Modeling Delivery Accuracy for Metering Operations to Support RNAV Arrivals

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***Abstract*—Interval Management (IM) is a future airborne spacing concept that aims to provide more precise inter-aircraft spacing to yield throughput improvements and greater use of fuel-efficient trajectories in arrival and approach environments. To participate in an IM operation, an aircraft must be equipped with avionics that provide speeds to achieve and maintain a desired spacing interval relative to another aircraft. It is not expected that all aircraft will be equipped with the necessary avionics, but rather that IM fits into a larger arrival management concept developed to support the broader mixed-equipage environment. Arrival management concepts are comprised of three parts: a ground-based sequencing and scheduling function to develop an overall arrival strategy, ground-based tools to support the management of aircraft to that schedule, and the IM tools necessary for the IM operation (i.e., ground-based set-up, initiation, and monitoring, and the flight-deck tools to conduct the IM operation). The Federal Aviation Administration is deploying a near-term ground-automation system to support metering operations in the National Airspace System, which falls within the first two components of the arrival management concept. The near-term system will include sequencing and scheduling functions and tools to help air traffic controllers in managing aircraft to meet their scheduled times of arrival (STAs) at meter points. This paper presents a methodology for determining the required delivery accuracy at the meter points in order to achieve desired flow rates, adequate separation at the meter points, and to enable aircraft to meet their STAs while remaining on their RNAV arrivals, which will reduce costly vectoring and holding. An example based on operations at Phoenix airport is presented to illustrate the analysis framework in a real-world context.**

Keywords*—*Interval Management, Metering, Delivery Accuracy, Optimal Profile Descent, NextGen.

I. Introduction

Interval Management (IM) is a NextGen concept that will enable more precise spacing between aircraft through the use of flight-deck avionics that aid a flight crew in achieving and maintaining a desired spacing interval relative to an assigned Target Aircraft. More precise inter-aircraft spacing is expected to enable throughput increases by reducing uncertainties which prevent air traffic controllers (ATC) from spacing aircraft more closely together. The use of IM may also enable greater opportunities for fuel-efficient trajectories, or Optimal Profile Descents (OPDs), by more precisely spacing aircraft prior to top of descent (TOD) and allowing an aircraft to manage its spacing relative to a Target Aircraft during the descent. Whereas IM operations are envisioned in a variety of environments where more precise spacing aids ATC in meeting their airspace objectives, much of the past IM-related research has focused on arrival and approach applications [1]–[3].

For the foreseeable future, it is not expected that all aircraft will be equipped with the ADS-B IN avionics to conduct IM. Furthermore, an aircraft must broadcast ADS-B OUT to act as a Target Aircraft. Prior to the ADS-B-equipage mandate in 2020, the number of qualified Target Aircraft will be a function of ADS-B equipage rates and will limit the opportunity to conduct IM operations. Therefore, any arrival management concept will need to address mixed-equipage operations [4].

Future arrival management concepts involving IM will be comprised of scheduling, ground-based, and flight-deck based capabilities [5]. The scheduling function develops an overall sequence and schedule of aircraft into the terminal area. Ground-based functions support the management of aircraft to the schedule and the initiation of IM operations as appropriate. Upon initiation of the IM operation, the ADS-B IN avionics helps the flight crew to achieve a relative spacing interval from its preceding aircraft in the sequence.

The Federal Aviation Administration (FAA) is currently developing an IM Concept of Operations (ConOps), IM avionics standards (jointly with EUROCONTROL), and ground-automation requirements to support the set-up and initiation of IM operations. In the near-term, the FAA is deploying an initial ground system that will support the management of aircraft to their Scheduled Times of Arrival (STAs) at flow-constrained points. The ground automation used to manage aircraft to their STAs includes trajectory modeling, scheduling and sequencing functions, calculation of speed advisories to meet the schedule, and controller displays. These new ground-based automation

functions are expected to help controllers increase the use of RNAV arrivals, including OPDs, by pre-conditioning the flow of aircraft prior to the TOD and increasing the delivery accuracy at en-route meter points and at the terminal meter fix. It is also expected that this ground functionality can be used to support the pre-conditioning of aircraft equipped for IM operations in the future.

Future ground-system deployments will include functionality to help the controller to identify candidate aircraft pairs for an IM operation, information to the controller to initiate the IM operation (e.g., the desired spacing interval), tools to help the controller monitor the relative spacing, and status information on whether an aircraft is actively conducting an IM operation.

Performance of arrival management systems has been analyzed in various contexts. In reference [6], the authors analyzed trajectory uncertainties to determine the minimum targeted spacing at the terminal meter fix on the en-route/terminal boundary that allows aircraft to continue their RNAV arrivals to the runway without controller intervention. In that work, the aircraft were metered to the terminal meter fix by conventional means, and errors in the inter-arrival time were modeled. That methodology was used to establish inter-arrival spacing for flight tests conducted at Louisville International Airport (SDF), and in reference [7], the performance predicted by the model was shown to agree well with measured performance. The delivery accuracy to STAs for saturated metering operations at Hartsfield-Jackson Atlanta International (ATL) operations was analyzed in reference [8]. With a similar motivation to the research in reference [6], the authors determined the delivery accuracy at the meter fix required to achieve certain efficient operations (i.e., no vectoring or extended final) in the terminal area. Finally, a tool was developed and applied in reference [9], which combined the effects of metering to the terminal meter fix and the runway, scheduled delay, and controller intervention rates to optimize the performance of a scheduler for the terminal area.

