Guidance Mode Identification and The Impact of Pilot Input Mismatch



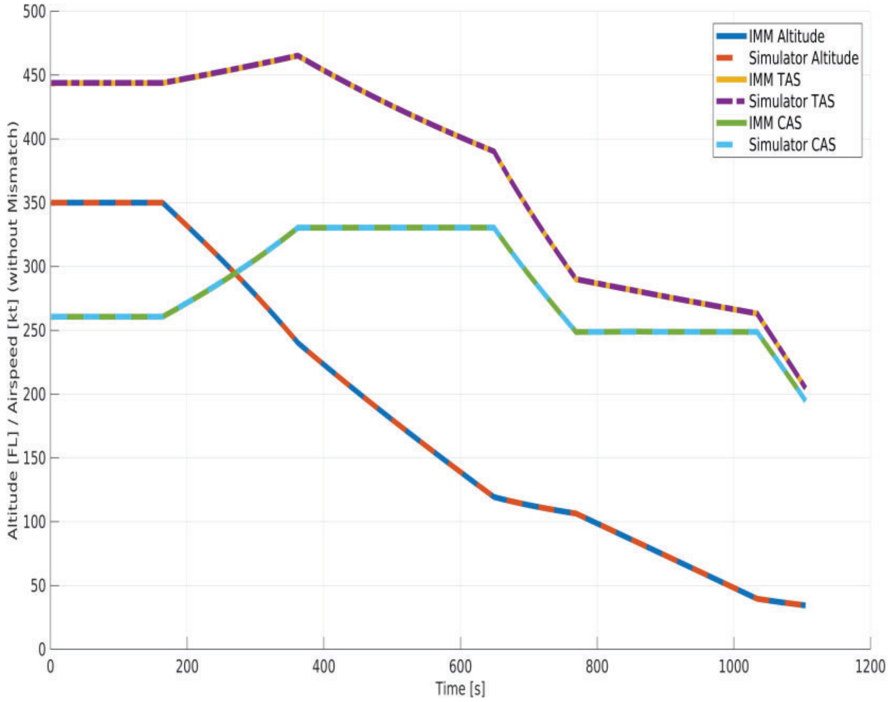
The standard IMM for guidance mode identification (without mismatch)



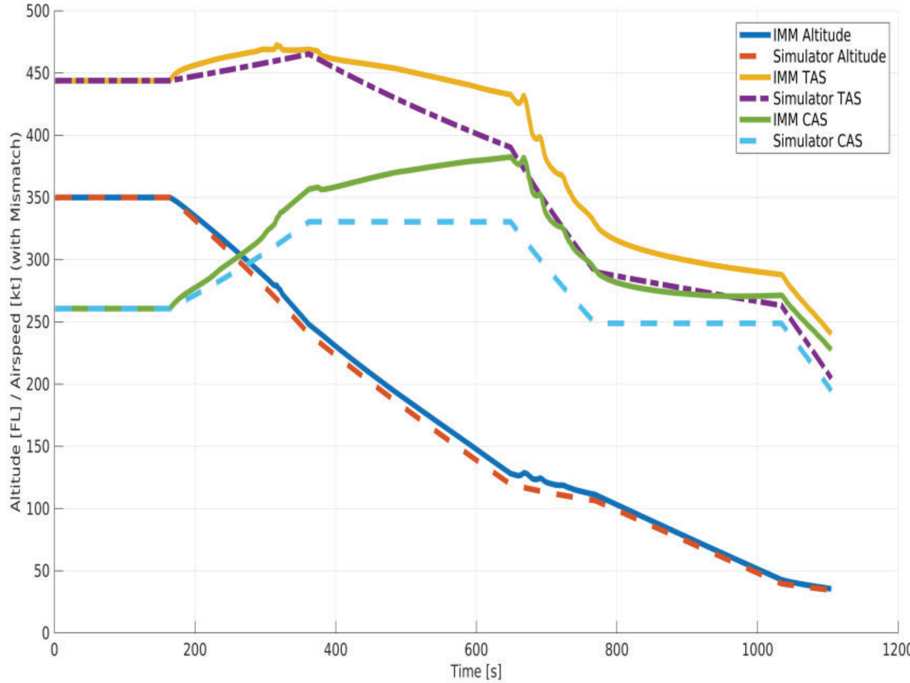
The standard IMM under model mismatch (Not Robust)

