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# Synchronization Likelihood in Aircraft Trajectories

## Assessment of relationships between trajectories, and their implications for safety

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***Abstract*—In the continuous effort for ensuring increasing levels of safety, it is of utmost importance to understand the reasons behind the occurrence of operational errors. In this contribution, we propose the use of the *Trajectory Synchronization Likelihood* metric for the analysis of two types of events: situations resulting in a reduced separation between aircrafts, and situations that might have resulted in similar conditions but were solved on time. Results indicate that unsolved events are associated with highly synchronized pairs of aircraft, which have been deviated from the usual expected trajectories. This opens new way for the development of more effective automated safety systems, capable of detecting in real time events that are known to have a high probability of resulting in a conflict.**

***Keywords-safety; Trajectory Synchronization Likelihood; data mining***

## I. INTRODUCTION

Separation assurance between aircraft is obviously an essential component in the safety of air transport. In the actual system, the maintenance of such separation is built upon two different concepts [1]: the organization of the airspace in routes, and the use of dedicated procedures. Human errors, especially in the design or execution of a given procedure, are possible and the separation results strongly depend on the human intervention.

In order to reduce operational errors and safety-related events, automated systems, like the Short-Term Conflict Alert (STCA) used by ground-based equipments, or the Traffic Collision Avoidance System (TCAS) on-board have an important role as a safety net. These systems use sensors data to predict conflicts between aircraft, in order to alert humans and provide guidance for the resolution of the situation. Yet, these systems are safety nets expected to be activated as rarely as possible, and only act in a narrow range of situations. There is an increasing interest in the identification of more general safety related scenarios [2-3], especially by means of data mining and machine learning techniques [4]. Mining historical data in the search of relevant patterns, and especially mining historical trajectories, presents several challenges. Firstly, the huge quantity of data to be analyzed, which requires highly optimized algorithms if results have to be obtained in real-time. Secondly, the shifting of the focus from the study of the behavior of individual aircraft, to the characteristics of the interactions between different aircraft; this last point requires a systemic approach and the adoption of non-standard techniques drawn from *complex systems theory*.

In this contribution, we tackle the problem of forecasting the occurrence of safety-related events, corresponding to situations in which separation minima is not respected, by analyzing real aircraft trajectories and planned intentions. Two are the main differences with respect to previous researches.

First of all, we investigate the differences between two groups of events: those events that may have evolved in safety-related events, but were avoided by the intervention of ATC, and those that actually ended up in a separation loss. In other words, we are interested in the understanding and forecasting of these events that are not safely managed by means of actual technology and procedures. It should be noticed that this idea of discriminating accidents from events that could have resulted in an accident has already been explored in the literature [5-6]; clearly, the discrimination of the characteristic features of each groups may have important implication for safety, as they are indicating which mechanisms are failing when an accident appears.

Second, we here propose the use of *complex systems* techniques to analyze the trajectories of aircraft. *Complexity science* is an interdisciplinary field, encompassing mathematics and physics, devoted to the analysis of systems composed of a large number of elements interacting in a non-linear way [7]. Examples span from communication networks [8], social networks [9], or biological systems [10-11]. The presence of such interactions between the constituting elements results in the appearance of *emergent behaviors*, i.e., macro-scale properties that cannot be forecasted by analyzing the dynamics of isolated individual elements. This has required the development of specific tools, for instance, aimed at the extraction of information from the non-linear dynamics of the single elements, or at the representation of large-scale relationship structures [12-13]. Aircraft trajectories indeed form a complex system, as their global evolution strongly depends on the interactions between flights. Due to this,

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classical data mining tools may not be able to detect the causes behind some macro-scale effects, like for instance, the presence of airspaces with higher probability of conflicts.

In this contribution, we adapt a well-known technique for the study of the dynamics of the brain, known as *Synchronization Likelihood* [14], to the study of the evolution of pairs of trajectories through time. Thanks to this metric, it is possible to assess whether two aircraft are "synchronized", in the sense that a deviation from the expected trajectory of one of them is always associated with a deviation of the second aircraft. The hypothesis we test is that synchronized aircraft have higher probability of developing safety-related events.