This paper adds to the analysis of arrival management performance by establishing a simple model to analyze the operational objectives of the initial ground system and the planned operations. Impacting ground automation performance in the overall decomposition of arrival management, this paper ties together traffic density, the determination of STAs, and the accuracy of delivering to those STAs. This modeling is envisioned to establish a baseline for arrival management performance analyses for near-term and future operations.

The paper is organized as follows. The initial ground system and the associated operations are described in more detail in Section II. The main results of the paper are presented in Section III, where key quantities are defined, the modeling approach and design objectives are presented, and finally, a numerical example is used to illustrate the analysis framework in a realistic operational context. Conclusions and future work are presented in Section IV.

II. Description of the Initial Ground System

The initial ground automation improvements to support the metering operations into the terminal area will be implemented on the Time-Based Flow Management (TBFM) automation platform, which currently includes trajectory modeling, scheduling, and controller display functions [10]. The near-term concept will leverage the Extended Metering concept and the provision of speed advisories to help controllers increase the use of OPDs by allowing aircraft to stay on their RNAV arrivals without vectoring and by increasing the delivery accuracy at constraint satisfaction points (CSPs). CSPs may be en-route meter points, En-route Flow Management Points (ERFMPs), or the Arrival Flow Management Point (AFMP) at the terminal boundary [5]. After an aircraft crosses a “freeze horizon” associated with a CSP, the sequence and schedule is frozen at that CSP, and speed advisories will be provided to the controller such that the aircraft will meet the frozen STA within some tolerance. Figure 1 illustrates two ERFMPs and the AFMP and their associated freeze horizons.

The Extended Metering concept enables the metering operation to begin hundreds of miles from the AFMP. TBFM Coupled Scheduling (TCS), the automation functionality upon which the Extended Metering concept is to be built, determines STAs at ERFMPs and the AFMP with delays distributed between the CSPs to ensure acceptable flow rates to the terminal area and adequate separation at each CSP [11][1]. By starting the metering operation farther from the AFMP, the controller is better able to meet the STAs using a speed solution alone; i.e., fuel-inefficient solutions, such as vectoring or holding patterns, are less frequent. Furthermore, the flow of aircraft will be pre-conditioned prior to TOD, allowing for more frequent use of OPDs without the controller intervening in the procedure due to separation concerns.

Speed advisories will also assist the controller in meeting the frozen STAs by calculating a precise speed, based on the ground-derived Estimated Time of Arrival (ETA) for the aircraft at the CSP. When the aircraft is going to the AFMP, the speed advisory algorithm is designed to support the use of OPDs by calculating both a cruise speed and a descent speed that will meet the STA [12]. Speed advisories are calculated by the ground automation and are provided to the controller on the en-route displays. If the speed advisory is acceptable to the controller, the controller will provide the speed clearance to the flight crew via voice communications.

The key functions in the ground automation are: trajectory modeling, sequencing and scheduling, and schedule problem prediction and resolution. The trajectory-modeling function calculates an ETA at the ERFMP or the AFMP for each aircraft each time new surveillance data is received. Prior to a freeze horizon, the ETAs are used as input to the sequencing and scheduling function to determine a de-conflicted schedule at the CSP. Once the aircraft has passed the freeze horizon

[1] Scheduled delays have the effect of changing the expected flight time between two CSPs. The maximum delays that may be applied to an aircraft’s schedule are known by the automation system, and delays up to the maximum delay are applied to the next STA in order to prevent conflicts at the CSP.

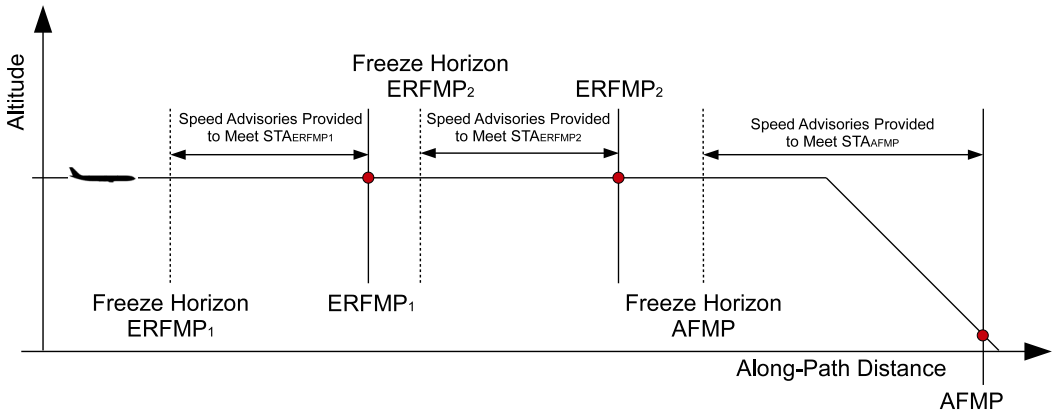

Fig. 1. ILLUSTRATION OF EXTENDED METERING WITH TWO EN-ROUTE METER POINTS AND A TERMINAL METER FIX.

and the schedule has been frozen, speed advisories will be generated by the schedule problem prediction and resolution function if the difference in the aircraft's ETA and the frozen STA exceeds a specified threshold. The speed advisory is a function of the aircraft's ETA at the CSP and is designed to deliver the aircraft to the CSP at the frozen STA.