* Interacting Multiple Model (IMM)

Preliminary Results: The IMM-based Guidance Mode Identification and The Impact of Pilot Input Mismatch



The IMM based estimated values and their corresponding real trajectories Without model mismatch



The IMM based estimated values and their corresponding real trajectories Under model mismatch

* Interacting Multiple Model (IMM)

IMM and MC

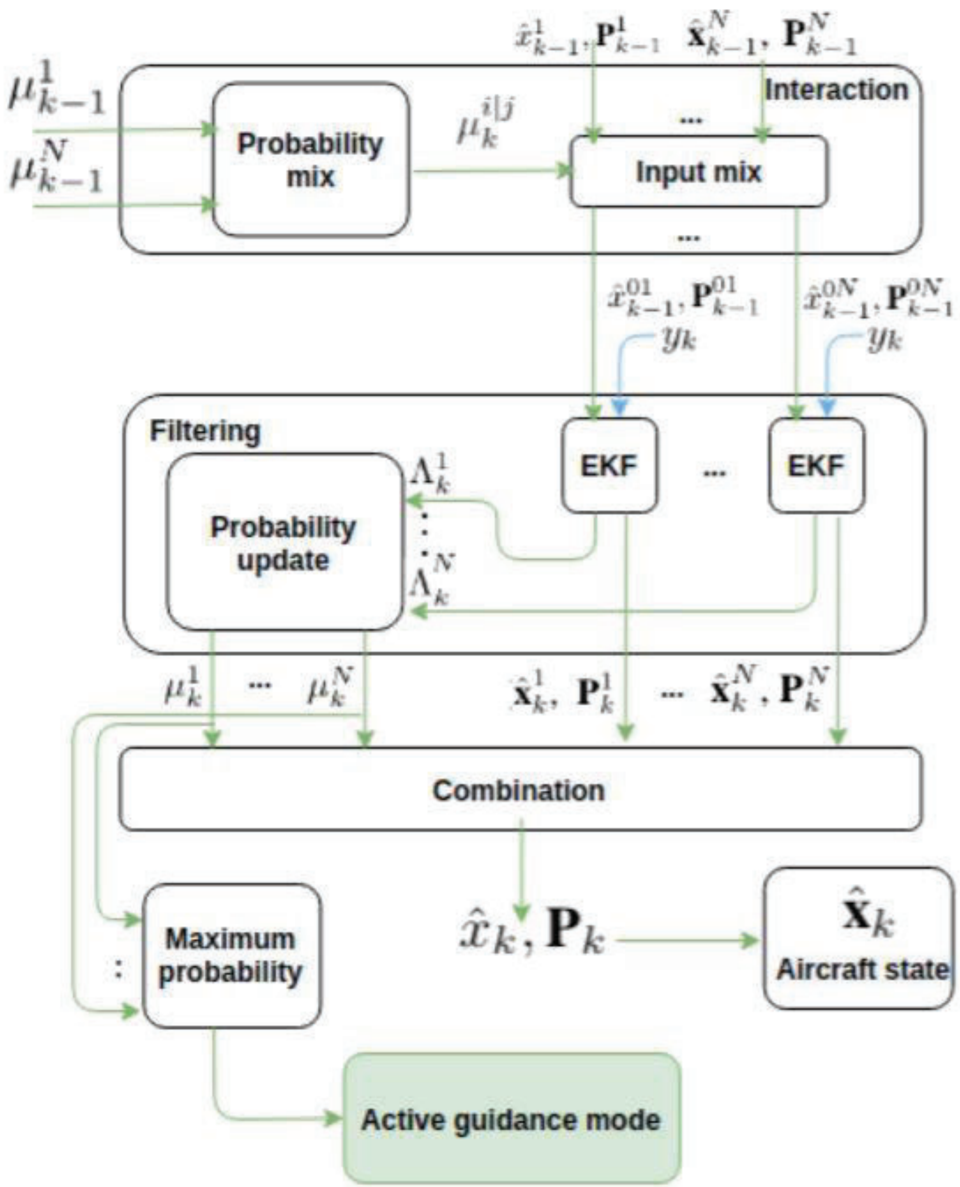


Fig. 2: The EKF-IMM flow diagram.