

Prediction of Top of Descent Location for Idle-thrust Descents

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Abstract—To enable arriving aircraft to fly optimized descents computed by the flight management system (FMS) in congested airspace, ground automation must accurately predict descent trajectories. Development and assessment of the trajectory predictor and the concept of operations requires models of the prediction error due to various error sources. Polynomial approximations of the along-track distance of the top of descent (TOD) from the meter fix are given in terms of the inputs to the equations of motion. Polynomials with three different levels of complexity are presented, with the simplest being linear. These approximations were obtained by analyzing output from one predictor. While this predictor’s thrust and drag models do not seem to agree well with those used by the FMS, both laboratory and operational data using commercial FMSs support the conclusion that, for given models of thrust and drag, the TOD location is roughly a linear combination of cruise altitude, descent CAS, aircraft weight, wind, and altitude and speed at the meter fix. The laboratory data include 14 descents each in a Boeing 737-700 simulator and a Boeing 777-200 simulator, using a test matrix that varied aircraft weight and descent speed. The operational data include approximately 70 descents each in commercial Boeing 757 and Airbus 319/320 aircraft.

Keywords – *idle-thrust descents; trajectory prediction; top of descent; equations of motion; flight management system*

I. INTRODUCTION

In congested airspace today, controllers direct aircraft to descend in steps. Since air density, and hence drag, increase as the aircraft descends, significant reductions in fuel consumption and emissions would result if aircraft stayed at cruise altitude longer and then descended smoothly at idle thrust. The flight management system (FMS) on a large jet can compute the location of top of descent (TOD) assuming an idle-thrust descent. To merge aircraft, however, controllers impose level flight segments, which make it easier for them to estimate the relative speeds of two aircraft given their calibrated airspeeds (CAS). The ultimate goal of the research described in this paper is to enable more fuel-efficient descents in congested airspace. This requires development of a trajectory predictor as well as its error models so that aircraft can be given clearances with a low probability that a revision will be needed later.

Due to the potential fuel savings and emissions reductions,

enabling continuous descents is being pursued by several research groups. Most of the previous research has focused on prediction of arrival time at a waypoint. While this helps with lateral separation, accurate prediction of the vertical profile is also essential to ensure vertical separation from aircraft at different altitudes, including crossing traffic. In fact, error in prediction of the vertical profile is likely to have more impact than along-track prediction error on procedures and data exchange necessary to enable more fuel-efficient descents in congested airspace. Only the TOD location is analyzed in this paper, which is the first step in understanding the entire vertical profile. Once prediction of TOD location is sufficiently accurate, analysis will be expanded to the rest of the vertical profile.

The purpose of this paper is to identify the factors that affect the TOD location and how they affect it. This is done by developing several approximations, with varying accuracy, of the distance between TOD and the meter fix. After presenting the background, related literature, and equations of motion in Sec. II and Sec. III, this paper presents results from three different types of analysis. Sec. IV describes analysis of the equations of motion to obtain polynomial approximations of TOD location. These approximations give insight into how factors affect TOD location. They must, however, be validated with FMS data. Use of a laboratory environment makes it possible to use a test matrix, which simplifies analyzing and visualizing the effects of individual factors. On the other hand, an important concern is the amount of randomness that will occur under operational conditions. This can only be determined by analyzing operational data. Therefore, Sec. V presents analysis of data from both FMS test bench experiments and commercial operations.

II. BACKGROUND

There are many possible operational concepts to increase fuel efficiency of descents. A few of these will be described here. For example, Klooster, Del Amo, and Manzi [1] proposed assigning each aircraft a controlled time of arrival at either the meter fix or runway threshold. The FMS chose its speed profile and vertical profile to meet that time. Their flight trials had good compliance with the assigned times, but “controllers frequently

asked the crews for the planned airspeeds to enable downstream conflict detections.” In congested airspace, controllers would likely also need information about the vertical profile.

Giving considerably less flexibility to the aircraft, Ren and Clarke [2] used a stochastic technique to determine minimum spacing at a control point, which is roughly the meter fix, and then assigned the same speed profile to all aircraft and used altitude constraints at several points below the meter fix. This would increase controller situation awareness but might decrease the fuel savings. Separation, especially vertically, was assisted by segregating arrival flows, but the system performance ultimately depends upon controllers’ ability to deliver aircraft to the control point with the specified separation and then to maintain separation to the runway without excessive reduction of fuel efficiency.

With a level of flexibility in between these two concepts, Copenbarger et al. [3] are developing the Efficient Descent Advisor (EDA), a controller decision support tool to determine the speed profile and any necessary path stretching to meet scheduled times at the meter fix. The tool also does rudimentary conflict detection and resolution. The concept is intended for voice-based communications, so it requires only minimal information exchange between the controllers and the flight crew.

Closely related to the previous approach, the Tailored Arrivals concept [4] assumes a data link capability, which allows many more speed and altitude constraints. Some of these constraints may be dynamically determined to avoid other traffic. The constraints are loaded into the FMS, which then computes and flies an optimized descent trajectory to the runway. Suitable adaptations of the concept have been used by commercial flights in the United States, Australia, and the Netherlands. In the United States, full operational capability would require enhancement of the EDA automation tool. For the Netherlands, the Speed and Route Advisor (SARA) tool [5] is under development to provide speed and route advisories to controllers in order to improve accuracy in meeting a specified time of delivery at the initial approach fix, but its design does not include any conflict detection and resolution functionality.