The objective of this paper is to determine how accurately the near-term ground automation system should deliver aircraft to their assigned STAs at the ERFMP and AFMP in order for aircraft to remain on their RNAV arrival routes, and thus conduct OPDs in mid- to high-traffic densities. In doing so, limitations on delay allocation are revealed and future analysis is identified.

III. Main Results

In this section the main results of the modeling and analysis are presented. The ability for an aircraft to remain on its RNAV arrival procedure between the ERFMP and AFMP is modeled by the satisfaction of two operational constraints. In modeling these constraints, performance parameters are identified and related, such as the delivery accuracy to each CSP, the delay allocated between the CSPs, the separation minimum, and the Airport Arrival Rate (AAR).

A. Definitions

The delivery error to a CSP is defined to be the difference between the actual arrival time of an aircraft at a CSP and the frozen STA determined by the ground scheduling function for that CSP. In particular, if ATA_{CSP} is the actual time of arrival at the meter point, and STA_{CSP} is the scheduled time, then the delivery error is defined as

$$\epsilon_{CSP} = ATA_{CSP} - STA_{CSP}. \quad (1)$$

Late arrival to the CSP is a positive error, and early arrival is negative.

The delivery error can also be conceptualized as a position error. Define the distance-based delivery error as the distance-to-go to the meter point at the time $t = STA_{CSP}$. Note that with these definitions, the sign of the errors in distance

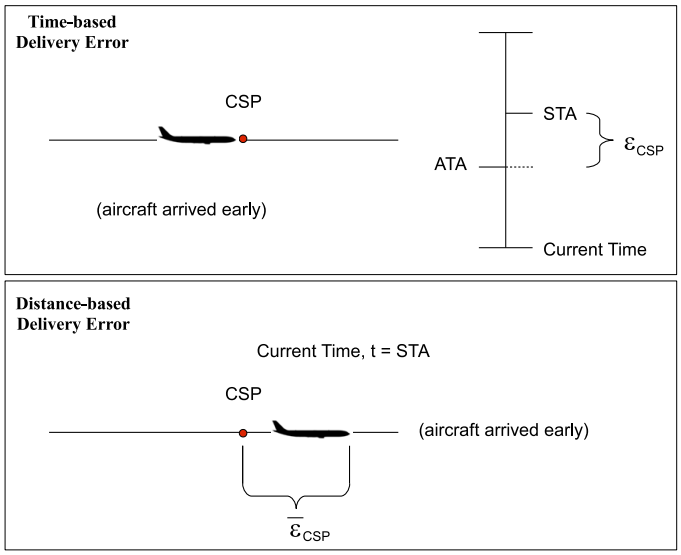

Fig. 2. TIME- AND DISTANCE-BASED DELIVERY ERRORS.

corresponds with those in time – when the aircraft is delivered late to the meter point, the delivery error is positive as there is still a positive distance to go at time $t = STA_{CSP}$. The distance-based delivery error is denoted as $\bar{\epsilon}_{CSP}$.

Over the population of aircraft, the delivery error is a random variable whose stochastic behavior depends on a number of factors, including both the performance of the ground system as well as environmental effects. For this paper, the delivery accuracy is defined as the standard deviation of that distribution. For simplicity, it is assumed that there are no biases in the system, and that the delivery accuracy follows a Gaussian distribution with zero mean (i.e., the average arrival time is the STA itself). The variance about the mean can be used as an operational requirement to drive system design, and may be used to determine whether the system meets its stated objectives.

B. Modeling GIM Delivery Errors

The operational objective analyzed for the initial ground system is to ensure a reasonable probability of remaining on the RNAV arrival procedure (i.e., no vectoring to lengthen the aircraft's path) in traffic density η without intervention other than speed control between the ERFMP and AFMP. This constrains the delivery accuracy at the ERFMP and the AFMP in two ways. An aircraft can only remain on its RNAV arrival path if both of the following hold:

- successive aircraft are conflict-free at each meter point with a given probability; and
- the aircraft can fly the trajectory constrained by the STAs without the need for path-length adjustments with a given probability.

In the first case, for a given traffic density η, there is a minimum amount of uncertainty at a CSP such that the probability of a separation infringement at that point is sufficiently low. In the second case, delivery errors to the ERFMP and AFMP can be seen as equivalent to modifications of the trajectory time, similar to delay. Many RNAV arrivals are designed for OPDs between the ERFMP and AFMP, and limited delay absorption is a characteristic of OPDs. One conclusion of this analysis is that the scheduler must not over-allocate delay between these two CSPs.

C. Constraint 1: Conflict-Free Delivery at the Meter Point

The first constraint is modeled by determining a minimum targeted spacing interval s_{tar} (in time) between STAs for any lead and trail aircraft across a CSP. At a CSP, if the lead and trail aircraft have STAs, STA^l_{CSP} and STA^t_{CSP}, respectively, such that $s_{tar} = STA^t_{CSP} - STA^l_{CSP}$, then the actual spacing S between the aircraft is as shown.

$$\begin{aligned} S &= ATA^t_{CSP} - ATA^l_{CSP} \\ &= \left(STA^t_{CSP} + \epsilon^t_{CSP}\right) - \left(STA^l_{CSP} + \epsilon^l_{CSP}\right) \quad (2) \\ &= s_{tar} + \epsilon^t_{CSP} - \epsilon^l_{CSP} \quad (3) \end{aligned}$$

This relationship is illustrated in Figure 3, where M is the applicable separation minimum, converted to a time-based value.

