Markov Model for Benefits Analysis of Air Traffic Technologies

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***Abstract*—A critical step in the design and development of new tools and systems for air traffic management is the estimation of potential benefits of the added technology. The current methodology of estimating the added benefit of a new tool is based on a combination of simulation and field observations, requiring either an extensive model of the system or a fielded prototype. This paper contributes a Markov model for benefits estimation, which allows for quick assessment of benefit uncertainty and rapid evaluation of different operational scenarios. In this paper, a Markov model is employed to estimate the benefits of a strategic departure management tool. The model probabilities are derived from a historical archive of Route Availability Planning Tool (RAPT). Monte Carlo simulations are performed to estimate the range of benefit for uncertainties in model parameters and technology performance accuracy. Using this model we also provide an illustration of how different decision procedures can be accommodated, and their impact on benefits.**

I. Introduction

The estimation of potential benefits is an important component of investment analysis and system design for air traffic management automation and decision support capabilities. The typical process of estimating benefits can incorporate the following steps. First, a system shortfall is identified through a combination of operational review and analysis of performance metrics. Next, a potential benefits pool, which includes the scope of operations that are affected by the shortfall and the external factors (e.g. weather) that contribute to the shortfall, is defined. A system capability is proposed which addresses the shortfall by providing automation, decision support, and procedures that enable air traffic service providers and operators to make better decisions and implement them more efficiently. The likelihood that the proposed capability will successfully address the shortfall must be assessed, and potential benefits can be estimated by discounting the size of the benefits pool by the anticipated effectiveness of the solution. Finally, the benefits estimation methodology should identify the aspects of the proposed capability that are most critical to achieving the expected benefits, and guide requirements for the accuracy, precision, and timeliness of data and forecasts incorporated in the proposed capability [1].

As an example, the difficulty in strategic planning for arrival traffic flows into major metroplexes during periods of significant convective weather impacts is considered a representative traffic management shortfall[2]. The potential benefits pool would focus on a set of metroplexes and days where convective impacts are significant, as well as the flights and traffic flows affected. The potential benefits pool is discounted by estimating the lost effectiveness due to weather forecast uncertainty, and identifying the likelihood and cost of poor decisions based on inaccurate forecasts. The resulting benefits would assist in determining the need for requiring a high degree of forecast certainty before decisions can be made and implemented.

Modeling forecast uncertainty and properly accounting for its impact on the effectiveness of proposed system capabilities is particularly difficult due to the complexity of the air traffic management process and the external factors, such as weather, that affect it. It requires the estimation of probabilities that forecast errors will result in missed opportunities to make a beneficial decision or wrong decisions based on overly optimistic forecasts. It can be further complicated by the availability of actionable forecast information with sufficient lead time to implement the appropriate action. Realistic assumptions about user confidence in forecasts and time lags between the appearance of forecast information and air traffic management actions must be made. It is optimistic to assume that air traffic decisions will always be made and implemented at the earliest appearance of forecast opportunity, or that decision makers will always be able to recognize the difference between a good forecast and a poor one.

A variety of approaches are used to estimate potential benefits of aviation technologies: queuing models [3], [4], extrapolation from field observation [5], [6] and simulation [7]. Queuing models are predominantly used to predict and analyze delay performance in the system, although some studies have monetized the delay. Benefits estimates extrapolated from field observation are based on detailed observations of the decisions that are made during live operations using a prototype of the proposed capability. Post event data analysis estimates the improvement in performance metrics from outcomes of circumstances when the prototype technology is present and absent. Achievable benefits are estimated by extrapolating from limited duration observation instances, to a multi-year duration of operation. Field observation eliminates the need to make assumptions about the effectiveness of the capability in driving beneficial decision making, the feasibility of implementing decisions based on the capability, and the degree of forecast certainty and user confidence required to achieve benefits, since

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all of these factors are observed directly during live operations. However, the reliability of benefits estimate derived from operational observations is limited by the accuracy of the post event outcome modeling and extrapolation from a relatively small data set. Furthermore the extrapolation methodology requires a working prototype, and cannot be used to estimate benefits for a proposed capability that has not been implemented.

Simulations of the air traffic operations, either fully automated for Monte Carlo simulation or with human-in-the-loop (HITL), are also used to estimate performance improvements enabled by the incorporation of the proposed capability into the traffic management system [8]. Monte Carlo simulation eliminates (in principle) the problem of benefits extrapolation from a small number of cases; the number of days studied and variations in operational behavior are limited only by the available computational resources. Sensitivity analysis can support the specification of requirements. However, the accuracy of benefit estimation depends upon the fidelity of the simulation in capturing the factors that affect system performance and accurately modeling the complex interactions between human decision makers responding to events as they unfold. Validating the fidelity of simulations during complex operational scenarios can be very difficult. Benefits estimation based on HITL simulation is characterized by the shortcomings of both extrapolation and simulation methodologies, since HITLs rarely model complex operational scenarios with sufficiently high fidelity to estimate benefits, and estimates must be extrapolated from the limited number of HITL instances that is practically feasible.

This paper presents an approach for modeling benefits, and examining the impact of uncertainty on the operational costs and benefits of introducing a new air traffic management technology. We consider uncertainty from factors such as: forecast accuracy of the technology, variations in external factors (e.g. weather), variations in user decision making, and variations in cost. Our approach employs a stateful model of a flight's operation. The choice of states is based on treating flight phases that are relevant to changes in decisions and outcomes facilitated by the introduction of a new decision support technology. In section II we present a general framework for probabilistic modeling of costs and benefits using a stateful approach, with state transitions that follow the Markov property. The example diversion mitigation technology, to which we apply our cost and benefits model, is described in section III. In section III we consider alternative decision policies that may be employed in concert with the diversion mitigation technology. The application described in section III is sufficiently tractable to allow for analytic expressions for cost and benefits, which are provided in section IV. In section V we describe how uncertainty arising from the technology, the decision making, and external factors (e.g. weather and cost) are incorporated into our framework. The estimation of model probabilities for our application, as well as empirical data for assessing the validity of our model are discussed in section VI. The statistical values for cost and benefits computed from the application of our model are presented in section VII.