Ideally, each aircraft would be able to fly its preferred descent trajectory. Except when congestion is very light, however, this would lead to conflicts, and using tactical separation maneuvers to avoid conflicts could actually result in increased fuel consumption. Assigning controlled times of arrival at some point would decrease conflicts but not eliminate them. In particular, maintaining vertical separation will require either accurate predictions of the vertical profile or sufficient empty space above and below the predicted descent. Furthermore, controllers need situation awareness to know what to expect each aircraft will do. The most efficient way to avoid future conflicts is to modify the aircraft’s preferred trajectory only as much as seems necessary in order to avoid other traffic in the vicinity. The ability to perform such “what-if” calculations would require a trajectory predictor. Furthermore, the most effective way to accommodate the prediction error would be to require the probability of a future revision to be sufficiently low. Estimating this probability requires probabilistic models

for the trajectory prediction error.

The costs and benefits of a concept that includes a trajectory predictor will be determined by the error of that predictor — whether FMS, ground automation, or controller — and how the concept of operations accommodates that error. For example, for a given prediction error distribution, increasing the buffer around trial trajectories will decrease the probability of a future revision but also decrease throughput. On the other hand, allowing a greater probability of a future revision will increase workload, which may also decrease throughput. Evaluation of the concept with some predictor thus depends critically upon development of error models for that predictor. Papers such as [5–8] have tried to quantify some of the trade-offs in concepts to enable idle-thrust descents, but none of these included a trajectory predictor to help with vertical separation based on the current traffic. Furthermore, many papers that assess concepts to enable fuel-efficient descents focus on the descent between the meter fix and the runway, whereas the research described in this paper focuses on the descent to the meter fix. Of published results, Boeing’s Trajectory Analysis and Modeling Environment (TAME) [9, 10] provides the capabilities closest to those required for assessments along the lines described above. It does not, however, model errors in the vertical profile prediction.

Fig. 1 shows the error for operational data of the EDA trajectory predictor. The data will be described further in Sec. V. Over 90% of the meter fix crossing time predictions have absolute error less than 30 sec, which is a common estimate of this accuracy requirement (see [5], for example). The TOD location prediction error, on the other hand, is much worse than desired, with fewer than half the predictions having absolute error less than 5 nmi. This is a rough estimate of this accuracy requirement based on the 1000-ft vertical separation requirement and the likelihood that many aircraft will descend more than 1000 ft in 5 nmi after TOD, as indicated by consideration of both a theoretical 3° glide slope and the descents used in Fig. 1. The clear need to address this problem motivated the current research focus on prediction of TOD location. Possible approaches include improving the predictor, modifying procedures, and enhancing data exchange. Choosing the most cost-effective concept of operations requires understanding the tradeoffs between possible solutions. This in turn requires models of the prediction error for each alternative considered, which means not only modeling distributions as in Fig. 1 but also modeling the effect of different sources of error.

Thus, models of the trajectory predictor error are not only essential in analyzing the concept performance but are also important in implementation. Furthermore, determining what to do about the large TOD prediction errors shown in Fig. 1 requires modeling the error due to different error sources. Unfortunately, most of the previous research has focused on the prediction of arrival time at a waypoint. References [4, 11–14] investigated the prediction of the vertical profile using operational data. To model the error, larger samples are needed, especially considering the inherent randomness in operational data. None of these samples contained more than 20 flights per

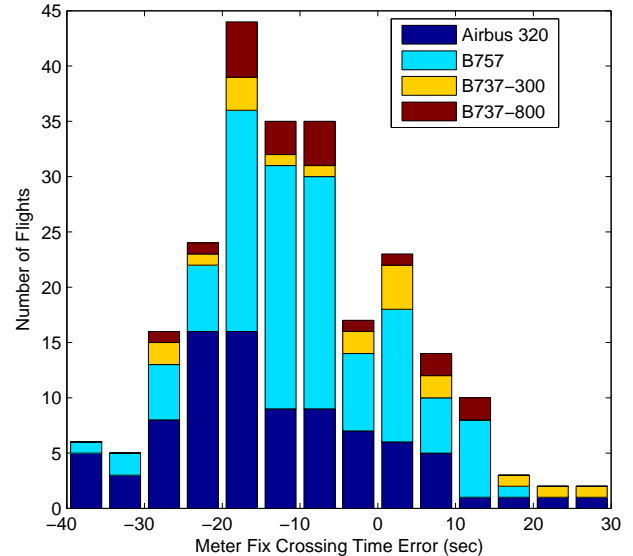
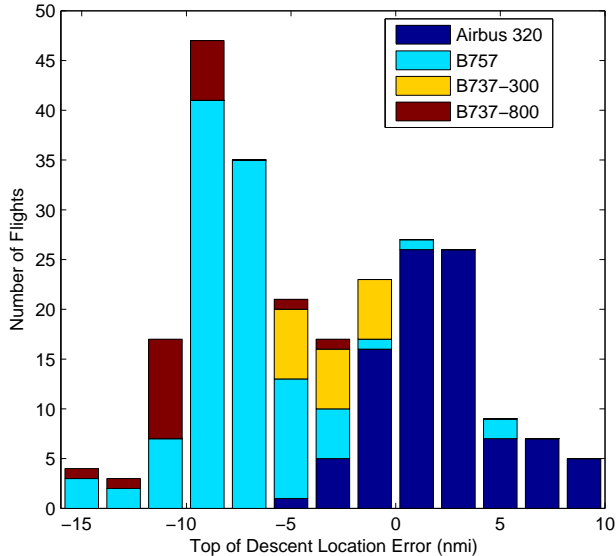


Figure 1: Error in prediction of TOD location and meter fix crossing time.