For a fixed value of s_{tar}, there is a statistical behavior of the delivery errors of lead and trail which ensures that the actual spacing is greater than the separation minimum M with some probability.

Using the relationship in (3), the probability of violating the minimum separation, M, at the CSP should be less than the parameter ω:

$$Pr\left(s_{tar} + \epsilon^t_{CSP} - \epsilon^l_{CSP} < M\right) < \omega. \quad (4)$$

Here, the applicable separation minimum M is assumed to be 5 nautical miles (nmi) for simplicity and is converted to time by dividing by the appropriate groundspeed. The AFMP is assumed to be the most flow-constrained CSP. That is, speeds and spacing at the ERFMP are such that separation issues are not as much of a concern as at the AFMP, where the traffic

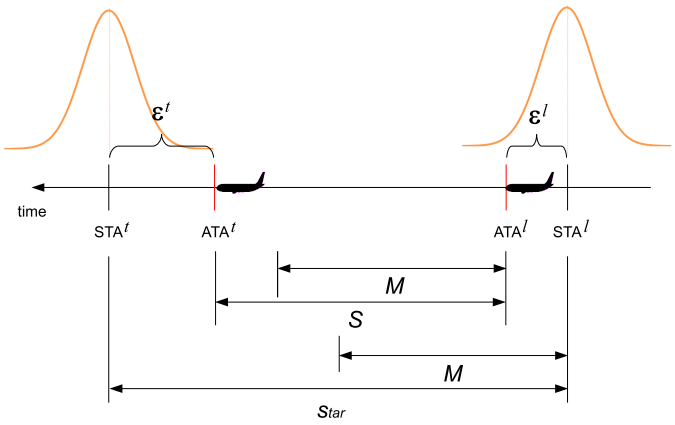

Fig. 3. DEPICTION OF CONSTRAINT 1.

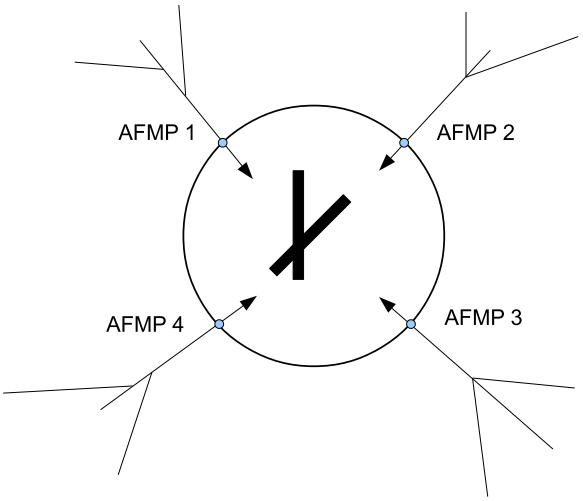

Fig. 4. TERMINAL AIRSPACE WITH FOUR METER FIXES THROUGH WHICH FLOWS ENTER THE TERMINAL AREA.

stream has compressed such that inter-aircraft spacing is closer to the separation standard.

In order to determine s_{tar}, a simplified model of traffic with a facility operating at density η is applied. For a facility operating at a fraction η of its maximum Airport Arrival Rate (AAR), the AFMP with the highest flow rate in aircraft per hour is determined based on an assumption of traffic balancing. Given a flow rate over an AFMP, s_{tar} can be determined based on an assumption of how the STAs are distributed with s_{tar} as a minimum spacing. For simplicity, the STAs at the AFMP are assumed to be regularly distributed and spaced at s_{tar}.

The value of s_{tar} as a function of traffic density η is highly dependent on the facility and traffic conditions. Consider an idealized airport model, with two runways and four meter fixes to the terminal airspace. Figure 4 shows this idealized facility with AFMPs labeled as 1, 2, 3, and 4.

During certain times of the day, it is typical to see a bias in the traffic volume from a particular direction, depending on the facility. To represent the unbalanced flow over the AFMPs,

it is assumed that 40% of the traffic is distributed to each of AFMP 1 and 3, and 10% is distributed to each of AFMP 2 and 4. Equal runway balancing is assumed for simplicity. Finally, it is assumed that at maximum capacity, aircraft can land with as small as a 90-second spacing interval.

With two runways operating 90-second spacing intervals, this yields a maximum airport capacity of

$$2 \cdot \frac{3600\frac{s}{hr}}{90\frac{s}{ac}} = 80\frac{ac}{hr}. \quad (5)$$

Taking the fraction η of maximum capacity and distributing the traffic to the AFMPs as above, the maximum flow rate at AFMPs 1 and 3 is $\rho_{max} = 32\eta$ ac/hr.

Adding a runway to the same TRACON configuration increases the modeled AAR to 120 ac/hr. Using the same relative AFMP traffic distribution, the corresponding maximum flow rate is $\rho_{max} = 48\eta$ ac/hr at AFMPs 1 and 3. Equivalently, if there are two runways in use, but only three meter fixes with loading of 60%, 20%, and 20% (60/20/20) on each, then the maximum flow rate at an AFMP is also $\rho_{max} = 48\eta$ ac/hr.

Particular facilities can also be analyzed for a maximum flow rate over an AFMP. For example, the highest published AAR at PHX under Instrument Meteorological Conditions (IMC) is 48 ac/hr. There are four AFMPs for PHX; therefore, a 40/40/10/10 distribution of traffic yields a maximum flow rate of $\rho_{max} = 19.2\eta$ ac/hr. The highest AAR under Visual Meteorological Conditions (VMC) conditions is 78 ac/hr, which corresponds to a maximum flow rate of $\rho_{max} = 31.2\eta$.