This contribution is organized as follows. Section II introduces the *Synchronization Likelihood* metric, and its main applications in biomedical problems, while Section III proposes a modification for the study of aircraft trajectories. Section IV presents the data set used in this work, comprising real trajectories of flights over Europe, and Section V reports the main results obtained. Finally, Section VI draws some final conclusions and future lines of work.

## II. Synchronization Likelihood

The concept of *Synchronization Likelihood* (SL in short) was firstly introduced by Stam and van Dick as a way to assess whether two (chaotic) systems were synchronized in a general way [14]. When two dynamical systems are in the so-called *generalized synchronization* condition, the dynamics of one of them is partly defined by the other, but their outputs maintain some specific characteristics [15]. Mathematically, if two systems $x$ and $y$ are represented by the two vector of variables $(x_1, x_2, \ldots, x_N)$ and $(y_1, y_2, \ldots, y_M)$, they are said to be in a generalized synchronization regime if, after an initial transitory evolution, it exists a function $\Theta$ such that:

$$[y_1, y_2, \ldots, y_M] = \Theta(x_1, x_2, \ldots, x_N). \tag{1}$$

Assessing the existence of this function is, of course, not straightforward: although an analysis of the Lyapunov exponents of the coupled system can be performed, its computational cost makes such approach unfeasible in any real-world situation.

The SL measure is a proposal for the fast calculation of the level of generalized synchronization between two systems, based on the analysis of the time series generated by them. The basic intuition behind it is that, whenever the time series associated to $x$ repeats a given pattern, the output of system $y$ should also repeat another pattern. Notice that both patterns may be different, as the two systems are not homogeneous, and therefore no complete synchronization is expected.

This idea is illustrated in Fig.1, where two systems ($x$ and $y$) are represented by two different time series ($x(t)$ and $y(t)$ respectively). In a qualitative manner, one can observe that, whenever the dynamics of $x$ is repeated (see the two green boxes), the system $y$ responds with a different, yet constant dynamics; therefore, we expect $x$ and $y$ to be somehow related. As proposed in [14], both time series can be expressed as embedded vectors

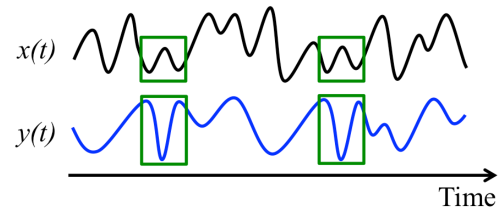


Figure 1. Example of the SL calculation process.

$$X_{k,i} = (k_i, k_{i+l}, k_{i+2l}, \ldots, k_{i+(m-1)l}), \tag{2}$$

$k$ being one of the two time series (that is, $x$ or $y$), $l$ the lag, and $m$ the embedding dimension. Notice that this is equivalent to represent a set of $m$ values of the time series as a single point in an $m$-dimensional space. In the simple case where $l=1$, each vector is equivalent to the time series of length $m$ starting at time $i$. Afterward, the closeness of two different $X$s, respectively starting at time $i$ and $j$, can be easily computed as follows:

$$P^{\varepsilon}_{k,(i,j)} = \theta\left(\varepsilon - \left|X_{k,i} - X_{k,j}\right|\right). \tag{3}$$

$|.|$ is the Euclidean distance in a $m$-dimensional space and $\theta$ the Heaviside step function. Notice that $P$ is a function that returns one only when the two embedded vectors (i.e., the time series starting at $i$ and $t$) are within a radius $\varepsilon$.

Once the function $P$ has been defined, it may be used to calculate the final SL function. Specifically, for each pair of time steps $i$ and $j$, we calculate the proportion of pairs $i$ and $j$ simultaneously fulfilling $P^{\varepsilon}_{Y,(i,j)} = 1$ and $P^{\varepsilon}_{X,(i,j)} = 1$.

### A. Applications of the SL

Since its introduction in 2002, Synchronization Likelihood has attracted a special interest in the field of neuroscience, and specifically in the study of neurological diseases. Different groups of neurons generate electromagnetic signals during their activity, which can be recorded by means of an electroencephalography (EEG) or a magnetoencephalography (MEG). A standard assumption in neuroscience is that two neural ensembles show some kind of synchronization in their activity when they share information, or, in other words, that synchronization is an essential element for neural computation. Therefore, by analyzing the degree of correlation between different parts of the brain during a cognitive task, it is possible to unveil where and how information is being processed [11].