aircraft type, and they cannot be combined because the aircraft types are different. The ADAPT2 project [15] analyzed prediction of TOD location for 51 commercial flights in B737-600 and B737-800 aircraft. If these two aircraft types have similar TOD behavior, then this is a useful sample size. Their results confirmed the difficulty of predicting TOD location within 5 nmi, but they did not indicate a remedy or provide insight into the causes of the large errors. Laboratory experiments can also be useful in developing the error models, especially in determining the effect of individual factors on error. Tong, Boyle, and Warren [16] found a difference of 15 nmi in TOD solely due to the de-icing setting for B777-200 but did not consider any other factors. Herndon et al. [17] compared vertical profiles for a test matrix consisting of two descent speeds along with 14 combinations of aircraft and FMS, but the results do not indicate aircraft type, which means they do not indicate predictability of TOD location for a given aircraft type.

Analytical or numerical integration of the equations of motion can also provide insight. Several researchers [18–21] have used various integrators to study how individual factors affect descent trajectories, although none of them considered TOD location as the dependent variable.

In summary, the goal of the research described in this paper is to develop error models for TOD location prediction, including the effect of different error sources. The approach uses a combination of analytical techniques with analysis of both laboratory and operational data. These models will later be used to determine the procedures and data exchange that will most effectively enable fuel-efficient descents in congested airspace. The research is currently only considering the descent down to the meter fix. The remainder of the descent in the terminal area is even harder to predict and is likely to offer more potential for fuel savings, but starting with the simpler problem seems prudent.

III. EQUATIONS OF MOTION

In the EDA trajectory predictor, the equation of motion in the direction parallel to the path is

$$m\dot{V}_t = T - D - mg\gamma_a + \frac{dW}{dt}, \quad (1)$$

where

V_t is the true airspeed,

T is thrust,

D is drag,

m is aircraft mass,

g is gravitational acceleration,

γ_a is the flight path angle with respect to the air mass

W is the wind parallel to the path, and

t is time.

The equations used by the FMS are proprietary but believed to be close to those used by EDA, although the FMS probably approximates the wind derivative by zero. If this is the case, then [22] explains the effect of wind on TOD location, and it can be separated from the other inputs to the predictor. Furthermore, the FMS and ground automation could use the same wind forecast if necessary. Therefore, wind is essentially assumed to be zero in the analysis presented here. For simplicity, let $\omega = mg$ be the aircraft weight and define a parameter P as

$$P = \frac{\omega}{T - D}, \quad (2)$$

which is dimensionless. The equation of motion in the direction parallel to the path is now reduced to

$$\frac{1}{g}\dot{V}_t = \frac{1}{P} - \gamma_a. \quad (3)$$

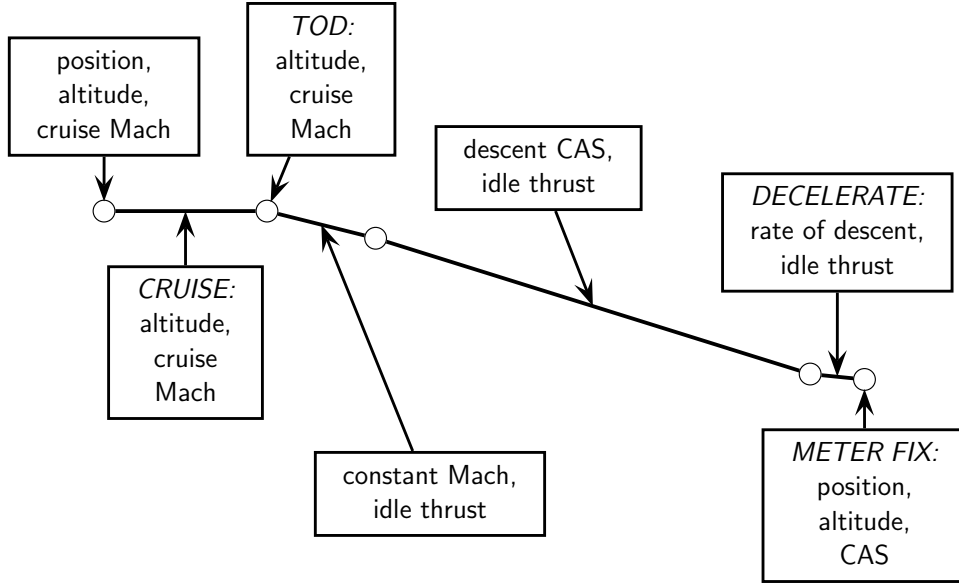


Figure 2: Schematic of typical idle-thrust descent vertical profile and constraints.

If $\dot{V}_t + g\gamma_a$ is zero, then P is undefined; but, as explained below, this will not happen in the descents considered here.

The other two equations of motion relevant to the computation of TOD location are

$$\frac{ds}{dt} = V_t \quad (4)$$

$$\frac{dh}{dt} = \gamma_a V_t, \quad (5)$$

where

s is the ground path distance relative to the meter fix and
 h is the altitude.

The descent trajectory must also satisfy the initial conditions given by the cruise speed and altitude, as well as the speed and altitude constraints at the meter fix.