At DFW, there are also four AFMPs. The highest AAR associated with a runway configuration suitable for VMC or IMC operations is 96 ac/hr; AARs around 50 ac/hr are used in strong wind conditions. These AARs correspond to flow rates of 27 and 14 ac/hr, respectively.

Under the assumed traffic model, the minimum targeted spacing s_{tar} corresponds to the inverse of the flow rate ρ_{max}:

$$s_{tar} = \frac{3600\frac{s}{hr}}{\rho_{max}}. \quad (6)$$

Using the constraint in (3), s_{tar} is treated as a constant and the delivery errors, ϵ, are treated as independent, identically distributed normal random variables with mean zero. Under these assumptions, the delivery accuracy which satisfies the constraint for a range of values of ω is plotted in Figure 5 for a range of maximum flow rates.

This trade off between how accurately the ground automation needs to perform and how often the controller can be expected to manage a potential separation violation occurs around $\omega = 0.01$, or one in 100 operations. For larger probabilities of separation violation, the corresponding delivery accuracy begins to be so large that ATC can intervene and meet the required accuracy easily without any assistance from automation. For smaller probabilities of separation violation, the incremental cost to automation in terms of increased delivery accuracy is not high. We will choose $\omega = 0.001$, or one separation violation in 1,000 operations, to explore the sensitivity to groundspeed and flow rates.

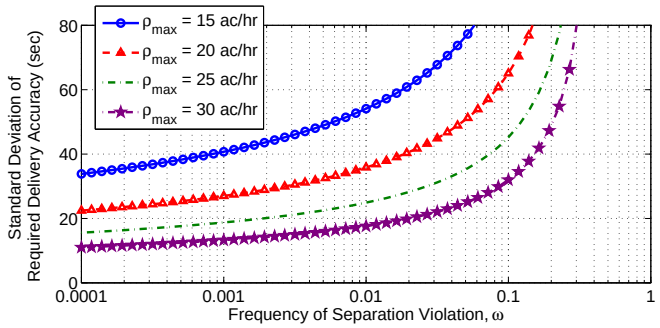

Fig. 5. DELIVERY ACCURACY VS. FREQUENCY OF SEPARATION VIOLATION FOR DIFFERENT FLOW RATES ρ_{max}.

For this choice of ω and using the properties of the normal distribution, the constraint in (3) will be satisfied when

$$s_{tar} - M > 3.09\sqrt{2}\sigma, \quad (7)$$

where σ is the standard deviation of the delivery errors; the delivery errors for the lead and trailing aircraft are assumed to be identically distributed. The term $s_{tar} - M$ can be thought of as the buffer applied to the separation minimum. Note that the separation minimum, M, is typically given in nmi, in which case it must be converted to time when s_{tar} and the delivery variation are given in time. This conversion is performed by dividing the separation minimum by a reference groundspeed, approximating the amount of time that it would take the aircraft to traverse the distance M.

Using a 5-nmi separation minimum, Figure 6 shows the delivery accuracy required at the AFMP to yield the desired flow rate. The flow rates at the most heavily-loaded AFMP at DFW, assuming $\eta = 0.7$, for two different configurations and associated AARs are shown with dashed lines for a variety of groundspeeds to the AFMP.

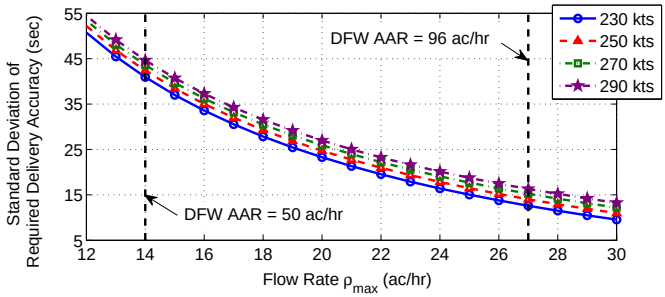

Fig. 6. DELIVERY ACCURACY AS A FUNCTION OF FLOW RATE ρ_{max}.

D. Constraint 2: Feasibility of the OPD

In considering the second constraint, the observation is that delivery errors to the ERFMP and AFMP correspond to an adjustment in the trajectory time between those points. For example, late arrival to the AFMP implies a longer trajectory

time than on-time arrival. This constraint is modeled by consideration of how the STAs are determined and how delivery errors affect the trajectory. A schedule is assumed with frozen STAs at the ERFMP and AFMP, denoted as STA_{ERFMP} and STA_{AFMP}. It is assumed that there is a nominal trajectory time, t_{nom}, to fly from the ERFMP to the AFMP for each aircraft. The nominal trajectory time, t_{nom}, is a theoretical value that represents the best estimate that could be made by the ground automation for the time to fly between two CSPs. As such, it includes uncertainty and the actual trajectory time, t_{act}, of the scheduled aircraft is modeled to vary around the mean t_{nom}.