Yet, it has to be noticed that different parts of the brain are not homogeneous: the number of neurons monitored by each sensor of the machine may vary, their connectivity can also be different, and finally neurons themselves can have different characteristics [16]. Due to these, it is not realistic to expect a linear (i.e., Pearson's) correlation between their activities; instead, we may look for a complex function $\Theta$ defining a generalized synchronization by means of SL.

Examples of successful application of SL to neuroscience problems include the analysis of different neurological diseases (e.g., schizophrenia, epilepsy, autism, Alzheimer's, or

Parkinson's), which seems to be partly caused by abnormalities in the temporal coordination of information through the brain [17-18]; or the analysis of brain dynamics, and its efficiency in managing information at different time scales [19-20].

## III. SL BETWEEN TWO TRAJECTORIES

The Synchronization Likelihood, as previously defined, cannot be directly applied to the study of aircraft trajectories, as these have some specific characteristics that should be taken into account.

Firstly, each trajectory can be seen as a multivariate time series, composed of 3 different components (i.e., latitude, longitude, and flight level). Yet, these three components are not independent, and should be analyzed altogether; for instance, the evolution of the latitude of two aircraft cannot be studied without taking into account their longitude.

Secondly, and within the scope of the present work, we are interested in co-occurring modifications in the trajectories, while the actual modification is not interesting per se. For instance, suppose that two flights have to be re-routed due to adverse en-route weather. Our aim is to detect whether both trajectories have been re-routed at the same time, i.e. if their dynamics is somehow synchronized, while the actual re-routing strategy (turn left or right) is not relevant.

In what follows, we define a Trajectory Synchronization Likelihood (TSL in short) fulfilling these two requirements.

### A. TSL overview

Generally speaking, we define that two aircraft are synchronized when both of them present significant deviations from their usual trajectories at the same time. Therefore, and as a first step, it is necessary to define what is the usual trajectory of an aircraft, and how to measure if a deviation is statistically significant.

Let us denote the trajectories of two aircraft, $x$ and $y$, as $\overline{X}(r,t)$ and $\overline{Y}(r,t)$ respectively. These trajectories are defined, in a discrete way, as a set of points, describing the position of the aircraft at time $t = 1,2,\ldots,t_{\max}$. In what follows, we consider each point as composed of two coordinates, longitude and latitude (such that $\overline{X}(r,t) = [X_{lat}(r,t), X_{lon}(r,t)]$ and $\overline{Y}(r,t) = [Y_{lat}(r,t), Y_{lon}(r,t)]$); in the sake of simplicity, the flight level is discarded from the analysis. Each flight is also identified by an index $r = 1,2,\ldots,r_{\max}$, which allows us distinguishing between different repetitions of the same flight across several days.

An example may help clarifying the latter index. Suppose that we have identified a safety related event on a given day, in which two flights were involved: AIR0001 and AIR0002. The same two flights may have been operated in previous days, and this can be easily checked by looking at flights with the same code, operating between the same pair of airports and with the same planned departure time. These other historical flights are then used to compute the expected position of each aircraft at the time of the considered event. Thus, $\overline{X}(r,t)$ will represent of position of flight $x$ at time $t$, and in the day $r$.

The expected position of each flight can be approximated by the mean value of aircraft positions at a given time, calculated for different $r$:

$$\overline{E}_{\overline{X}}(t) = \frac{1}{r_{\max}} \sum_{i=1}^{r_{\max}} \overline{X}(i,t). \tag{4}$$

Notice that $\overline{E}_{\overline{X}}(t)$ is a vector of two coordinates, latitude and longitude; furthermore, a similar expression can be calculated for the second flight, by substituting $\overline{X}$ by $\overline{Y}$ [1].