The preceding equations have multiple solutions. To have uniqueness, two of the following three parameters are typically specified: thrust, speed profile, and descent rate. The descent may be divided into segments, however, with different pairs of these parameters specified in different segments. Fig. 2 is a schematic of the constraints used in this paper, which include idle thrust from TOD to the meter fix. The parameters shown in each box are specified for that segment or that point of the descent. The first segment after TOD has constant Mach number, which is the same as the cruise Mach number. As the altitude decreases, the CAS increases until it reaches the target descent CAS. The next segment is then flown at that CAS. (Sometimes, however, the descent CAS is slower than the cruise speed; in which case the aircraft decelerates at the cruise altitude, and there is then no constant Mach segment after TOD.) Finally, the aircraft typically pitches up to decelerate to the meter fix speed constraint at a given descent rate. The EDA predictor assumes this deceleration occurs at level flight. The descent rate used by

the FMS in the deceleration segment is proprietary, but it is not constant for all FMSs, aircraft types, and conditions.

Another common approach is called a geometric descent, which means the inertial flight path angle is specified for the descent. This has been advocated by some researchers [6, 16] because it eliminates the uncertainty in TOD location. If the descent is still at idle thrust, however, then the uncertainty in speed may be prohibitive [6]. On the other hand, if the speed profile is specified, then the fuel savings will be reduced. Since the best choice of constraints in the descent is not obvious, the research in this paper assumes those depicted in Fig. 2 because they are the most common for jet aircraft today. Once the necessary trajectory prediction error models have been developed, appropriate trade studies should provide better guidance.

The meter fix crossing time depends only upon wind speed and the aircraft's airspeed. Since the target speed profile was known for the descents used in Fig. 1, the meter fix crossing time prediction error depends only on speed compliance and wind forecast error. These are not likely to depend upon aircraft type, and they were apparently both good for this sample.

The TOD location is more complicated. In the constant Mach and constant CAS segments, V_t is a known function of h , so dividing (3) by (5) and solving for γ_a gives

$$\gamma_a = \frac{1}{\left(\frac{1}{g} V_t \frac{dV_t}{dh} + 1\right) P}. \quad (6)$$

Dividing (5) by (4) gives

$$\frac{dh}{ds} = \gamma_a, \quad (7)$$

so the combined length of these two segments is

$$\Delta S_{\text{spd}} = \int_{h_{\text{crz}}}^{h_{\text{fix}}} \left(1 + \frac{1}{2g} \frac{dV_t^2}{dh}\right) P dh, \quad (8)$$

where the upper limit of integration assumes the deceleration segment is at level flight. Note that P is defined as long as neither γ_a nor $\left(1 + \frac{1}{2g} \frac{dV_t^2}{dh}\right)$ is zero. In the constant Mach and constant CAS segments, γ_a is never zero. In the constant CAS segment, dV_t/dh is positive, so the term in parentheses is positive. In the constant Mach segment, dV_t/dh may be negative, but it is sufficiently close to zero that the term in parentheses is still positive.

In the deceleration segment, dividing (3) by (4) gives

$$\frac{1}{g} V_t \frac{dV_t}{ds} = \frac{1}{P} - \gamma_a, \quad (9)$$

where γ_a is specified; so the length of the segment is

$$\Delta S_{\text{dec}} = \frac{1}{g} \int_{V_c}^{V_{\text{fix}}} \frac{V_t}{1 - \gamma_a P} P dV_t, \quad (10)$$

where

V_c is the speed in the constant CAS segment and V_{fix} is the meter fix speed constraint.

Since $\gamma_a \leq 0$ and $dV_t/ds < 0$ in the deceleration segment, P is defined. In the following, γ_a is assumed to be zero in this segment. Experimentation with the EDA predictor indicates that using more realistic values might move the TOD location by as much as 1 nmi, but (10) could be used to analyze the effect more accurately. Assuming γ_a equal to zero means the deceleration segment has constant altitude h_{fix} , so V_t is a monotone function of only CAS along a given deceleration segment. Therefore, the variable of integration can be changed to CAS.

Note that the length of the descent depends strongly on P . Unfortunately, the aircraft weight is not currently available to ground systems; so EDA uses a default weight for each aircraft type, while the FMS estimates the correct weight. More importantly, the values of T and D used by the FMS are proprietary and not available for use in EDA. Everything else in (8) and (10) is known accurately except γ_a , but that cannot cause the large errors in TOD prediction shown in Fig. 1. The along-track wind component will also affect TOD location, but the contribution of this to the errors in Fig. 1 was also found to be relatively small. Therefore, the primary cause of these errors must be differences in P between EDA and the FMS.

IV. ANALYTICAL APPROXIMATION

This section presents polynomial approximations of ΔS_{spd} and ΔS_{dec} in terms of P , h_{crz} , h_{fix} , V_c , V_{fix} , and ω (which is assumed constant from TOD to the meter fix). This is possible because (8) and (10) show that these lengths are smooth functions of these parameters. Unfortunately, deriving the approximations requires knowledge of how P depends upon the variables of integration in these equations, which is not obvious. For a given thrust model and a given drag model, P can probably be approximated well by a Taylor polynomial of sufficiently high degree. Because T and D are complicated, the polynomial approximations of P used here are obtained empirically rather

than analytically. Since the thrust and drag models used by the FMS are proprietary and were not available for this research, the derivations of the approximations are based on the output of the EDA predictor. While the EDA values of P are not the same as those used by the FMS, the forms of the polynomial approximations of P may be the same. Even if this is not the case, the following derivation and approximations provide useful insights. Since P is handled empirically, the entire derivation is also handled empirically, using the EDA predictions described next. Standard Atmosphere (which affects the relation between Mach number, CAS, and true airspeed) is assumed; the effect of this assumption will be investigated in future research. The primary justification of the approximations is comparison with the predictor output shown in Fig. 3 at the end of this section. The lengths ΔS_{spd} and ΔS_{dec} are considered separately in the first two subsections, then Sec. C combines the approximations, shows the errors, and discusses possible applications. Laboratory and operational results in Sec. V suggest that the results are relevant to descents computed by the FMS.

