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Airspace Phase Transitions and the Traffic Physics of Interacting 4D Trajectories

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Abstract— **This paper presents early progress in the development of a modeling and simulation capability derived from advancements in complexity science coupled with advancements in computational platforms for the simulation and analysis of emergent phenomena in the airspace. We present a research effort to test concepts of collective dynamics of large numbers of heterogeneous aircraft (thousands to tens of thousands) in the NAS undergoing continuous 4D trajectory replanning in the presence of noise and uncertainty while optimizing performance measures and deconflicting trajectories. We use a combination of modified genetic algorithms and pseudopotential methods acting on extended objects (trajectories) rather than on aircraft themselves to implement this capability. This is a natural way to preserve intent while deconflicting aircraft. Subjects under investigation include measures of fullness of the airspace, emergent structures arising from interacting trajectory optimization, tradeoffs between centralized and distributed optimization, and phase transitions in collective behavior ("traffic physics"). Our work is concentrated in the enroute airspace, but can in principle be extended to the terminal airspace. We describe the combined software and hardware platform we have built to realize a rapid-prototyping environment capable of investigating these questions at a realistic level of fidelity and in much greater than real time speed. Our simulation platform is built on the principle of minimum assumption and maximum emergence. There are no sectors, no flight level constraints, and control actions can be arbitrarily subtle and continuous in all four dimensions. Constraints up to and including the current NAS configuration can be "switched on" for comparison purposes. With this software simulation system, we can address implications for centralized versus decentralized control in a real-world system and explore alternative TBO concepts of operation, including applications such as game theory for economic considerations, bulk management of airspace phase state for capacity considerations, and well as policy and technology strategy evaluations.**

Keywords-phase transition; aircraft trajectory optimization; airspace capacity; optimal control, real-time optimization, air traffic conflict resolution, 4D trajectory; pseudopotential method; deconfliction.

ORGANIZATION

The body of this report is organized into the following sections:

1. An overview of the research program
2. An introduction to the key related concepts of traffic physics, satisfiability, and phase transitions and their relevance to this research.
3. An overview of the trajectory generation and resolution algorithms utilized in this study.
4. Desktop Supercomputing: The configuration of software and hardware created for this study is described.
5. Preliminary results
6. Conclusion

I. INTRODUCTION

This paper presents progress in the development of a modeling and simulation capability derived from advancements in complexity. The complexity science tools include agent-based modeling and traffic physics concepts. We wish to observe and understand collective phenomena arising from many agents representing aircraft trajectories optimizing their fitness functions in parallel, and ultimately to use this understanding to engineer a safer and more robust airspace. The computational platform tools include the application of Graphical Processor Unit (GPU) technology combined with a computational language structure developed for the dynamic management of fleets of 4D aircraft trajectories. The coupling of these tools leads to the ability to simultaneously accommodate competing, conflicting, and evolving constraints in faster than real-time. The constraints include trajectory conflicts, weather, restricted airspace, economic objectives, and traffic flow considerations, from ramp-to-ramp. This capability is designed to support the study of airspace phase state (between "empty" and "full", as well as subtler demarcations) during Trajectory-Based Operations (TBO) envisioned in NextGen and SESAR.

The unique attribute of this modeling capability lies in the extremely rapid computation of thousands of deconflicted trajectories. The desired outcomes include managing the flows of all trajectories to make the best use of scarce resources, while striving to satisfy the objective function for each aircraft (including environmental constraints). The speed of computation for this new toolset is designed to enable continual re-negotiation of all trajectories to achieve desired airspace-wide management outcomes. The modeling tool conceptually supports more than tens of thousands of trajectory conflict resolutions per second (demonstration of these capabilities is planned during the project). With these capabilities, it is possible to explore alternative TBO concepts of operation, including applications such as game theory for economic considerations, bulk management of airspace phase state for capacity considerations, as well as policy and technology strategy evaluations.