The following equation expresses how the difference between t_{act} and t_{nom} depends on delay and delivery errors:

$$\begin{aligned} t_{act} - t_{nom} &= (ATA_{AFMP} - ATA_{ERFMP}) - t_{nom} \\ &= (STA_{AFMP} + \epsilon_{AFMP}) - ... \\ &... - (STA_{ERFMP} + \epsilon_{ERFMP}) - t_{nom} \\ &= d + \epsilon_{AFMP} - \epsilon_{ERFMP} \end{aligned} \tag{8}$$

Here, the delay d introduced by the schedule is defined to be the difference between the nominal trajectory time and the scheduled trajectory time:

$$d = (STA_{AFMP} - STA_{ERFMP}) - t_{nom} \tag{9}$$

The relationship between delay, the nominal trajectory time, and the actual time flown is illustrated in Figure 7.

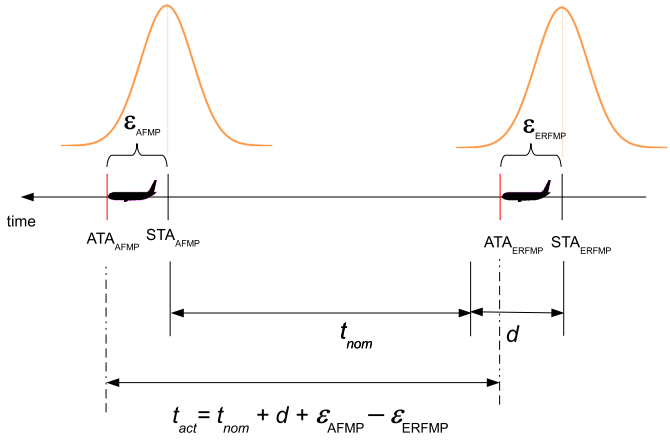

Fig. 7. DEPICTION OF CONSTRAINT 2.

The larger the magnitude of the delay d introduced by the schedule, the further the actual flight time, t_{act}, is likely to be from the assumed nominal trajectory time, t_{nom}, between the ERFMP and the AFMP. Delay may be introduced between the ERFMP and the AFMP to ensure that the AAR is not exceeded and that successive aircraft are de-conflicted at the AFMP.

It is noted that (8) is a simplification of how the trajectory is actually flown with respect to delivery errors, as there is generally a dependence between ϵ_{ERFMP} and ϵ_{AFMP}. In particular, if ϵ_{ERFMP} is so large that the aircraft does not have enough speed authority to correct for the error, then ϵ_{AFMP} will also be large. As random variables, ϵ_{ERFMP} can be viewed as independent from ϵ_{AFMP} as long as there exists a feasible trajectory which gets the aircraft to its STA at the AFMP, given ϵ_{ERFMP}. It is assumed that the scheduling function will perform such that this is the norm.

To that end, we introduce a parameter, Δ, that represents the maximum deviation of t_{act} from t_{nom}. If $|t_{act} - t_{nom}|$ is larger than Δ, then this is an infeasible trajectory to which the aircraft cannot adhere based on weight, airspeed, or some other factor. For a fixed value of d, we seek to analyze the statistical behavior of the delivery errors at the ERFMP and AFMP to ensure that $|t_{act} - t_{nom}| < \Delta$ with some probability to minimize the infeasibility of meeting the STA at the AFMP. In this paper, we constrain the delivery errors to be such that the difference in the actual flight time from the nominal flight time is greater than Δ with probability less than γ.

From equation (8), the following apply to each aircraft in the operation:

$$Pr\left(|t_{act} - t_{nom}| > \Delta\right) < \gamma, \text{ or}$$

$$Pr\left(|d + \epsilon_{AFMP} - \epsilon_{ERFMP}| > \Delta\right) < \gamma. \tag{10}$$

In order to solve for the delivery accuracy, a maximum magnitude of delay, $|d| = d_{max}$, is modeled as being applied by the scheduler, and the worst-case delay allocation is considered. It is assumed that the scheduler either applies maximum delay absorption or maximum delay reduction, modeled as shown below.[2]

$$\begin{aligned} &Pr\left(d_{max} + \epsilon_{AFMP} - \epsilon_{ERFMP} > \Delta\right) + ... \\ &...+ Pr\left(-d_{max} + \epsilon_{AFMP} - \epsilon_{ERFMP} < -\Delta\right) < \gamma \end{aligned}$$

For simplicity, we assume the errors associated with arriving early or late are similar and can be modeled equivalently. We then constrain delivery accuracy by

$$Pr\left(d_{max} + \epsilon_{AFMP} - \epsilon_{ERFMP} > \Delta\right) < \frac{\gamma}{2}. \tag{11}$$

It can be shown that if the average groundspeed of an aircraft is changed by a fraction α, then the trajectory time will also change by approximately α. The feasibility of a trajectory time depends on whether the speeds required to fly the trajectory are within aircraft performance limits. Modeling this as a maximum percentage change in groundspeed from nominal corresponds to a percent change in the nominal trajectory time.

Nominal trajectory times have been calculated for a range of approaches, including EWR, LAX, LAS, and PHX. These nominal trajectory times are in the range of 1400 to 1800 seconds. We take the smallest of the trajectory times and set $\Delta = \alpha t_{nom} = \alpha \cdot 1400$ sec. For simplicity, we will set $d_{max} =$

[2]It should be noted that the scheduler in the current ground-automation system only applies delay absorption when setting the STAs; i.e., the STA at the next CSP may only be set to increase the trajectory time from the last CSP. Future systems may also include delay reduction, as modeled here, in order to best utilize the airspace.