The distance of any flight from its expected position, and therefore the deviation with the expected trajectory, is given by

$$d_{\overline{X}}(r,t) = \left| \overline{X}(r,t) - \overline{E}_{\overline{X}}(t) \right|, \tag{5}$$

$|\cdot|$ being the Euclidean distance in the considered 2-dimensional space. Assuming a Gaussian distribution of distances, its variability is given by

$$\sigma_{\overline{X}}(t) = \sqrt{\frac{1}{r_{\max}} \sum_{i=1}^{r_{\max}} \left[ d_{\overline{X}}(i,t) - \mu_{\overline{X}}(t) \right]^2}, \tag{6}$$

$\mu_{\overline{X}}(t)$ being the mean value of the deviations, i.e.,

$$\mu_{\overline{X}}(t) = \frac{1}{r_{\max}} \sum_{i=1}^{r_{\max}} d_{\overline{X}}(i,t). \tag{7}$$

Using the previous definitions, it is possible to define a Z-Score of the deviations of each trajectory:

$$Z_{\overline{X}}(r,t) = \frac{d_{\overline{X}}(r,t)}{\sigma_{\overline{X}}(t)}. \tag{8}$$

For each flight, and for each point in the trajectory, $Z$ is a

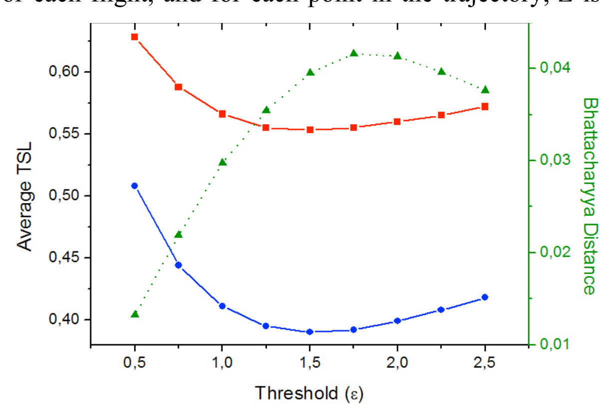


Figure 2. Average TSL, for solved and unsolved iEvents, and Bhattacharyya distance, as a function of the threshold ε.

[1] If not otherwise stated, all following equations hold for the second flight, by substituting $\overline{X}$ by $\overline{Y}$.

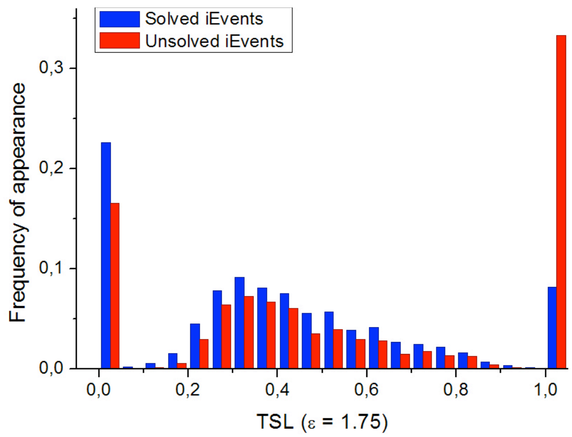


Figure 3. Histogram of TSL for solved and unsolved events.

number defined in the interval $[0,\infty)$ describing how close was that flight to its expected (historical) position. Specifically, we expect most of flights to have $Z < 2$, while values of $Z > 2$ designate extreme deviations from the normal trajectory[2]. Furthermore, notice that the intrinsic variability is naturally taken into account: for instance, if a flight frequently suffers from delays at take-off, its corresponding σ will increase, returning a value of $Z > 2$ only for unusual high delays.

The TSL is finally defined as the number of times the trajectories of both aircraft are deviated more than a given threshold, divided by the number of times the trajectory of the first aircraft is deviated:

$$TSL = \frac{\sum_{i=1}^{r_{\max}} \sum_{t=1}^{t_{\max}} \theta\left[Z_{\bar{Y}}(i,t) - \varepsilon\right] \cdot \theta\left[Z_{\bar{X}}(i,t) - \varepsilon\right]}{\sum_{i=1}^{r_{\max}} \sum_{t=1}^{t_{\max}} \theta\left[Z_{\bar{X}}(i,t) - \varepsilon\right]} . \qquad (9)$$

TSL is therefore normalized between zero and one; the higher the value, the more synchronized are both trajectories, in the sense that significant deviations from the expected route happen at the same time.