Both the US and Europe share common concerns for the efficiency, environmental effects, safety, and affordable expansion of 21st-century airspace system capacity.[1]. In order to handle these challenges, NextGen will introduce key transformations in Air Traffic Management (ATM). Three examples of the transformations are:

- Increasing information sharing through net-enabledinformation access.
- Making access to National Airspace System (NAS) resources dependent on aircraft equipage.
- Aircraft trajectory-based operations enabled by aircraft ability to precisely follow customized four-dimensional (4D) trajectories [1].

These capabilities enable a more optimal allocation of functions among the air traffic system agents, such as possibly shifting the ATM system towards a distributed architecture [2]. For example, NextGen is investigating delegating more responsibility for traffic separation to the pilot [2, 3] and delegating more responsibility to airline operation centers for traffic flow management [3, 4]. Enabling the gains of distributed decision-making depends on the ability of distributed actions to maintain safety and efficiency at acceptable levels[5].

In the longer-term future (2025-2050), the airspace may contain and even be dominated by a much more heterogeneous mix of vehicles than currently occupy it, incorporating large variations in speed, altitude, mass, and operating characteristics as well as including conventionally piloted, remotely piloted, and fully autonomous vehicles. NextGen capabilities should be “upwardly compatible” to these eventualities. It is also possible that the set of possible origins and destinations may grow to be much larger than the current set of commercially served airports, rendering the current jet route system irrelevant.

When creating a new aircraft, engineers are now able to simulate the aircraft behavior to a high level of fidelity before building a prototype, including the interaction of many subsystems both physical and computational. Many choices can thus be made before committing resources, and in fact it is prohibitively expensive to proceed in any other fashion. Increases in the power and availability of computer hardware and software now make it possible to “virtual wind tunnel test” entire airspace control concepts to reasonable levels of fidelity, incorporating subsystems such as differing levels of communications and control capabilities as well as the physical characteristics of aircraft in the airspace and hypothetical demand patterns. The ability to explore and optimize over a large spectrum of possible control concepts is key to developing a flexible and robust realization of the NextGen agenda.

In addition to the benefits of powerful simulation capabilities, the young science of “traffic physics” [6] promises to buttress simulation results with analytical rigor with insights derived from long-established principles of statistical physics applied to systems of self-propelled vehicles incorporating intent. These insights have already proved useful in ground traffic analyses, particularly on European freeways. [7] Since simulations cannot address all possible scenarios of airspace dynamics due to the overwhelming combinatorics of configurations, insights based on statistical physics techniques are important in assessing probabilities of entire classes of undesirable or unsafe configurations, such as congestion and loss of separation.

II. Science Background

A. *Traffic Physics and Phase Transitions*

The science of traffic physics is a new field emerging at the boundary of the study of agent-based modeling and statistical physics. It addresses the statistical properties of large numbers of self-propelled objects acting on their own behalf. To date, the science has largely been applied to roadway vehicle dynamics because of the significant societal and financial import and because the problem is simplified by geometrical constraints. In addition, road traffic systems are perceived to be highly suboptimal and offer ready access to large amounts of data [7]. This research has applicability to other many-agent systems in addition to roadways [6,8]. The utility of the science is the ability to define systemic measures that are independent of the particular behaviors of each agent in a traffic system, much as the pressure exerted by a gas on its container is independent of the details of motion of each individual molecule.

Physical systems consisting of many particles are often characterized in terms of *phase*, such as liquid, solid, or gaseous. The phase is a property of an entire system, rather than of any of its particular components. Systems of interacting agents in freeway traffic have been shown both theoretically and observationally to exhibit phases that correspond to free-flowing (“liquid”) or jammed (“solid”) traffic. Traffic also has phases that do not have analogues in common physical systems, such as backwards-flowing waves of stalled traffic mixed with moving traffic.