II. Markov Model

We use a state transition model to represent different phases experienced by a flight. The movement from one state to another is the result of a decision or response to an event. Our model associates a cost with each state, therefore cost is accrued when a flight visits a state. The probability of transition between state $X_{n-1} = i$ and $X_n = j$ is given by γ_{ij}, where n is the number of transitions. Collectively these transition probabilities are represented by the matrix:

$$\mathbf{P} = \begin{bmatrix} \gamma_{1,1} & \gamma_{1,2} & \cdots & \gamma_{1,K} \\ \gamma_{2,1} & \gamma_{2,2} & \cdots & \gamma_{2,K} \\ \vdots & \vdots & \cdots & \vdots \\ \gamma_{K,1} & \gamma_{K,2} & \cdots & \gamma_{K,K} \end{bmatrix} \tag{1}$$

The probability of a flight being in any particular state on the n-th transition is given by the vector:

$$\boldsymbol{\pi}_n^T = \begin{bmatrix} \pi_{1,n} & \pi_{2,n} & \cdots & \pi_{K,n} \end{bmatrix} \tag{2}$$

where $\pi_{k,n}$ is the probability of being in state k on the n-th transition. The state probabilities can be obtained iteratively by solving

$$\boldsymbol{\pi}_n^T = \boldsymbol{\pi}_{n-1}^T \mathbf{P} \tag{3}$$

The costs incurred by visiting any state are collectively given by

$$\mathbf{c} = \begin{bmatrix} c_1 & c_2 & \cdots & c_K \end{bmatrix} \tag{4}$$

The incremental cost accrued by a flight on the n-th transition is

$$\theta_n = \boldsymbol{\pi}_n^T \mathbf{c} \tag{5}$$

The total cost incurred by a flight is the sum over the incremental costs

$$S = \sum_n \theta_n = \boldsymbol{\pi}_0^T \left[\sum_n \mathbf{P}^n \right] \mathbf{c} \tag{6}$$

The benefit gained by introducing a technology is assessed by adjusting the transition probability matrix. The adjusted probability matrix can be written as

$$\mathbf{P} = \mathbf{P}_B + \boldsymbol{\Delta}\mathbf{P} \tag{7}$$

where $\mathbf{P}_B$ is the base transition probability matrix, and $\boldsymbol{\Delta}\mathbf{P}$ is the adjustment to the base resulting from the introduction of new technology. The cost benefit is defined as the difference between the base cost and the new cost.

$$\Delta S = S_B - S = \boldsymbol{\pi}_0^T \left\{ \sum_n \left[\mathbf{P}_B^n - (\mathbf{P}_B + \boldsymbol{\Delta}\mathbf{P})^n \right] \right\} \mathbf{c} \tag{8}$$

III. Application:Impact of Diversion Mitigation Technology

In this section we will illustrate the application of the Markov model to an air traffic control scenario involving diversion of aircraft due to weather blockage along the path to an arrival destination. In this scenario we examine the impact of a potential technology that can assist in accurately forecasting the weather blockage over a time horizon. We suppose that short flights, whose origins are within this time horizon, can exploit the blockage forecasting technology by

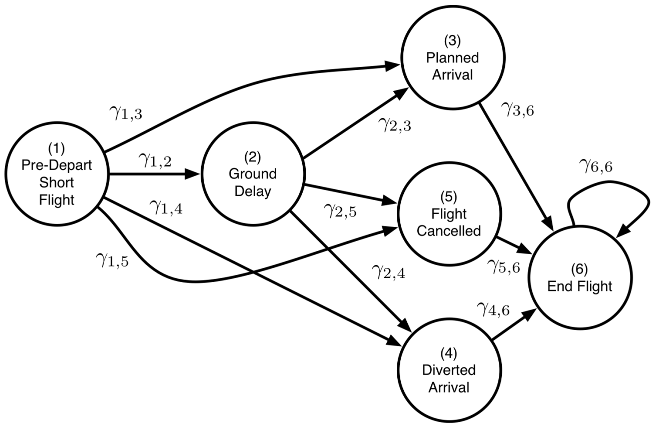

Fig. 1: State Transition Diagram For Diversion Mitigation

delaying their departure until the weather blockage has abated. Figure 1 provides the state transition diagram which models this simple diversion mitigation scenario. The model consists of six states which delineate phases that a flight may pass through: 1) `Pre-Departure`, 2) `Ground Delay`, 3) `Planned Arrival`, 4) `Diverted Arrival`, 5) `Flight Cancelled`, and 6) `End Flight`. State transitions in this model are based on considering a non-ideal diversion mitigation technology, which at times produces inaccurate forecasts. In order to describe the transition probabilities, we first consider the forecasted and actual probabilities of finding the arrival path open or closed. The random variable $X(t) \in \{\text{open, closed}\}$ gives the forecasted state of an arrival path at time t, and $Y(t)$ as the actual state of the path. We define the following probabilities:

$$\alpha_t = P(X(t) = \text{open}, Y(t) = \text{open}) \quad (9)$$
$$\alpha_f = P(X(t) = \text{open}, Y(t) = \text{closed}) \quad (10)$$
$$\rho_t = P(X(t) = \text{closed}, Y(t) = \text{closed}) \quad (11)$$
$$\rho_f = P(X(t) = \text{closed}, Y(t) = \text{open}) \quad (12)$$
$$\rho = \rho_t + \rho_f \quad (13)$$
$$\alpha = \alpha_t + \alpha_f = 1 - \rho \quad (14)$$