The following derivations are based on EDA predictions for one engine type for each of the aircraft types B737-700 and B777-200. These aircraft were chosen because they are the aircraft types used in the FMS test bench experiments described in Sec. V. The test matrix is shown in Table I, although the B737 descents with cruise altitude 40,000 ft and cruise Mach 0.73 failed because the airspeed was below operational limits.

TABLE I: EDA TEST MATRIX FOR APPROXIMATION OF ΔS_{SPD} , ΔS_{DEC} .

parameter	minimum	maximum	step size
cruise altitude (ft)	30,000	40,000	2000
B737 cruise Mach	0.73	0.78	0.01
B777 cruise Mach	0.76	0.81	0.01
descent speed (KCAS)	250	330	20
fix altitude (ft)	10,000	20,000	2500
fix speed (KCAS)	230	250	20
B737 weight (lb)	92,000	132,000	10,000
B777 weight (lb)	312,000	447,000	30,000

A. Approximation of ΔS_{spd}

Experimentation with the values of P , h , ω , and CAS at each EDA predictor integration step in the constant Mach and constant CAS segments found that P can be approximated reasonably well by

$$P \approx \mathcal{P}_{\text{spd}}(V_c, \omega) = \pi_{s,0} + \pi_{s,1} V_c + \pi_{s,2} \omega + \pi_{s,3} V_c \omega, \quad (11)$$

where the constants $\pi_{s,i}$ depend upon the aircraft type. This approximation is better in the constant CAS segment than the constant Mach segment, but it is accurate enough in the constant Mach segment and the constant Mach segment is short enough to use this approximation in both segments. This means P depends only weakly on h in (8), so it can be taken outside the

integration, which then gives

$$\Delta S_{\text{spd}} \approx \mathcal{P}_{\text{spd}}(V_c, \omega) \left(h + \frac{1}{2g} V_t^2 \right) \Big|_{h_{\text{crz}}}^{h_{\text{fix}}}. \quad (12)$$

The second factor should not depend upon aircraft type. This is confirmed by the EDA predictions analyzed here, which also show it is very well approximated by $(\Delta h)\mathcal{V}_{\text{spd}}(V_c)$, where

$$\begin{aligned} \Delta h &= h_{\text{crz}} - h_{\text{fix}} \\ \mathcal{V}_{\text{spd}}(V_c) &= v_{s,0} + v_{s,1}V_c. \end{aligned} \quad (13)$$

Therefore,

$$\Delta S_{\text{spd}} \approx -(\Delta h)\mathcal{V}_{\text{spd}}(V_c)\mathcal{P}_{\text{spd}}(V_c, \omega). \quad (14)$$

The speed of the constant Mach segment does not have much effect on ΔS_{spd} , probably because the constant Mach segment is usually much shorter than the constant CAS segment and because the range of Mach numbers used here is equivalent to a range of 30 kt true airspeed.

Experimentation revealed that (14) could be approximated well by an expression of the form

$$\Delta S_{\text{spd}} \approx -(\Delta h)(\beta_{s,0} + \beta_{s,1}V_c + \beta_{s,2}\omega), \quad (15)$$

where $\beta_{s,i}$ are determined by least squares. Finally, this can be further simplified to an approximation that is linear in Δh , V_c , and ω . This can be motivated by the first-order Taylor series

$$xy \approx -x_0y_0 + y_0x + x_0y, \quad (16)$$

but the coefficients given in Table II were actually computed by least squares. A positive coefficient means that increasing that variable increases ΔS_{spd} .

TABLE II: COEFFICIENTS OF LINEAR APPROXIMATION OF ΔS_{spd} .

term	B737-700	B777-200
V_c	-0.26 nmi/KCAS	-0.38 nmi/KCAS
ω	0.00027 nmi/lb	8.0×10^{-5} nmi/lb
Δh	0.0024 nmi/ft	0.0036 nmi/ft

B. Approximation of ΔS_{dec}

At constant altitude, V_t is approximately linear in CAS with coefficient linear in that altitude. The change of variable in (10) from true airspeed to CAS results in multiplying the integral by this coefficient. This was determined by fitting polynomials to the EDA predictor output, but could be derived analytically from the formula to convert from CAS to true airspeed. Experimentation with the EDA integration values in the deceleration segment found that P can be approximated reasonably well by

$$\begin{aligned} P &\approx \mathcal{P}_{\text{dec}}(V_c, \omega, h_{\text{fix}}) \\ &= \pi_{d,0} + \pi_{d,1}V_c + \pi_{d,2}h_{\text{fix}} + \pi_{d,3}V_c\omega, \end{aligned} \quad (17)$$

where the constants $\pi_{d,i}$ depend upon the aircraft type. Since P depends upon the variable of integration in (10), the first polynomial approximation for ΔS_{dec} is considerably more complicated than (12), even though γ_a is assumed to be zero. Some trial-and-error based on analogy with the approximation for ΔS_{spd} , however, led to

$$\Delta S_{\text{dec}} \approx -\frac{\Delta V}{g}\mathcal{V}_{\text{dec}}(\bar{V}, h_{\text{fix}})\mathcal{P}_{\text{dec}}(\bar{V}, \omega, h_{\text{fix}}), \quad (18)$$

where

$$\begin{aligned} \Delta V &= V_c - V_{\text{fix}} \\ \bar{V} &= \frac{V_{\text{fix}} + V_c}{2} \\ \mathcal{V}_{\text{dec}}(V, h_{\text{fix}}) &= v_{d,0} + v_{d,1}h_{\text{fix}} + v_{d,2}V, \end{aligned}$$

and the coefficients $v_{d,i}$ are determined by least squares fitting of $-g\Delta S_{\text{dec}}/[\Delta V\mathcal{P}_{\text{dec}}(\bar{V}, \omega, h_{\text{fix}})]$ for both aircraft types combined.