Just as molecules obey certain laws (conservation of energy and momentum and the equipartition of energy), the traffic “molecules” (agents) obey simple laws implemented in a fully distributed fashion – attempting to get where they are going as quickly as possible (with an upper limit) and interacting with other vehicles, such as avoiding collisions and following at a safe distance. In vehicle traffic, throughput (or capacity) of a roadway increases with density to a certain point after which a marked decrease is observed; hence, the emergence of a traffic jam. The following diagram shows the clustering of data in two distinct phases.

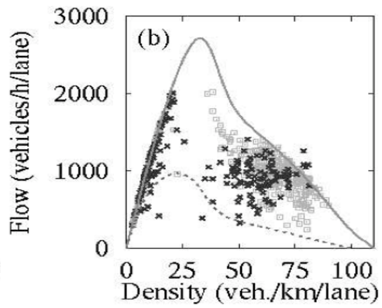

Figure 1. – Traffic Phase Diagram [7]

Applying these phase analysis techniques to the 4D motion of aircraft is one of the principal thrusts of our research. Formulating a traffic physics paradigm for aircraft is important in that it allows one to formulate a general answer to the following questions:

- When is the airspace 'full'? Can this be established independent of simulation details?
- Can the airspace capacity and safety be increased by a different choice of "particle interaction rules" (for example, conflict resolution protocols) and
- What are the trade-offs between capacity, complexity, and safety?

A key part of creating a safe, robust, flexible, and efficient air traffic system is defining metrics that are measurable and can be optimized. Phase transitions were discussed in the context of physical systems of particles and traffic in the section above, but phase transitions also exist in *logical* systems such as schedules or other hard optimization problems. In a general optimization problem, the number of possible solutions will decrease (unless it is already zero) as the number of constraints increases. The decrease is not gradual but rather sharp (and increases in sharpness with the problem size), and it looks like a typical physical phase transition such as that between water and ice with a sharp and well-defined boundary.

B. Satisfiability, Deconfliction, and Phase Transitions

It has been shown that all computationally NP-hard problems (such as the generalized deconfliction problem involving N aircraft[13,14]) can be reduced to a construct known as **3SAT**, short for "satisfiability" [9] and displaying a standard form:

$$(x_{11} \vee x_{12} \vee x_{13}) \wedge (x_{21} \vee x_{22} \vee x_{23}) \wedge (x_{31} \vee x_{32} \vee x_{33}) \wedge \ldots \qquad (1)$$

where the x_{ij} are Boolean variables and $\{\wedge, \vee\}$ represent the Boolean operations {**AND, OR**}. This formalism has been extended to continuous analogues of Boolean functions. Solving general 3SAT problems requires an amount of time exponential in the problem size, which in the deconfliction problem would be the number of aircraft impinging on a particular volume of space and time. This rapidly becomes impractical for congested airspace without simplifications such as clustering and prioritization.

In deconflliction problems, the situation is further complicated by the presence of uncertainty from causes including input data to decision algorithms, measurement error of wind fields and weather objects, and inaccurate response of aircraft. This does not necessarily incur a computational disadvantage. The presence of uncertainty means that the deconfliction problem statement changes from finding the best solution (or even a single good one) but rather finding a *solution space* that consists of many "good enough" solutions that are connected to each other by simple perturbations and that are likely to survive replanning cycles, and being able to assert that the probability of finding such a solution is within a very small but well-defined distance of 100%. We need to know that *a solution exists* even more than we need to know what that exact solution is, at least far in advance, because a particular solution computed far in advance is unlikely to be the one that is ultimately flown. This turns out to be an easier question to answer than the optimization question.

In recent years, this concept of a good solution space has been formalized, and like traffic physics, it maps onto the physics of phase transitions. Statistical ensembles of **3SAT** problems display a phase transition between soluble and insoluble that maps onto typical physical phase transitions [10].