Employing these definitions we assume that a flight will successfully pass to its desired arrival destination without encountering weather blockage or being delayed with probability $\gamma_{1,3} = \alpha_t$. The probability that a flight is diverted due to a false open forecast is $\gamma_{1,4} = \alpha_f$. The parameter $\gamma_{1,5} = \epsilon$ is introduced to allow for flight cancellations resulting from operational decisions after receiving information about the state of the flight path. The event that a flight is delayed on the ground for an interval T_g when the diversion mitigation technology forecasts the arrival path state to be $X(t) = \text{closed}$, has probability of $\gamma_{1,2} = (\rho_t^* + \rho_f)$, where the quantity $\rho_t^* = \rho_t - \epsilon$. The probabilities of transitioning to the `End Flight` (including the self-transition) are all set to one, $\gamma_{3,6} = \gamma_{4,6} = \gamma_{5,6} = \gamma_{6,6} = 1$. The `End Flight` state is an absorbing state, which has been added to assist in terminating the Markov process, without the need to explicitly track the number of transitions. The quantity $\gamma_{2,3} = \beta_t$ defines the probability of successfully arriving at the intended destination after accepting a ground delay. The probability of a being diverted due to an erroneous forecast is represented by $\gamma_{2,4} = \beta_f$, after being delayed. Jointly these represent the probability, $\beta = \beta_t + \beta_f$, of a flight departing after experiencing a ground delay. The probability of a flight being cancelled after being delayed is $\gamma_{2,5} = \phi$. The probabilities of transitioning from the `Ground Delay` state depend on decision policies employed by operations control. Here we consider two policies: i) fixed delay policy and ii) adaptive delay policy.

A. Fixed Delay Policy

In this case we assume that for operational simplicity a fixed duration ground delay, T_g, is imposed on a flight when the diversion mitigation technology forecasts the path to be closed. Based on this policy we define probability ϕ of transitioning to the `Flight Cancelled` state as

$$\phi = P(X(t + T_g) = \text{closed} | X(t) = \text{closed}) \quad (15)$$

The probability of not cancelling the flight is given by $\beta = \beta_t + \beta_f$.

$$\beta = P(X(t + T_g) = \text{open} | X(t) = \text{closed}) \quad (16)$$

A concrete expression for ϕ and β can be obtained by assuming $X(t)$ to be an embedded two-state discrete Markov process with geometrically distributed open and closed periods[9]. We will address the accuracy of this assumption in section VI. The two-state process is defined by letting ϑ be the probability of transitioning from closed to open state, and φ be the probability of transitioning from open to closed state. The transitions take place at fixed intervals $\Delta t = t_n - t_{n-1}$, with t_n representing the time of n-th transition. The state probability vector $\boldsymbol{\zeta}^T(t_n)$ is defined as:

$$\boldsymbol{\zeta}^T(t_n) = [P(X(t_n) = \text{closed} \quad P(X(t_n) = \text{open})] \quad (17)$$

The state probability vector is calculated by the recurrence relation

$$\boldsymbol{\zeta}^T(t_n) = \boldsymbol{\zeta}^T(t_{n-1}) \begin{bmatrix} 1-\vartheta & \vartheta \\ \varphi & 1-\varphi \end{bmatrix} \quad (18)$$

The equilibrium distribution $\boldsymbol{\zeta}$, as $n \to \infty$, is given by

$$\boldsymbol{\zeta}^T(\infty) = [\rho \quad \alpha] = \frac{1}{\vartheta + \varphi}[\varphi \quad \vartheta] \quad (19)$$

If we allow T_g to be expressed as a multiple of discrete time intervals, $m_g(\Delta t)$, then $\phi(m_g)$ and $\beta(m_g)$ are given by

$$\phi(m_g) = \rho + \alpha(1 - \vartheta - \varphi)^{m_g} \quad (20)$$
$$\beta(m_g) = \alpha[1 - (1 - \vartheta - \varphi)^{m_g}] \quad (21)$$

In this context, the probability of a false forecast leading to a flight being diverted after the ground delay given by β_f can be obtained in terms of the joint probabilities α_t and α_f. Substituting $\alpha = \alpha_t + \alpha_f$ into the above equation, we get

$$\beta_f(m_g) = \alpha_f[1 - (1 - \vartheta - \varphi)^{m_g}] \quad (22)$$
$$\beta_t(m_g) = \alpha_t[1 - (1 - \vartheta - \varphi)^{m_g}] \quad (23)$$

B. Adaptive Delay Policy

In this case we assume that the diversion mitigation technology provides a forecast of flight path being closed, as well as the duration for which it is closed. The adaptive policy requires a flight to be delayed only for the duration that the path is closed. As a result we set the transition probability $\phi = 0$, and while allowing flight cancellations, characterized by the parameter ϵ, to occur as a result of initially observing the closure duration of the flight path. In accordance with the adaptive policy, the transition probabilities β_t and β_f are defined as:

$$\beta_t = P(Y(t) = \mathsf{open}|X(t) = \mathsf{open}) = \frac{\alpha_t}{\alpha_t + \alpha_f} \tag{24}$$

$$\beta_f = P(Y(t) = \mathsf{closed}|X(t) = \mathsf{open}) = \frac{\alpha_f}{\alpha_t + \alpha_f} \tag{25}$$

IV. Calculating Cost and Benefits

In this section we provide analytic expressions of costs and benefits for the diversion mitigation scenario described in section III. Based on the definitions given in section III we can construct the probability transition matrices for the cases in which the diversion mitigation technology is absent and present. The later case can be further divided into two cases – ideal and non-ideal forecasts. In the non-ideal case the diversion mitigation technology provides erroneous forecasts of path blockage with finite probability. The transition probability matrix for the non-ideal diversion mitigation scenario is:

$$\mathbf{P} = \begin{bmatrix} 0 & \rho^* & \alpha_t & \alpha_f & \epsilon & 0 \\ 0 & 0 & \beta_t & \beta_f & \phi & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{26}$$