## IV. Analysis of Historical Data

In what follows, we present the analyses performed on actual historical traffic data, including planned and real trajectories over the European airspace, and their implications for the analysis of safety-related events.

### A. The data set

Trajectories data have been extracted from the *ALL_FT+* data set, collected by the EUROCONTROL PRISME group. This includes information about planned, regulated and executed trajectories for all flights crossing the European airspace. The data set covers the period from 1st March to the 31st December 2011, including a total of 10.3 million flights.

[2] Specifically, if a normal distribution of deviations is assumed, the 95% of realizations will have $Z < 2$.

### B. Definition of safety-related events

In order to assess a relationship between trajectory synchronization and the appearance of safety-related events, we have firstly identified a set of interesting events in the data set. These *iEvents* were identified by projecting the intentions of each aircraft (that is, the future route according to the filed flight plan) starting from a given position (obtained by the radar trajectory), and by detecting if two aircraft may break the separation minima in the near future. Notice that this is equivalent to the surveillance task performed by any Air Traffic Controller. Following this definition, iEvents include both events that may result in a safety-related condition (e.g., a reduction of the separation between aircrafts), and situations that might have resulted in similar conditions, but in which the intervention of the controllers (or of the pilots) solved the problem before its appearance. By analyzing the real evolution of both flights, all iEvents have been classified in these two groups, called "unsolved iEvents" and "solved iEvents" respectively.

### C. iEvents pre-filtering process

While analyzing the recorded iEvents, the characteristics associated to some of them have been noticed to be unrealistic, e.g., the case of commercial aircraft flying parts of the trajectory at supersonic velocities. This was mainly due to the low resolution of radar trajectories, or to errors and incongruence in the used data set. In order to clean the data on which calculations are made, events fulfilling one of the following conditions have been eliminated:

- Flights whose radar (i.e., real) and planned trajectories are exactly the same;
- iEvents for which the final real separation has been lower than 20 seconds. This is usually due to a low spatial or temporal resolution in the radar information available.
- Flights whose real trajectories included physically impossible segments, like supersonic velocities.

A total of 100.032 iEvents passed this selection, 4.316 of which have been classified as "unsolved iEvent".

### D. Assumptions and simplifications

In order to simplify the analysis, the following assumptions and simplifications have been considered when calculating the TSL between different trajectories:

- Only events between trajectories in the en-route phase have been considered, and their trajectories have been projected to the latitude-longitude plane. In other words, level change maneuvers have been disregarded.
- The number of realizations of the same flight (i.e., $r_{max}$) was the same for both aircraft involved in an iEvent.
- In the calculation of the TSL, only the ten points previous to the considered event are included; this reduces the computational cost of the algorithm, while at the same time excludes other non-relevant factors (e.g., runway configuration at take-off).

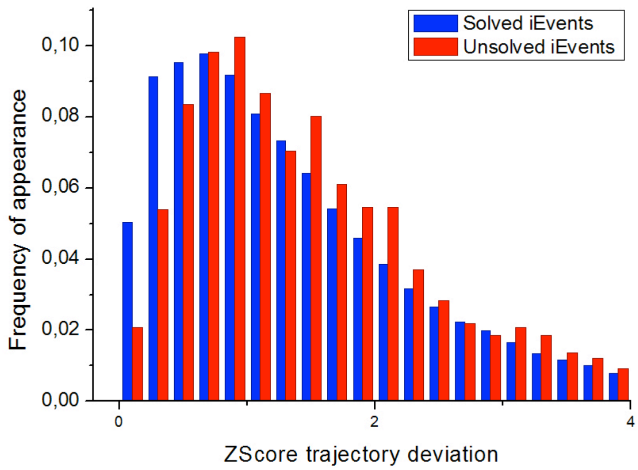

Figure 4. Histogram of the ZScore of trajectory deviation (see Eq. 8), for solved and unsolved events.

## V. ANALYSIS OF RESULTS

In this Section, we will review the main results obtained by applying the TSL to trajectories of flights involved in iEvents.