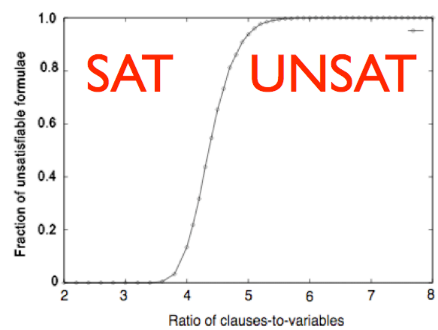

Figure 2. – Satisfiability Phase Transition [10]

The critical parameter for the phase transition is not temperature or pressure as it would be in a physical phase transition, but rather the density of constraints – the x-axis in Figure 2. Naturally one would desire deconfliction problems to be certifiably on the left side of the phase transition, or failing that, have a prescribed mechanism for mitigating predicted future unsatisfiable configurations.

Since we wish to not only generate viable configurations of trajectories but also want to use continuous versions of genetic algorithms to move towards viable configurations from unviable ones, we will also need a continuum analog of the discrete formulation of satisfiability that provides "satisfiability gradients" that can be exploited in an optimization process. . The original formulation of the SAT phase transition has been extended from the Boolean to the continuous case, allowing for more general applicability to continuous decision spaces.[11]

The logical and physical definitions for systems are intuitively connected if one takes the agent's point of view: If the molecule (or car or aircraft) has, on average, no options as to where to go next in space; then the system freezes up, or the traffic jams, or the system is "full". This connection between the theory of optimization and the phase transition between viable and non-viable solution regions can provide insight into the definition of metrics for robustness and flexibility, both of which are related to the presence and "on the fly" accessibility of alternate solutions for an aircraft's trajectory. Other attempts to address this question have used Lyapunov exponents[5], vector field divergence[12], cellular automata [13], and others. Each one of these addresses the satisfiability question of airspace fullness from a different perspective, and like NP-hard problems in general, should be unifiable under the rubric of satisfiability.

Robustness can be interpreted as the presence of many solutions to a problem, and flexibility has been interpreted in the aeronautical literature as the set of solutions reachable from a given airspace configuration[5]. The detailed analysis of general SAT systems provides compelling insight and analytical rigor [14,15] for these abstract concepts.

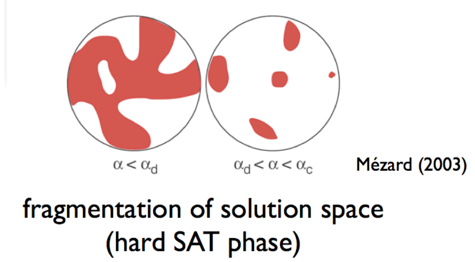

Figure 3. – Solution Space of SAT Problems – Near Phase Transition [14]

In Fig. 3, the solution space transforms from connected (on the left) to disconnected (right) as the phase transition boundary is approached but at a discrete and well-defined distance, meaning that flexibility disappears before robustness does. The near-phase-transition phenomenon is also appealing because it means that there is an advance warning of the onset of a phase transition, something extremely useful in systems where humans might intervene to avoid undesirable dynamics. The phenomenon hints at the possibility of an emerging role for NextGen air traffic management systems: Managing systemic behavior rather than micromanaging individual aircraft behavior.

III. Algorithms for Trajectory Generation and Deconfliction

In order to generate dynamic optimization and deconfliction of thousands of trajectories and observe realistic emergent collective phenomena, a number of algorithmic efficiencies must be employed. Because of the combinatorics inherent in the nature of large deconfliction problems and the nonconvexity associated with the general 4D (including wind fields) trajectory optimization problem, conventional optimization methods are limited in usefulness.[16-18] Scalable heuristics rooted in the physics of trajectories are employed.

A concise definition of automated conflict resolution is found in Erzberger *et al* [16]: "Automated conflict prediction and resolution designed to work at least ten minutes before a conflict can occur is considered a basic requirement for achieving "Free Flight." The basic problem of conflict prediction is inherent in the nature of trajectory prediction; namely, that errors in prediction are unavoidable. The farther in the future a prediction is made, furthermore, the greater the probability of error…The optimal time to initiate a conflict resolution maneuver is a tradeoff between efficiency and certainty. The farther in advance a resolution maneuver is initiated, the more efficient it is likely to be in terms of extra time and distance flown, but the less certain will be exactly what maneuver is required or whether a maneuver is required at all."