The assessment of benefits derived from introducing the diversion management technology is performed relative to *base* and *ideal* scenarios. The *base* case lacks the diversion mitigation technology, while in the *ideal* case the technology is assumed to provide perfect forecasts of the arrival path state. These two scenarios are modeled by the following probability transition matrices:

$$\mathbf{P}_B/\mathbf{P}_I = \tag{27}$$
$$\begin{bmatrix} 0 & \boxed{0\ /\ (\alpha_f+\rho_t)} & (\alpha_t+\rho_f) & \boxed{(\alpha_f+\rho_t)\ /\ 0} & 0 & 0 \\ 0 & 0 & \boxed{0\ /\ 1} & 0 & \boxed{1\ /\ 0} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In this example the starting state probability vector is $\boldsymbol{\pi}_0^T = [1\ 0\ 0\ 0\ 0\ 0]$, and the (excess) cost vector is $\mathbf{c}^T = N_f[0\ (\eta_g T_g)\ 0\ (\eta_d T_d)\ C_{cf}\ 0]$. Here the cost varies linearly with the number of total flights considered N_f, and considers relevant factors such as the ground delay duration T_g and the additional delay imposed by a diversion T_d. The quantities η_g and η_d are the cost rates for ground delay and diversion respectively. The cost of a flight cancellation is taken as C_{cf}. Although this is a simple cost vector, the model can be extended to accommodate more general costs, as well as the dependency of cost on the state probability π_n, which is relevant when considering traffic volume dependent cost.

The substitution of the model parameters for this application into Eqn. 6 results in the following expressions for the base cost S_B, ideal cost S_I, and non-ideal cost S:

$$S_B = (\alpha_f + \rho_t) N_f \eta_d T_d \tag{28}$$

$$S_I = (\alpha_f + \rho_t) N_f \eta_g T_g \tag{29}$$

$$\begin{aligned} S = &(\rho_t + \rho_f - \epsilon) N_f \eta_g T_g \\ &+ [\alpha_f + (\rho_t + \rho_f - \epsilon)\beta_f] N_f \eta_d T_d \\ &+ [\epsilon + (\rho_t + \rho_f - \epsilon)\phi] N_f C_{cf} \end{aligned} \tag{30}$$

The cost improvement of employing the diversion mitigation technology is calculated as the difference between S_B and S:

$$\begin{aligned} \Delta S = &-(\rho_t + \rho_f - \epsilon) N_f \eta_g T_g \\ &- [\epsilon + (\rho_t + \rho_f - \epsilon)\phi] N_f C_{cf} \\ &+ [(1 - \beta_f)\rho_t - \beta_f(\rho_f - \epsilon)] N_f \eta_d T_d \end{aligned} \tag{31}$$

The first term in this expression discounts the cost of ground delay from the total benefit, while accounting for aircraft delayed erroneously through the probability ρ_f. The second term diminishes the overall benefit by the cost of flight cancellations. The third term yields the benefit of avoiding diversions, while accounting for false forecasts.

V. Incorporating Uncertainty

In section IV we have treated the development of the transition probabilities $\mathbf{P}$, the initial state probabilities $\boldsymbol{\pi}_0^T$, and the cost $\mathbf{c}$ as deterministic parameters. In reality all elements of each of these array quantities are not fixed. We expect transition probabilities characterized by ρ and α to vary due to regional differences in weather conditions. The quantities ρ_f and α_f are expected to vary from region to region due to implementation and adaptation differences in the deployed diversion mitigation technology. The costs of diversion, ground delay, and flight cancellation can be expected vary with airline carrier and between geographic regions. Here we have considered a simple example which only considers short duration flights that can employ the diversion mitigation technology resulting in a fixed value for $\boldsymbol{\pi}_0$. A more representative scenario would involve short and long duration flight, whose relative proportion would be different from region to region. We propose to characterize these variations by treating elements of $\mathbf{P}$, $\boldsymbol{\pi}_0^T$, and $\mathbf{c}^T$ are random quantities that can be drawn from the joint distributions: $\mathbf{f_P}(\mathbf{P})$ $\mathbf{f}_{\boldsymbol{\pi}_0}(\boldsymbol{\pi}_0)$, and $\mathbf{f_c}(\mathbf{c})$. Assuming that $\mathbf{P}$, $\boldsymbol{\pi}_0$ and $\mathbf{c}$ can be treated as independent random array variables, the expected value of the total cost is given as

$$\mathsf{E}[S] = \mathsf{E}[\boldsymbol{\pi}_0^T] \left[\mathsf{E}[\sum_n \mathbf{P}^n] \right] \mathsf{E}[\mathbf{c}] \tag{32}$$

The variance in the cost can be written as:

$$\begin{aligned}\mathsf{var}(S) &= \mathsf{E}[(\boldsymbol{\pi}_0^T \sum_n \mathbf{P}^n \mathbf{c})^2] - (\mathsf{E}[S])^2 \\ &= \mathsf{E}[\mathbf{c}^T \sum_n (\mathbf{P}^T)^n \boldsymbol{\pi}_0 \boldsymbol{\pi}_0^T \sum_n (\mathbf{P})^n \mathbf{c}] - (\mathsf{E}[S])^2\end{aligned} \tag{33}$$

In order to characterize the distribution of costs and benefits, as well as computing the expected value and variance, we describe how uncertainty arises in our application scenario. Some of the state transitions in our scenario are an outcome of decisions based on information provided to actors controlling the flight. The decision information in our application is provided by a diversion mitigation technology, which forecasts the impact of weather along the flight path. Errors in forecasting are represented by the quantities ρ_f and α_f. We expect that errors produced by geographically disparate deployments of the technology to be distributed over a range of values. As result we treat ρ_f and α_f as random quantities, and assume them to be independent. For ease of analysis we will draw ρ_f and α_f from the uniform distributions $\mathcal{U}(\rho_{f,\min}, \rho_{f,\max})$ and $\mathcal{U}(\alpha_{f,\min}, \alpha_{f,\max})$ respectively. The actual probability of the flight path being closed is represented by $q = P(Y(t) = \mathsf{closed})$. Since the likelihood of finding a flight path closed due to weather is expected to vary between geographic regions, q is treated as an independent random variable described by the uniform distribution $\mathcal{U}(q_{\min}, q_{\max})$. The parameter ϵ may also be allowed to vary randomly with the distribution $\mathcal{U}(\epsilon_{\min}, \epsilon_{\max})$, to capture user decisions on cancelling a flight. We note that in order to maintain conservation of probability the maximum and minimum values for $q, \alpha_f, \rho_f, \epsilon$ cannot be selected independently. If $\alpha_{f,\min}$ and $\rho_{f,\min}$ are selected to be zero, then the inequalities $q_{\min} \geq \alpha_{f,\max}$, $q_{\max} \leq (1 - \rho_{f,\max})$, $\epsilon_{\min} \geq 0$ and $\epsilon_{\max} \leq q_{\max}$, must be satisfied. Employing the definitions given in Eqn. 9, the quantities α_t and ρ_t can be written as linear combinations of the independent random quantities q, α_f, and ρ_f.