Experimentation again gave an approximation of the form

$$\Delta S_{\text{dec}} \approx -\frac{\Delta V}{g}(\beta_{d,0} + \beta_{d,1}h_{\text{fix}} + \beta_{d,2}\omega), \quad (19)$$

where $\beta_{d,i}$ are determined by least squares. Using least squares to compute the coefficients shown in Table III of a model linear in ΔV , h_{fix} , and ω gives reasonable accuracy, which is again motivated by (16).

TABLE III: COEFFICIENTS OF LINEAR APPROXIMATION OF ΔS_{DEC} .

term	B737-700	B777-200
h_{fix}	0.00012 nmi/ft	0.00019 nmi/ft
ω	2.2×10^{-5} nmi/lb	6.3×10^{-6} nmi/lb
ΔV	0.079 nmi/KCAS	0.11 nmi/KCAS

C. Approximation of Total Length of Descent

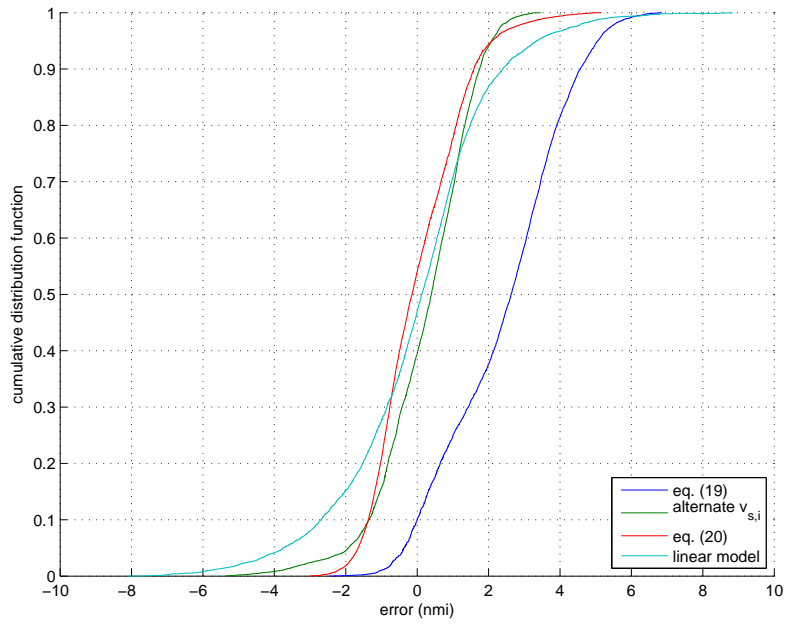
Combining the results of the previous two subsections, the TOD location specified as path distance relative to the meter fix can be approximated by

$$\begin{aligned} S_{\text{TOD}} &= -(\Delta S_{\text{spd}} + \Delta S_{\text{dec}}) \\ &\approx (\Delta h)\mathcal{V}_{\text{spd}}(V_c)\mathcal{P}_{\text{spd}}(V_c, \omega) \\ &\quad + \frac{\Delta V}{g}\mathcal{V}_{\text{dec}}(\bar{V}, h_{\text{fix}})\mathcal{P}_{\text{dec}}(\bar{V}, \omega, h_{\text{fix}}) \end{aligned} \quad (20)$$

$$\begin{aligned} &\approx (\Delta h)(\beta_{s,0} + \beta_{s,1}V_c + \beta_{s,2}\omega) \\ &\quad + \frac{\Delta V}{g}(\beta_{d,0} + \beta_{d,1}h_{\text{fix}} + \beta_{d,2}\omega). \end{aligned} \quad (21)$$

Furthermore, S_{TOD} can be approximated well by a function linear in Δh , ΔV , V_c , ω , and h_{fix} . Fig. 3 shows the error distributions of these three approximations for both aircraft types, with negative error meaning the EDA prediction of S_{TOD} is farther from the meter fix than is the approximation. These plots also show the error when the coefficients $v_{s,i}$ in (13) are determined by applying least squares to $-\Delta S_{\text{spd}}/[(\Delta h)\mathcal{P}_{\text{spd}}(V_c, \omega)]$