Following Jardin [19], the strategic trajectory optimization problem for a single aircraft *i* may be stated as a cost function minimization problem with dynamic constraints and constraints on initial and final aircraft state:

$$C_i = \int_{t_s}^{t_f} L_i(\vec{x}_i)\,dt \qquad \begin{aligned} \vec{x}(t_s) - \vec{x}_0 &= 0 \\ \vec{x}(t_f) - \vec{x}_f &= 0 \\ \dot{\vec{x}} &= f(\vec{x}_i, t) \end{aligned} \tag{2}$$

where $\vec{x}(t)$ is the 3D state vector for the aircraft, C_i is the integrated trajectory cost, and the objective function L_i is customarily defined as the time rate of change of the direct operating cost (DOC), a linear combination of fuel and time costs for commercial aircraft operation. The three equations on the right make explicit the fixed endpoints and the path dependence of the velocity. (a nonholonomic constraint) The single-aircraft optimization problem in is usually decoupled into separate vertical and horizontal trajectory optimization problems. The vertical problem is recast as a convex optimization problem in an energy state form and solved for the optimal speed and altitude profile vs. path distance for the case of zero winds [20,21].

The primary result is that optimal long-range vertical profiles for commercial jet transport aircraft consist of optimal ascent and descent segments connected by a long cruise-climb or step-climb segment. Optimal horizontal routes are not as easy to compute because the variations in the wind field lead to a non- convex nonlinear optimization problem with potentially many regions of local minima. As a result, approximate optimization solution approaches must be considered even before the added complexity of deconfliction is factored in. The air traffic control optimization problem is characterized by high system complexity and is thought to be in the NP-hard class of problems [16,17].

Because of this fact, approximation schemes are essential to the full 4D wind-field path planning problem in the presence of potential conflict. A variety of different heuristics have been applied in the literature, including virtual wind fields [19], discrete genetic algorithms [22], dynamic programming[23], path-planning by analogy with optics using refractive indices [24], and others.

In our simulation, we wish to generate optimal trajectories while maintaining viable separation and obstacle avoidance, thus achieving a balance between long-range interactions (intent) and short-range interactions (separation assurance). To do so, we borrow a concept from theoretical particle physics, the notion of an ensemble of interacting extended objects ("strings") first put forth as a possible explanation of the interaction of nuclear particles and proven to display a phase transition[25]. We identify these strings with potential 4D aircraft trajectories. Strings are endowed with a fictitious "charge" so that they repel each other (an effect generated by quantum mechanical effects in the original physics paper), and the charge is sufficient such that required separation is maintained. This charge generates a monotonically decreasing force of finite range and of tunable strength and distance dependence.

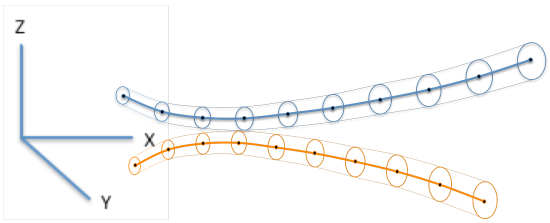

Fig. 4: Charged string concept.

In Fig. 4, the two strings represent aircraft trajectories and the sequences of rings around them represent time slices. The effect of optimization in the absence of other trajectories acts as a "string tension" that "pulls the string taught", subject to constraints of curvature that represent aircraft capabilities and operating constraints. The representation of elements of the strings as cubic splines is a natural way of incorporating constraints while generating smooth trajectories. The presence of wind and weather and closed volumes of airspace induce additional constraint forces.

This pseudopotential method applied to extended objects is computationally more efficient than applying pseudopotential methods to points representing aircraft, as we are considering sets of points constrained to move together rather than free to move independently, thus reducing the number of degrees of freedom in the optimization. In addition, charged strings are easily extended to configurations incorporating uncertainty, as charge can be distributed over a volume in space and time as easily as it can be distributed over a string or a point. In addition, volumes of high pseudopotential contain useful information about the "fullness" of a space-time volume and promise to provide an additional measure to correlate with satisfiability and phase transition metrics.