$$\alpha_t = 1 - q - \rho_f \tag{34}$$

$$\rho_t = q - \alpha_f \tag{35}$$

The quantities β_t, β_f, and ϕ are also dependent on q, α_f, and ρ_f, through the expressions defined for the fixed and adaptive delay policies in section III.

The duration of ground delay, T_g, is another parameter that impacts the cost. For the fixed delay policy T_g is given as a deterministic value. However, for the adaptive policy, the duration T_g depends on the remaining duration of forecasted weather blockage, conditioned on being in $X(t) = \mathsf{closed}$ state. Here we assume that the diversion mitigation technology provides forecasts at discrete intervals $t_k = (k\Delta t)$, and the ground delay is also measured as a multiple of (Δt). In order to derive the probability distribution for $T_g = (m\Delta t)$, we first wish to characterize the probability of a path remaining closed for a time interval $(m\Delta t)$ in the future, given it is currently closed. Such an event, conditioned on the total duration of blockage $(b\Delta t)$, is given as

$$\begin{aligned}\psi_{m|c,b} = \{X(t_k \leq\, & t < t_k + m\Delta t) = \mathsf{closed} \\ & |X(t_k) = \mathsf{closed}, B = b\}\end{aligned} \tag{36}$$

The quantity B is the random variable which characterizes the duration of a blockage event. The probability of this event is defined as:

$$P(\psi_{m|c,b}) = \begin{cases} \dfrac{b-m+1}{b} & b \geq m, m \geq 1 \\ 0 & else \end{cases} \tag{37}$$

The probability of a path remaining closed for duration m in the future is given by

$$P(\psi_{m|c}) = \sum_b P(\psi_{m|c,b}) P(B = b) \tag{38}$$

The probability distribution $P(B = b)$ may be obtained experimentally, as presented in section VII. In which case the probability distribution $P(M = m)$ is given as

$$P(M = m) = \frac{P(\psi_{m|c})}{\sum_k P(\psi_{k|c})} \tag{39}$$

If we assume the blockage durations to be derived from a two-state Markov process, as given in section III, then $P(B = b)$ is expressed as:

$$P(B = b) = \vartheta(1 - \vartheta)^{b-1} \tag{40}$$

The probability of a path remaining closed for a duration m is in this case given by

$$P(\psi_{m|c}) = (1-\vartheta)^{m-1} - (m-1)\frac{\vartheta}{1-\vartheta} \int \frac{(1-\vartheta)^{m-1}}{\vartheta} d\vartheta \tag{41}$$

VI. Estimating Model Probabilities

In this section we will characterize the model probabilities using historical data from a planned operational system. The Route Availability Planning Tool (RAPT) system provides decision support to air traffic managers on the impact of weather on flight operations [10]. This system forecasts the blockage state of a set of flight paths near an airport in $\Delta t = 5$ minute increments. This system has been prototyped at the New York Area and Chicago airports, during live operations. It is currently being integrated into the FAA's Traffic Flow Management System (TFMS) for more widespread deployment within the National Airspace System (NAS). Figure 2 illustrates the user display for RAPT in the New York area. RAPT assigns a status color to indicate the severity of weather impact on a flight path: RED (blocked), YELLOW (moderate weather), DARK GREEN (light weather), GREEN (clear). RAPT is capable of forecasting the impact of weather on each route up to 30 minutes into the future. The status is determined by combining the deterministic precipitation and echo-top forecasts from the Corridor Integrated Weather System (CIWS) with a route blockage algorithm that incorporates a model for departure airspace usage. The RAPT system employs historical data to determine a set of median path trajectories employed by flights for a particular airport. The forecasted weather impact is computed by examining the intersection of the forecasted weather field over a flight path segment that is visited by the flight based on a predetermined speed profile. The CIWS precipitation intensity forecast is presented to the user over a weather map for the region of interest, as well as a set of timelines for individual flight paths (rows) with whose status is given at discrete time intervals of 5 minutes (columns). In

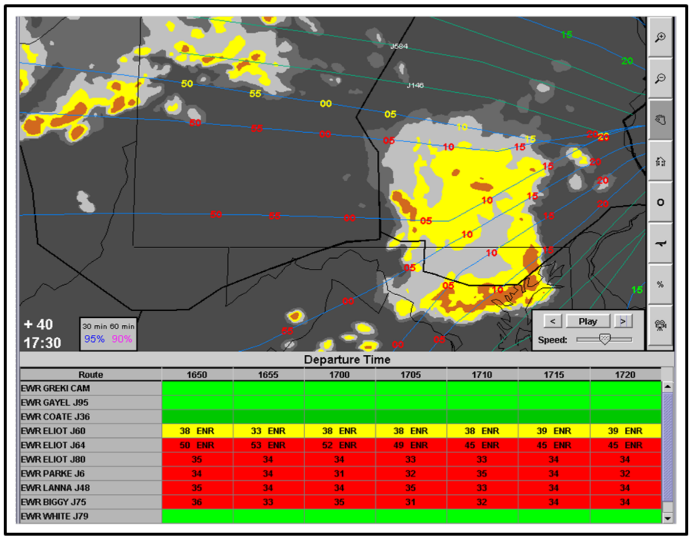

Fig. 2: User interface for Route Availability Planning Tool (RAPT)

addition to the forecasted status of blockage on flight paths, the RAPT system also maintains the truth status of weather impact in order to assess the errors in the forecasted product.