B737-700



B777-200

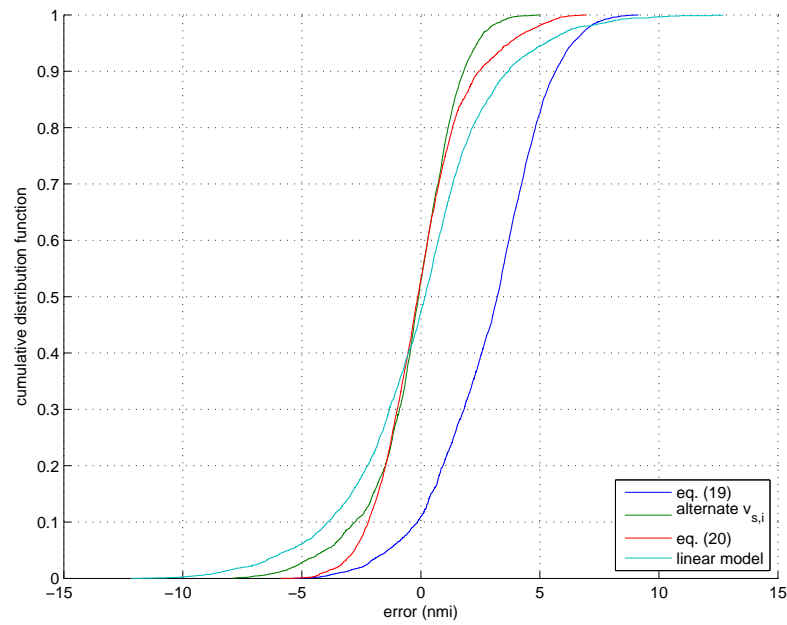


Figure 3: Cumulative distribution functions of error in approximations of S_{TOD} .

for both aircraft types combined rather than to the second factor in (12). For both aircraft types, over 95% of the absolute errors are less than 5 nmi for the approximations from (20) with the alternate $v_{s,i}$ and from (21). Adding interaction terms to the least squares model gives results consistent with (21); and adding quadratic terms does not appreciably reduce the error, which is also consistent with (21).

These approximations still depend strongly on P , of course, so using any of them as a kinematic predictor would not avoid the need for values of P sufficiently close to those used by the FMS. With suitable FMS data, one might consider \mathcal{V}_{spd} and \mathcal{V}_{dec} to be known in (20), and then use least squares to estimate the coefficients in \mathcal{P}_{spd} and \mathcal{P}_{dec} . In other words, it might be possible to reverse engineer a suitable approximation for P from FMS-computed values of S_{TOD} . This does not seem to be trivial, however, and will be considered in future research. There are other possible applications of the approximations presented above. For example, (20) should be used to design the test matrix in future FMS test bench experiments so as to minimize the uncertainty in the coefficients estimated from the data. Such applications are left for future research.

V. COMPARISON WITH DATA

The polynomial approximations of TOD location presented above are based on EDA trajectory predictions, so they might not apply to the TOD location computed by the FMS. In this section, both laboratory and operational data are shown to have TOD location that is roughly a linear combination of cruise altitude, descent CAS, aircraft weight, wind, and altitude and speed at the meter fix. The experiments and much of the analysis have been published previously; so they are only outlined here, emphasizing connections to the approximations presented above. Validation of (20) and (21) using FMS-computed TOD locations is only lightly touched upon here. Even for the linear approximation, the sample sizes are too small to claim the result has been validated. These are preliminary results that must be supported by much more data in the future.

A. FMS Test Bench Experiment

As described in [22, 23], the experiments in this study were run in simulators — a B737-700 and a B777-200 — operated by Boeing Phantom Works. These simulators were custom-built and each included a commercial FMS, which had different manufacturers for these two aircraft types. The test matrix consisted of only three values each of V_c and ω , except that eight values of V_c were used for the middle value of ω ; and the ranges of V_c and ω were approximately the same as in Table I. All the other inputs — altitude and speed both in cruise and at the meter fix as well as wind speed — were the same in the runs considered here. The actual TOD locations computed by the FMS were obtained from Automatic Dependent Surveillance-Contract (ADS-C) messages.

Least squares fit of the FMS-computed TOD location with a model linear in V_c and ω gave absolute errors less than 2 nmi for both aircraft types, which is much better than the linear models in Fig. 3. This is essentially because holding all the parameters

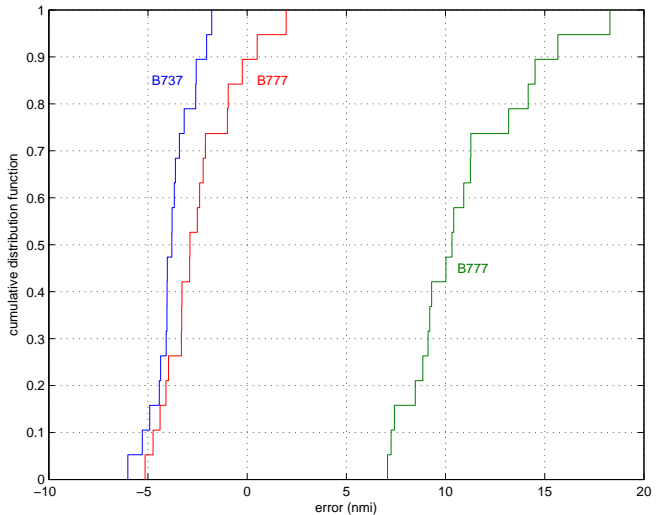


Figure 4: Error distributions for EDA predictions of TOD location.

other than V_c and ω constant means (21) reduces to

$$S_{\text{TOD}} \approx (\gamma_{s,0} + \gamma_{s,1}V_c + \gamma_{s,2}\omega) + (\gamma_{d,0} + \gamma_{d,1}V_c)(\gamma_{d,2} + \gamma_{d,3}\omega), \quad (22)$$

which is almost linear. This does not completely explain the accuracy of the linear approximation, however, because it is clearly better than the accuracy of (21) for the B777. The accuracy of the linear approximation for these data is thus misleading, which highlights the importance of choosing a test matrix that adequately covers the parameter space in order to analyze predictability of TOD location. Adding the interaction term $V_c\omega$ to the least squares model did not improve the fit noticeably, which is consistent with (22) and the relative unimportance of the deceleration segment.