Endowing aircraft with agency and using genetic algorithms to generate ensembles of deconflicted paths has been demonstrated [5] and produces good results, but it is computationally expensive and difficult to scale. Dynamic programming methods have also been combined with optical methods (refractive bending) to generate paths[24], but do not have as natural a connection with 4D optimization and the physics of aircraft motion. We believe that string optimization can combine the best of both techniques while providing insights into system-wide questions such as capacity.

IV. Experimental Setup

In order to explore the phase structure of plausible future instantiations of the NAS, we have built a computational laboratory for simulation of large numbers of aircraft flying through enroute airspace. More precisely, we are simulating 4D trajectory paths ("strings"), not merely aircraft. Although there are on the order of 2000 IFR aircraft in the NAS at typical peak periods, for our research, our goal is to be able to simulate several times as many aircraft (>10000) flying enroute trajectories simultaneously. Doing so requires a combination of optimized hardware and code. We plan to simulate the NAS with actual airports and traffic patterns roughly similar in proportion to current NAS usage patterns-but with the "volume turned up" to generate much denser use of the airspace.

Our trajectory paths in the model are fully dynamic, meaning the paths are dynamically recomputed (replanned) throughout the flight at regular intervals, while continuously maintaining deconfliction. This regular interval, or "heartbeat" of the simulation, is a constant that is chosen to be small compared to the characteristic time scale of changing external conditions-in our case, moving weather and other external inputs (such as an airport being closed) and the consequent changing trajectories of other aircraft. This means the trajectories can maintain a close to optimal response to a changing environment. Some replanning computations make no change to the prior planned trajectory, while other re-plans are dramatic and affect many other aircraft, perhaps causing avalanches of replanning across the system. The computational challenge is to be able to replan the entire system faster than a heartbeat of at most a few minutes.

Our initial test system operates within the simplified geometry of a fixed (1000 km) diameter cylinder of airspace. Aircraft enter and exit enroute airspace at the perimeter of the cylinder already at cruise altitude and speed. For this initial version of the model, climb and descent are ignored, as are terminal airspace segments of the trajectory.

Simulating 4D interacting trajectories with replanning is essentially a 5D problem, with 3 space dimensions and two time dimensions, one to represent the "current" simulated time and one to describe the "future" as seen from that time. The aircraft trajectories are abstracted as 4D "paths" consisting of a set of "path nodes" interpolated with cubic splines. Each path node consists of 7 values: time, 3D (x,y,z) position, and 3D velocity. A three degree-of-freedom aircraft energetics model is used. Arbitrary precision of the description of a path can be accomplished by using large numbers of path nodes, though at some point this becomes counterproductive because of overfitting uncertainty. [18] We intend to allow the path node number and density to vary in a context-dependent way so as to optimize compute resources.

In practice we only have one path node per re-computation (replanning) point of the trajectory, as the time interval between the simulated current time and the next path node is the heartbeat of the simulation. For the future as seen from the present time of the aircraft, we may even elect to have fewer path nodes per unit time, declining in path node density in proportion to the importance weighting of more distant times. This is because we desire more precision in planning the near future of the next few minutes than the more distant future that will most likely experience several replanning changes before it is actually flown. As a consequence, we want to devote more computing resources to (re-)planning the near future than the far future.

Each frequent periodic re-computation (re-planning) of the aircraft trajectory paths attempts to optimize a set of cost functions. There are 3 main types of cost functions: separation (minimum distance between aircraft, and away from weather cells), aircraft performance constraints (comfortable rates and limits of climb capabilities, descent, turn radii, etc.), and economic (VCI, on-time arrival, etc.). Performance constraints and aircraft characteristics are informed by BADA data and by airline standard practice.