The computation of model probabilities employs the forecasted and truth status data generated while operating the prototype RAPT system in the New York and Chicago regions. The model probabilities are based on a set of 31 weather impacted days, of which 17 days are from the New York area and 14 days from Chicago. The selected days include significant convective weather impacts during the time period of 0900Z to 0000Z. The cumulative set of data over the selected days and the regional flight paths consists of 33,660 samples from New York and 70,560 samples from Chicago.

In order to compute the model probabilities we first map the RAPT status as: $[\{\text{GREEN}, \text{DARK GREEN}, \text{YELLOW}\} \rightarrow \text{open})]$ and $[\{\text{RED}\} \rightarrow \text{closed}]$. The probabilities $(\alpha_t, \alpha_f, \rho_t, \rho_f)$ are calculated by counting the different events enumerated by $(X(t_k) \in \{\text{closed}, \text{open}\}, Y(t_k) \in \{\text{closed}, \text{open}\})$, and normalizing by the total count. The probability q is calculated as the sum of α_f and ρ_t. The transition probabilities for the two-state path blockage process as computed from considering the one step forecast events $(X(t_k), X(t_{k+1}))$.

$$\vartheta = \frac{|\{X(t_k) = \text{closed}, X(t_{k+1}) = \text{open}\}|}{|\{X(t_k) = \text{closed}\}|}$$
$$\varphi = \frac{|\{X(t_k) = \text{open}, X(t_{k+1}) = \text{closed}\}|}{|\{X(t_k) = \text{open}\}|} \tag{42}$$

The results of these calculations are summarized in Table I.

Region	α_t	α_f	ρ_t	ρ_f	q	ϑ	φ
New York	0.8299	0.0343	0.1195	0.0163	0.1538	0.1	0.017
Chicago	0.9142	0.0115	0.0628	0.0115	0.0743	0.1118	0.009

TABLE I: Empirically derived path blockage and technology performance probabilities

The computation of distributions for path blocked and available durations are given by considering the events

$$\begin{aligned} U_B(b) = \{&X(t_{k-1}) = \text{open}, X(t_k) = \text{closed}, \\ &\ldots, X(t_{k+b-1}) = \text{closed}, X(t_{k+b}) = \text{open}\} \\ U_A(a) = \{&X(t_{k-1}) = \text{closed}, X(t_k) = \text{open}, \\ &\ldots, X(t_{k+a-1}) = \text{open}, X(t_{k+a}) = \text{closed}\} \end{aligned} \tag{43}$$

The probability distribution are given as:

$$P(B = b) = \frac{|U_B(b)|}{\sum\limits_b |U_B(b)|}$$
$$P(A = a) = \frac{|U_A(a)|}{\sum\limits_a |U_A(a)|} \tag{44}$$

The probability distributions $P(B = b)$ and $P(A = a)$ for the New York region are shown in Figures 3 and 4 respectively In these plots the symbol "o" curves represent the distributions computed from empirical data, and the solid lines represent a least squares fit to a geometric distribution. Figure 3 illustrates that a geometric distribution serves as a reasonable fit for the probability of closed durations. The empirical distribution of path open durations, as shown in Figure 4, deviates significantly from a geometric distribution. The aggregate decay of the empirical distribution is however captured by a geometric model.

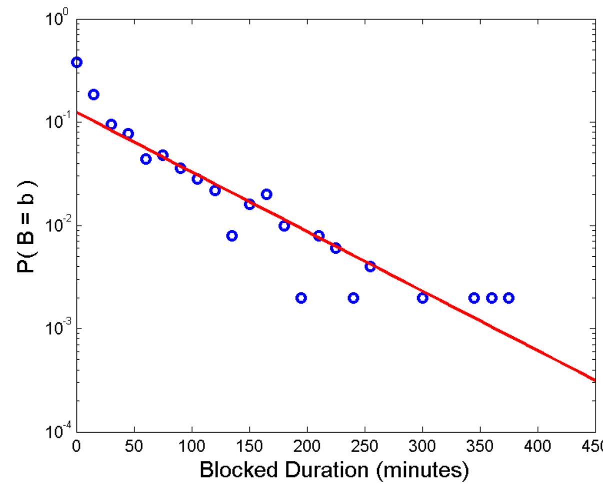

Fig. 3: Distribution of $X(t) = \text{closed}$ durations for New York region

The expression for calculating the distribution of residual duration of a blockage event, conditioned on being in a blockage event was given by Eqn. 39. Using the results for $P(B = b)$, the distribution $P(M = m)$ is calculated. Figure 5 shows the empirically derived distribution for the New York region using "o" symbols, and the solid line is the least-squares fit to a geometric model, with mean $\bar{m} = 40.16$. The expected values for m in the New York and Chicago regions, calculated directly from the empirical distribution, are found to be $\bar{m}_{\text{NY}} = 32.96$ and $\bar{m}_{\text{CHI}} = 32.75$ respectively.