As the objective of this research is the identification of characteristics able to discriminate solved and unsolved events, special attention has been devoted to the assessment of the distance between the two TSL distributions, one for each group of events. Results are presented in Fig. 2, where the blue points (red squares) represent the average TSL for solved (unsolved) events, for different thresholds ε. Interestingly, the average TSL is always greater for unsolved iEvents, which seems to suggest that pairs of aircraft involved in unsafe situations have "synchronized" trajectories.

In order to confirm the significance of such differences, a two-sample *t*-tests has been performed [21], indicating that, for all ε, the two distributions (i.e., corresponding to solved and unsolved iEvents) are not the same with a p-value smaller than 0.01. Furthermore, the distance between both distributions has been calculated by means of the *Bhattacharyya distance* [22-23]. This measure is based on the calculation of the overlap surface between two different distributions, so that the higher the value, the more different are the considered samples. Results are represented in Fig. 2 by green triangles. The maximum of the Bhattacharyya distance corresponds to $\varepsilon = 1.75$: this is therefore the threshold that best discriminates both groups of events, and that will be considered in the following analyses.

In Fig. 3 are shown the two histograms representing the probability of finding an event associated to a given TSL in each one of the two categories considered. It is worth noticing that both distributions are quite similar, except for the extreme values of TSL = 0 and TSL = 1: in other words, it seems that the difference resides in the pairs of trajectories displaying no synchronization (mostly associated to solved iEvents) or a complete synchronization (associated to unsolved iEvents).

At this point, it is necessary to recall the meaning of the proposed TSL, in order to understand the properties of events here highlighted. A high degree of synchronization between two flights indicates situations in which they are used to travel along the same trajectory every day; therefore, ATC officers are also used to manage the separation of these two flights. Nevertheless, from time to time, one of them is rerouted, and it happens that the trajectory of the second is also modified; the two flights will now cross in a new airspace, thus generating a scenario that is not the usual one. Under these circumstances, ATC officers in charge of this airspace may have a higher probability of incurring in errors, as they are not familiar with this traffic pattern.

In order to confirm this hypothesis, Fig. 4 reports the ZScore of the deviation of trajectories associated with the iEvents (see Eq. 8). It can be noticed that, although similar, the two distributions are not the same (p-value < 0.01), and that unsolved iEvents are associated with higher ZScore. Therefore, unsolved iEvents have a high probability of appear far away from the expected position of the aircraft.

## VI. CONCLUSIONS AND FUTURE WORK

Summing up, we have presented a new metric, based on a biomedical technique for analyzing the dynamics of the brain, able to assess the degree of "synchronization" between the trajectories of two aircraft. Here we define that two flights are synchronized if they are simultaneously deviated from their expected trajectories, the latter defined as the mean trajectory obtained from historical data.

By using the proposed Trajectory Synchronization Likelihood, it has been possible to detect differences between two classes of safety-related events, here called iEvents: situations resulting in a reduced separation between aircrafts, and situations that might have resulted in similar conditions, but that were solved by controllers or pilots.

Results indicate that unsolved iEvents appear when two aircraft are simultaneously deviated from their expected trajectories, so that they create a conflict in an airspace which is not the usual one. Therefore, the controller has to face a situation that is new (and unknown), increasing the probability of an operational error.

It has to be noticed that this insight is not new, as controllers are well-aware of the dangers associated with deviations from the operational routine; yet, this awareness is a merely qualitative intuition. In this contribution, we have presented a framework for the mathematic and statistic assessment of the relevance of these situations in the development of unsafe events. Beyond the mining of historical data, TSL may also be used in real-time operation analysis, in order to forecast events that are known to have a high probability of resulting in a conflict, therefore improving the efficiency of the available automated safety systems.

Future works will be aimed at the application of the complex networks theory [12-13] to the characterization of iEvents. Complex networks have been extensively used in the past to analyze air transport in general [24], or safety events [25]. Within this framework, the traffic crossing one sector may be represented by a network, where each node is an aircraft, and pairs of them are connected if the corresponding TSL is higher than a given threshold. The analysis of the resulting networks, by means of suitable data mining techniques [26], will allow shedding light on features that may better explain the appearance of unsolved safety events.

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