Our model employs methods drawn from complexity science to search the combinatorial space for an optimal set of trajectories, as the scope of this problem is beyond standard convex optimization due to the existence of rugged fitness landscapes. We use fitness landscape search methods including gradient descent, differential evolution[26], and particle swarm optimization[27]. The simulation engine "flies" large number of trajectories while keeping statistics on the behavior of the ensemble of dynamic trajectories. In particular, for our research on the phase structure of the airspace, we are particularly interested in two aspects of the overall process: compute intensity and trajectory correlations.

Most centrally to our research, our model keeps track of the "amount" of computation required to arbitrate these trajectories via the dynamical re-planning process (per time instant and geographical location). We may well see a phase transition in the compute cycles required as the airspace "heats up" as we increase the density of the airspace utilization, a phenomenon that has been observed in other superdense simulations of dynamic deconfliction. As noted elsewhere in the paper, this may aid in estimating the overall capacity of the airspace.

Another important feature on our model's dashboard is a measure of the correlations of nearby flight trajectories. Such correlations may be indicators of other phases of the airspace. If found, this may indicate "flocking"[28] or other emergent activity in certain phases of the airspace phase space. Such behavior has been observed in highly simplified computational models of self-propelled self-avoiding agents [29], and we believe should remain as simulations become more realistic and agents more complex.

Simulating large numbers of dynamically replanned aircraft trajectories in faster than real time requires considerable compute power. For ~100 aircraft, we can do an acceptable job with conventional CPU (multi-core, one machine) computer hardware. In order to simulate a complete airspace with 10^3-10^5 aircraft we use GPU (Graphics Processor Unit) technology. This is the same technology used in some supercomputers (for instance, the current world computing champion, the Chinese 2.52 petaflop "Milky Way")[30]. Modern GPUs have 400+ computing streams ("cores") running in parallel on each board. Our system utilizes an Nvidia GTX470 GPU with 448 cores. Using a water-cooled case, we can operate 3 GPUs in one desktop computer, or about 1350 cores, achieving a performance of about 2 teraflops at a cost of about $2 per gigaflop. This is more than a thousand times cheaper than a decade ago and continues an exponential path that has remained unbroken for 50 years [31]. Within another decade, it is conceivable that this amount of compute power could reside in an aircraft's cockpit. With a single GPU, we estimate we can gain about 100x performance increase over conventional CPU single-core hardware architecture.

GPUs enable dramatically more computation for our modeling but with the caveat that our algorithms had to be adapted to the parallel processing paradigm of the GPU. The GPU enables millions of software threads, up to 400+ threads operating simultaneously, but we had to reconfigure our algorithms to take advantage of this power. Fortunately, thousands of aircraft running simultaneous re-planning algorithms maps very well to the GPU parallel processing architecture. A bonus of using modern GPUs is advanced graphics, since GPUs were developed for video game applications. The role of high fidelity visual output is often underestimated, many scientific discoveries have come from long contemplation of a system's dynamics.

V. Results

We have implemented a simplified version of our algorithms (curvature constrained 4D trajectories) to a conventional computer and are currently porting them to the GPU environment. Although this project is at a very preliminary stage, we have achieved simultaneous deconfliction and weather-induced replanning with 10 aircraft at a replanning compute frequency of once per second, simulating a once per minute updating. In the next several months we intend to scale the simulation up to thousands of aircraft utilizing three degree-of-freedom energetic models, more complex weather scenarios, and full 4D path optimization with a heterogeneous fleet of aircraft.

VI. Conclusion

Building on advances in parallel computing technology, we have built a rapidly configurable test environment for exploring concepts of emergent collective behavior in the NAS with realistic simulations. We expect to find phase structure in the behavior of large numbers of aircraft concurrently optimizing their trajectories and avoiding conflicts with each other. We believe that mapping this "phase space" and its dependencies is the first step toward designing a powerful control methodology to ensure safe and efficient operation of the NAS while allowing for more heterogeneous aircraft characteristics and behavior in the future.

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