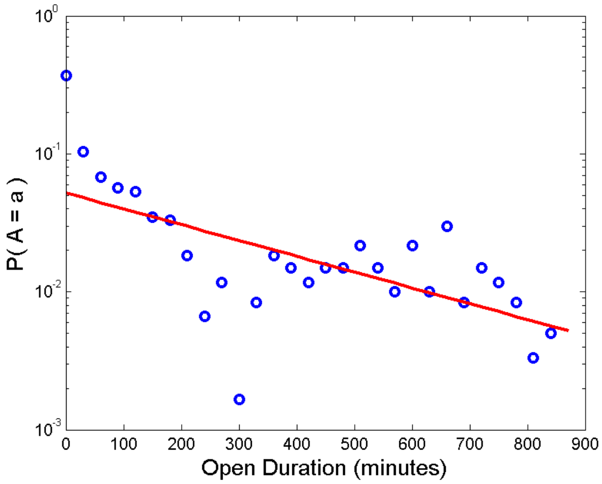

Fig. 4: Distribution of $X(t)$ = open durations for New York region

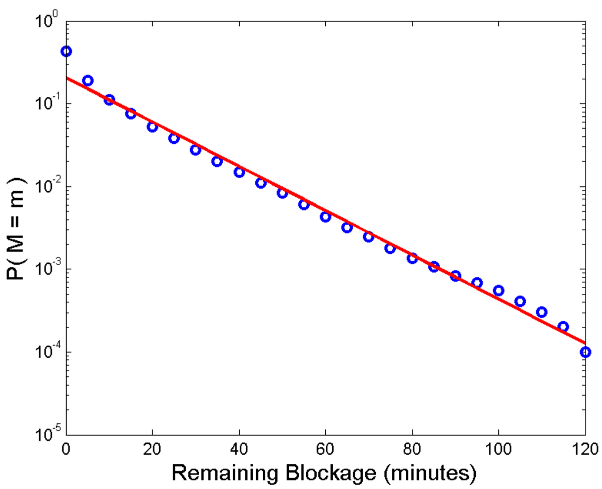

Fig. 5: $P(M = m)$ for New York region

VII. Simulation Results

In this section we will examine the statistics of costs and benefits as we vary parameters related to the proposed Markov model. The values for the parameters associated with flight path blockage and diversion mitigation technology performance are listed in Table II. The three case studies considered examine the effects of increasing the mean and variance of forecast errors α_f and ρ_f and the range of q. In addition we assume that the parameters of the embedded two-state process defining the path blockage vary such that their sum, $(\vartheta + \varphi)$, remains fixed. We justify this assumption by noting that the sum of these parameters for New York and Chicago regions is nearly the same. Maintaining the sum $(\vartheta + \varphi)$ to be a constant also ensures that the characteristic decay rate of the correlation of the forecasts in time is invariant.

The average input costs used in the simulation of benefits are given in Table III. In order to capture the effect of variation in cost, we will allow each input cost to vary uniformly by ± 10 percent.

The results for benefits for the three cases identified in Table II, assuming the fixed delay policy, are presented in Figures 6-8. In each case distributions of benefits are plotted

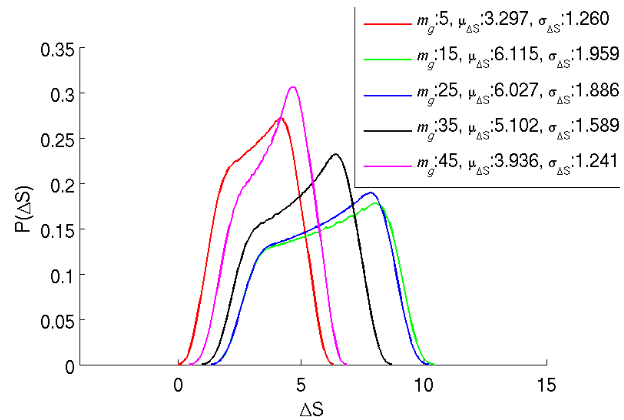

Fig. 6: Probability distribution of ΔS for case I

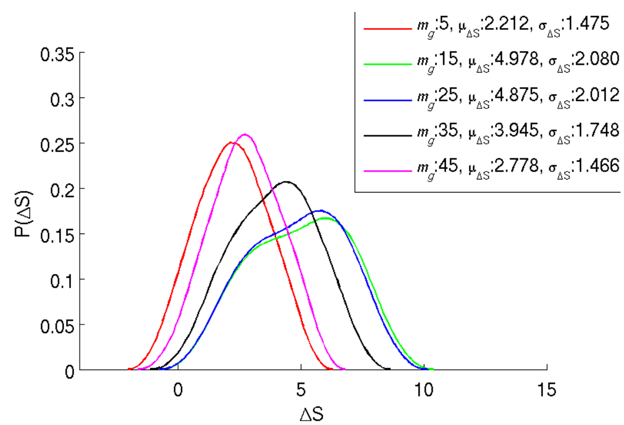

Fig. 7: Probability distribution of ΔS for case II

for $\{m_g : 5 \text{ - } 45\}$. These results show distributions that are a significant departure from the uniform distributions of the input parameters. The results of each of the three cases show the greatest cost benefit for values of $m_g = 15$, hence providing a mechanism for selection of $T_g = (m_g \Delta t)$. Increasing m_g beyond this value, results in a degradation in the benefit due

Case	$\alpha_{f,\min}$	$\alpha_{f,\max}$	$\rho_{f,\min}$	$\rho_{f,\max}$	$q_{\min}$	$q_{\max}$	$\epsilon_{\min}$	$\epsilon_{\max}$
I	0.0	0.02	0.0	0.02	0.06	0.2	0.0	0.01
II	0.0	0.04	0.0	0.04	0.06	0.2	0.0	0.01
III	0.0	0.04	0.0	0.04	0.08	0.5	0.0	0.01

TABLE II: Path blockage and diversion mitigation technology performance parameters