$\alpha/2 \cdot t_{nom}$, essentially allocating half of the allowable deviation to delay allocation. This simplifies the constraint to

$$Pr\left(\epsilon_{AFMP} - \epsilon_{ERFMP} > \alpha \cdot 700\right) < \frac{\gamma}{2}. \quad (12)$$

Choosing $\gamma = 0.002$ and $\alpha = 0.1$, for example, and applying the properties of the normal distribution, the constraint is satisfied if

$$3.09\sqrt{2}\sigma < 70 \text{ seconds, or } \sigma < 16 \text{ seconds.} \quad (13)$$

Figure 8 shows a series of parametric curves that illustrate the minimum delivery accuracies for different values of α and γ (for a fixed nominal trajectory time of 1400 seconds, as shown in the example above).

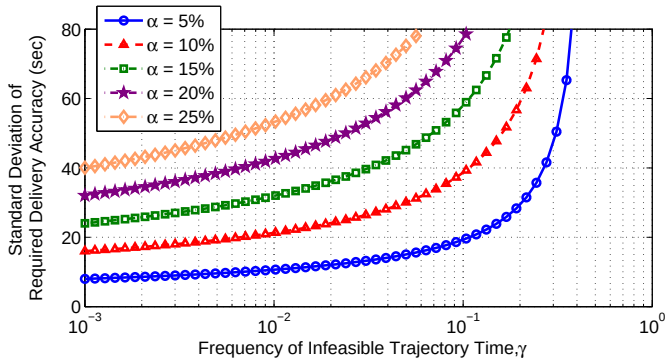

Fig. 8. MINIMUM DELIVERY ACCURACY AS A FUNCTION OF γ AND α.

The delivery accuracy (one standard deviation) has a characteristic non-linear shape as the probability of an infeasible trajectory time increases, making the OPD procedure infeasible. A 20% increase in the standard deviation for delivery accuracy corresponds to an increase between 0.1% and 1% of flying an infeasible OPD. The requirement scales directly linearly with α (the maximum fraction of the nominal trajectory that can be added or subtracted to achieve the STA).

E. Numerical Example

The EAGUL5 OPD arrival at PHX is used to demonstrate the framework presented in the last section for determining the delivery accuracy that supports staying on the RNAV procedure. This example is for illustrative purposes, and does not represent requirements at PHX or elsewhere.

The EAGUL5 procedure was designed with altitude and speed constraints to increase consistency in altitude profiles and flight times between aircraft types, thus enabling sufficient spacing to be established between aircraft prior to TOD and reducing the need for ATC intervention during the descent. A 270-kt indicated airspeed (IAS) constraint during the descent on the EAGUL5 procedure limits the maximum change that can be realized in the trajectory time to the AFMP. An aircraft trajectory simulator was used to generate the nominal trajectory time assuming a 0.78-Mach cruise speed and the 270-kt descent speed. Table I shows the trajectory times from different distances to the AFMP for the nominal trajectory and for fast and slow trajectories assuming 0.82- and 0.74-Mach cruise speeds, respectively, and also with the 270-kt descent speed. The maximum allowable deviation Δ from t_{nom} is determined from the minimum difference in the fast and slow trajectory times relative to t_{nom}.

The portion of Δ allocated to the delay is determined as a percentage of Δ. Assuming a feasibility requirement of $\gamma = 0.002$, the delivery accuracy is determined from the remaining allocation after the delay allocation has been removed.

The resulting delivery accuracies in Table I reveal the feasibility of placing the freeze horizon at different distances from the AFMP. For example, assuming the ground automation and procedures for the operation can support a delivery accuracy of 15 seconds or larger, the freeze horizon must be around 300 nmi from the AFMP if no portion of Δ is allocated to delays between the ERFMP and the AFMP. If some portion of Δ is allocated to delay, then the freeze horizon must be even farther from the AFMP. The distances to the AFMP and the delay allocations that yield at least a 15-second delivery accuracy are highlighted in green in Table I.

Trajectory times for the PHX OPDs were also generated assuming that the speed constraints are not applicable during the operation. Table II shows the nominal, fast, and slow trajectory times when the descent airspeed is allowed to vary from the 270-kt constraint during the descent. As expected the allowable deviation from the nominal trajectory time is larger, which enables greater delay allocations and less stringent requirements on the delivery accuracy (shown here for $\gamma = 0.002$). The green-highlighted cells in Table 2 show those conditions yielding at least a 15-second delivery accuracy; the elimination of the speed constraints allows greater allocations to delays at shorter distances to the AFMP without making the delivery accuracy requirements overly stringent.

Figure 9 compares the resulting delivery accuracy for different values of γ and two values of d_{max} for the trajectory with and without the speed constraints. The black dashed line indicates the 15-second delivery accuracy to provide a reference for comparing the different conditions.

Figure 9 shows that there are trade offs with allocating delay, the probability of a successful OPD, and the delivery accuracy that must be considered in developing automation systems that support the operation. In addition, the application of procedural speed constraints during the operation must also be understood as the speed constraints limit the ability to change the aircraft's trajectory time relative to the nominal trajectory.

Consider the delivery accuracy driven by constraint 1. Assuming the maximum flow rate for PHX of $\rho_{max} = 19.2\eta$, where η is the percentage of the maximum capacity, the needed delivery accuracy to support 100% of the maximum capacity is 28.7 seconds. Therefore, it is likely that constraint 2, needed to ensure the likelihood of adhering to the RNAV arrival, yields the more stringent delivery accuracy requirement.