The EDA predictor used the same aircraft weight as the FMS used, even though EDA typically uses the same weight for each aircraft type. Fig. 4 shows the distributions of the EDA TOD location prediction errors, with negative error meaning the FMS-computed location is farther from the meter fix than is the EDA prediction. EDA has two significantly different tables for $(T - D)$ for the B777 for two different engine types. The one with worse errors (which is the one used in previous analyses) was believed to be the most common B777 engine when the EDA database was created. Unfortunately, that was over 10 years ago. Thrust ratings have changed considerably since then, and some B777-200 engines and thrust ratings are not in the EDA database. This highlights two major obstacles to the development and deployment of a trajectory predictor that will enable fuel-efficient descents as outlined in Sec. II: the aircraft database must be maintained and the TOD location may depend upon factors such as engine type or thrust rating that are not currently available to ground automation. Use of the BADA database [24] would simplify maintenance, but it is not currently sufficiently detailed to address the second problem. Using a predictor based on kinematic models will not overcome these problems either. If engine type significantly changes TOD

location, then the predictor must use a model that depends upon engine type. If engine changes over time significantly change TOD location, then the predictor's models must change over time. It might, however, be possible to overcome these problems by increasing data exchange between aircraft and ground and by using prediction algorithms that dynamically update.

B. Operational Data

The experiment procedures and data are briefly described here, with more details in [25]. The data collection occurred September 8–23, 2009, in the Denver (ZDV) Air Route Traffic Control Center (ARTCC). Two airlines participated. Eligible flights were Airbus 319/320 and Boeing 737-300, 737-800, and 757-200 descending to Denver International Airport (DEN). For simplicity in this document, the Airbus 319/320 category will simply be referred to as Airbus, and the Boeing 757-200 will be referred to as B757.

EDA was not actually used during data collection, but the required inputs to the EDA trajectory predictor were recorded or extracted from radar data for later analysis. Pilots also were requested to complete data forms that included aircraft weight, the winds programmed into the FMS, and any comments; but many pilots did not return their data forms. The TOD location computed by the FMS was not directly available for these descents, so the actual TOD location extracted from the track was used instead. As long as the aircraft flew the FMS-computed descent as instructed, the actual TOD location should be within a few nautical miles of the location computed by the FMS. If the pilot initiates descent too early, the autopilot maintains a shallow descent rate of 1000 ft/min until the aircraft intersects the intended vertical profile. Such descents were fairly easy to identify and were discarded. The only way to confirm that an aircraft descended late, on the other hand, was from comments on the pilot data sheet or recorded by an observer at ZDV.

The descents used in Fig. 1 were those deemed to have followed the specified procedures from TOD down to the meter fix, regardless of whether the pilot data form was returned. This includes about 110 descents each for Airbus and B757. Differences in the wind used by EDA and the FMS were deemed to be a relatively small cause of the error in the EDA prediction of TOD location for two reasons. For those flights for which the pilot data form was returned, these difference would have contributed at most 1–2 nmi to the error in EDA TOD prediction. Furthermore, the EDA TOD error depends strongly on aircraft type; but this is not likely to be the case for the wind differences, especially since the Airbus and B757 flights were all flown by the same airline.

The primary goal of the data analysis was to investigate how the actual TOD location depended upon the altitude and speed in cruise and at the meter fix, descent CAS, winds, and aircraft weight. To accomplish this, multiple regression was applied to the actual FMS locations. This required the aircraft weight and the winds programmed into the FMS from the pilot data forms. Airbus and B757 each had about 70 good descents with all the necessary data. The other aircraft types had fewer than

20 descents each, which is too few for this regression analysis.

Assuming the theoretical dependence on wind noted in Sec. III, a model linear in Δh , V_C , and ω gave regression residuals with absolute value less than 5 nmi for all but four (6%) of the Airbus descents and six (8%) of the B757 descents. This is consistent with the results in Sec. IV. This suggests that the variation in TOD location due to human randomness or “hidden” factors in these trajectories is usually less than 5 nmi and is *not* the cause of the EDA TOD location prediction errors, which are larger. The goodness of this fit also indicates that P is roughly the same over all descents of the same aircraft type in this sample. This sample, however, only has two aircraft types flown by one airline into one airport over a three-week interval and the descent parameter values used do not adequately cover the operational range of values.

VI. CONCLUSIONS AND FUTURE WORK

For given models of thrust and drag, the along-track distance of TOD from the meter fix for descents of the type depicted in Fig. 2 can be approximated well by a fairly simple polynomial in cruise altitude, descent CAS, aircraft weight, wind, and altitude and speed at the meter fix. In fact, a linear combination of these factors is sufficiently accurate for some applications. The analytical results were shown to be consistent with data not only from FMS test bench experiments but also from commercial operations. In addition, the observed TOD locations for the operational descents did not contain excessive noise.

The ultimate goal of the research described in this paper is maximizing the use of fuel-efficient descents in congested airspace. The most efficient way to do this is to develop a trajectory predictor for ground automation so that an aircraft's preferred trajectory would be modified only as much as seems necessary in order to avoid other traffic in the vicinity. The greatest obstacle to accomplishing this seems to be that the proprietary thrust and drag models used by the FMS are not available for use by ground automation, which results in large errors in the predicted vertical profile. Furthermore, these models are affected by factors that are not currently available to ground automation. Examples of such factors are engine type, winglets, and de-icing setting. Overcoming this obstacle requires better knowledge of the factors affecting the thrust and drag models used by the FMS. This clearly depends upon obtaining data that reflects the effects of these factors. The approximations presented in this paper can assist in designing experiments to obtain such data, analyzing the data, and indicating how to improve ground automation trajectory prediction.

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