$< N_f \eta_d T_d >$	$< N_f \eta_g \Delta t >$	$< N_f C_{cf} >$
100	1	100

TABLE III: Average input costs

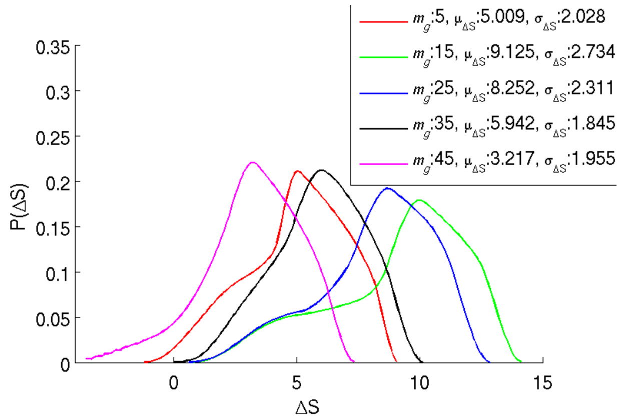

Fig. 8: Probability distribution of ΔS for case III

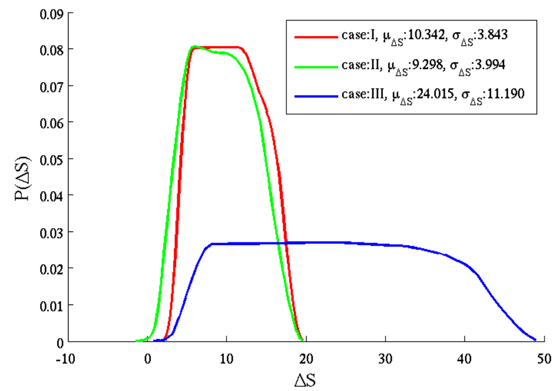

Fig. 9: Probability distribution of ΔS for adaptive delay policy, cases I-III

the increased cost of delay. On the other hand, for values of m_g below 15, the benefit degrades due to increased likelihood of a cancellation by attempting to depart while the flight path is still blocked. The shift in the distributions of benefits with m_g show a greater sensitivity to loss of benefits when $m_g < 15$, while the rate of benefits loss is lower for $m_g > 15$.

The expected base cost is invariant in m_g and given by a constant average value of $(\alpha_f + \rho_t)N_f\eta_d T_d$. The dominant terms in S that exhibit change with m_g are $Q_\beta = (\rho - \epsilon)\beta_t N_f \eta_g T_g$ and $Q_\phi = (\rho - \epsilon)\phi[N_f \eta_g T_g + N_f C_{cf}]$ with β_t and ϕ increasing and decreasing respectively with m_g. For small values of m_g, $Q_\phi >> Q_\beta$, resulting in a low expected value of $\Delta S = S_B - S$. With increases in m_g, Q_ϕ decreases, converging to a constant value, whereas Q_β increases to a comparable value. These features, lead to a maximum in the expected value and variance of ΔS in the region around $m_g = 15$.

The benefits distributions for case II shown in Figure 7 capture the impact of increasing $\alpha_{f,\text{max}}$ and $\rho_{f,\text{max}}$ by a factor of two. In comparison to case I, the reduction in the average benefit for $m_g = 15$ in case II, with greater forecast inaccuracy, is approximately 18%, without appreciable change in the variance. In operational terms, the reduction in benefit is a result of increased likelihood of diversion and unnecessary ground delay due to forecasting errors. This type of information can be relevant when assessing the return on investment in improving the diversion mitigation technology to provide greater forecast accuracy. For other values of m_g there are greater differences in the distributions for case I and II.

The results for case III are shown in Figure 8. Here we have increased the range over which the distribution of q varies, thus resulting in greater likelihood of path blockages. A broadening of the distribution for q yields a corresponding broadening of the benefits distribution. In addition, the distribution exhibits a negative skew, with the mass of the distribution favoring high benefit values. The results also show a shift in the sensitivity to a greater loss of benefits for values of $m_g > 15$.

Results for the adaptive policy for each of the three cases in Table II are presented in Figure 9. For the adaptive delay policy, m is drawn from the empirical distribution presented in section V. In general, the distribution of benefits for each case deviates slightly from a uniform distribution. The statistics for each case show a mean and variance that is larger than the corresponding values for the fixed delay policy. In particular, the mean benefit for the adaptive policy exhibits increases of 72%, 87%, and 162% over the $mg = 15$ fixed policy cases I,II and III, respectively. The decisions to enter a ground delay in the adaptive delay case exploit the forecasted duration of the path blockage in order to depart as soon as the blockage has abated. This allows for minimizing the cost of ground delays, as well as reducing the likelihood of cancellations as seen with the fixed delay policy. It is also apparent from case III that the likelihood of blockage has a large effect on the benefit of the tool, especially for the adaptive delay policy. This point highlights the need for accurate weather forecasts in the NAS so that flight-specific traffic management can be enabled. A comparison of case I and II distributions for the adaptive policy case also show a resilience of benefits to an increase in forecast errors, as was found for the fixed delay policy.

VIII. Conclusions and Future Work

In this paper we have developed an approach for cost and benefits modeling when introducing a new air traffic management technology into operations. This approach is capable of incorporating uncertainty in factors that influence flight operations and air traffic decision processes, without performing high-fidelity simulations or experiments. We have applied this approach to a candidate diversion management technology. In this context, we have described the impact uncertainty has on the statistics of costs and benefits. We have also compared the relative value of different operational procedures (fixed or adaptive ground delay), when employed in concert with the diversion management technology. Although, the technology example we have analyzed is limited, the approach presented is generally applicable. We expect to employ this approach for understanding benefits for more complicated operational scenarios, as well as for comparison of alternate technologies.

Acknowledgments

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Nomenclature

$\mathbf{c}$	Cost vector
C_{cf}	Cost of flight cancellation
η_g	Cost rate of ground delay
η_d	Cost rate of diversion
N_f	Total number of flights
T_d	Additional delay due to diversion
T_g	Duration of ground delay
Δt	Measurement and forecast time intervals
m_g	Duration of ground delay in discrete number of Δt intervals
$\mathbf{P}$	Transition probability matrix
$\boldsymbol{\pi}$	State probability vector
S	Total cost
$X(t)$	Forecasted path blockage state
$Y(t)$	Actual path blockage state
ΔS	Total benefit
θ_n	Incremental cost
α_f	False forecast probability of path open
α_t	True forecast probability of path open
α	Forecast probability of path open
ρ_f	False forecast probability of path closed
ρ_t	True forecast probability of path closed
ρ	Forecast probability of path closed
β_f	False forecast probability of path open during T_g if presently closed
β_t	True forecast probability of path open during T_g if presently closed
β	Forecast probability of path open during T_g if presently closed
ϕ	Forecast probability of path closed during T_g if presently closed
ϵ	Probability of flight cancellation
q	Actual probability of path being closed
$\boldsymbol{\zeta}$	State probability vector for embedded path blockage process
ϑ	Probability of closed to open transitions
φ	Probability of open to closed transitions
B, b	Duration of blockage or closed period
A, a	Duration of available or open period
M, m	Remaining duration in blockage or closed period

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