Alternatively, based on the results in Table II, assume that for a 200-nmi freeze horizon to the AFMP and for a 30-second allocation to schedule delays, the required delivery accuracy

TABLE I
TRAJECTORY TIMES AND REQUIRED DELIVERY ACCURACY (WITH SPEED CONSTRAINTS)

Distance to AFMP (nmi)	t_{slow} 270-kt IAS 0.74 Mach	t_{nom} 270-kt IAS 0.78 Mach	t_{fast} 270-kt IAS 0.82 Mach	Max Deviation Δ (sec)	$d_{max} = 0.50\Delta$ (sec)	Delivery Accuracy σ (sec)	$d_{max} = 0.25\Delta$ (sec)	Delivery Accuracy σ (sec)	$d_{max} = 0$ (sec)	Delivery Accuracy σ (sec)
100	942.0	933.0	927.0	6.0	3.0	0.7	1.5	1.0	0.0	1.4
200	1793.5	1740.5	1695.0	45.5	22.8	5.2	11.4	7.8	0.0	10.4
300	2645.0	2548.0	2463.0	85.0	42.5	9.7	21.3	14.6	0.0	19.5
400	3496.0	3355.5	3231.0	124.5	62.3	14.2	31.1	21.4	0.0	28.5

TABLE II
TRAJECTORY TIMES AND REQUIRED DELIVERY ACCURACY (NO SPEED CONSTRAINTS)

Distance to AFMP (nmi)	t_{slow} 250-kt IAS 0.74 Mach	t_{nom} 270-kt IAS 0.78 Mach	t_{fast} 300-kt IAS 0.82 Mach	Max Deviation Δ (sec)	$d_{max} = 0.50\Delta$ (sec)	Delivery Accuracy σ (sec)	$d_{max} = 0.25\Delta$ (sec)	Delivery Accuracy σ (sec)	$d_{max} = 0$ (sec)	Delivery Accuracy σ (sec)
100	1002.5	933.0	852.5	69.5	34.8	7.5	17.4	11.2	0.0	14.9
200	1854.5	1740.5	1621.0	114.0	57.0	12.3	28.5	18.4	0.0	24.5
300	2706.0	2548.0	2389.5	158.0	79.0	17.0	39.5	25.5	0.0	34.0
400	3557.5	3355.5	3157.5	198.0	99.0	21.3	49.5	31.9	0.0	42.6

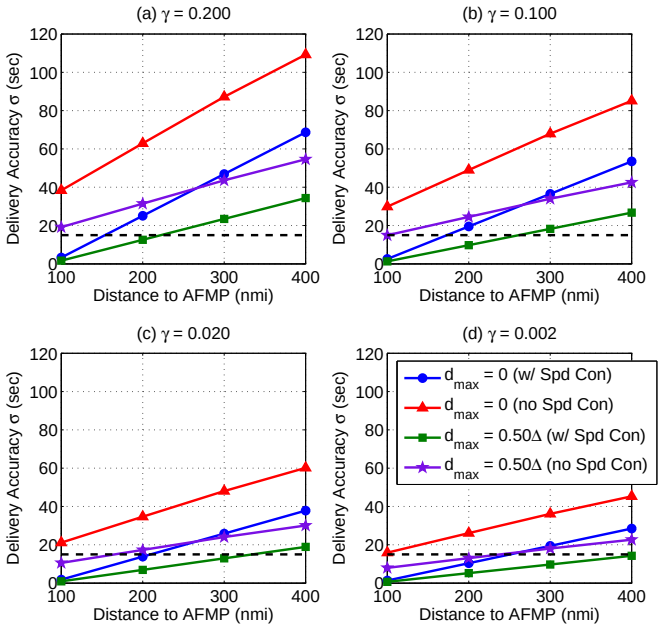

Fig. 9. DELIVERY ACCURACY AS A FUNCTION OF DELAY ALLOCATION AND γ.

is 19.2 seconds. Therefore, s_{tar} = 146.1 seconds at the more heavily loaded AFMPs at PHX, which results in a maximum flow rate of ρ_{max} = 24.6 ac/hr.

IV. CONCLUSIONS

In this paper a basic modeling approach of delivery accuracy and other relevant parameters to an initial ground automation system for arrival management operations has been developed. Two constraints related to the operational objectives of those operations are used to draw conclusions on the trade off between delay allocated by the scheduler, the accuracy of delivering to a STA, and the maximum arrival rates that can be supported.

Future work will expand this basic modeling approach in a couple of directions. First, sensitivity to the simplifying modeling assumptions will be explored. The assumption of a normal probability distribution for delivery errors is likely not accurate; for example, there may be a bias in how controllers deliver to the STA. The assumption of equally distributed STAs for a given non-saturated arrival rate is another simplification, and more realistic distributions of STAs would translate differently to a probability of separation violation. A similar problem was addressed through modeling of the unadjusted and adjusted inter-arrival spacing values prior to the AFMP in reference [6]. Finally, replacing a maximum value of delay with a distribution that models how the delay allocation in the scheduler is a function of the random arrival times of individual aircraft may reveal a different trade off with delivery accuracy.

Future work should also include extending the analysis to future arrival management operations, in particular the IM operation. Metering in the terminal area and using ADS-B IN avionics to control to relative spacing each have unique operational constraints on the same performance parameters studied here: delivery accuracy, delay allocation, and arrival rates. Additional performance parameters may be identified as other arrival management components are added. Modeling these other operations, and the interaction between concurrent operations in a mixed-equipage environment, is an important step in determining the overall performance necessary for a robust arrival management system.

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