

Impacts of Reporting Rules and Facility Consolidation on Recorded Operational Errors in TRACONs

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Abstract—This paper focuses on the occurrence of aircraft separation minimum violations as documented in the form of operational errors (OEs) at terminal radar approach control (TRACON) facilities. Poisson regression was used to analyze the daily count of OEs at various facilities. The occurrence of OEs was found to increase approximately with the square of daily traffic at TRACON facilities. At TRACON facilities, where separation violations are not automatically reported, an increase in reporting was seen after a new severity metric was introduced in 2007. It was also found that large, consolidated TRACON facilities tend to behave like a sum of several smaller facilities rather than a single larger facility with respect to the occurrence of OEs vs daily traffic. Weather effects such as visibility and wind were found to influence the occurrence of OEs as well. The model prediction for TRACON facilities is very good for the most severe OE types and very poor for the least severe OE types, indicating many unobserved factors contributing to the reporting of the least severe OE types in the terminal environment.

Keywords—safety, operational error, separation

I. INTRODUCTION

An operational error (OE) occurs when there is a violation of aircraft separation minimums due to air traffic control or from allowing an aircraft to enter another controller's airspace without notification. These events, while infrequent, often pose a serious safety risk.

Considerable work has been done on the incidence of OEs, which is presumed to reflect the risk of catastrophic mid-air collisions. Due to their infrequent nature, they are often studied using models that can handle large sets of sparse data. One of the most common among these is Poisson regression. Of particular interest to us are the effects of traffic, weather, and policy changes on the incidence of operational errors at terminal radar approach control (TRACON) facilities. In this paper we first review some of the studies involving operational errors and then analyze the incidence of operational errors at TRACON facilities using Poisson regression. Conclusions and recommendations for future work will follow.

II. BACKGROUND

A. Literature Review

Although the occurrence of operational errors has been studied extensively, most studies have aggregated OE data across time and/or space as a starting point for the analysis. Hansen and Zhang [1] modeled the daily count of operational errors at all TRACONs with negative binomial regression, Gosling [2] studied the occurrence of OEs at separate Area Route Traffic Control Center (ARTCC) facilities aggregated over an entire year, and Panagiotakopoulos et al [3] modeled the rate of OEs per month at a single facility using extreme value theory. We propose to use the daily count of operational errors at specific facilities between 2004 and 2009. While a daily count at a specific facility is still aggregated across the entire day, it should present insights into the occurrence of OEs that is not seen with more aggregate data.

One of the primary questions of interest that motivated this research is the relationship between OE occurrence and traffic. It has been suggested that the relationship should be roughly quadratic ($\# \text{ OEs} \sim \text{traffic}^2$) because the number of possible path intersections roughly increases with the square of the number of aircraft in a sector. Murphy [4] has shown that the exponent for traffic should be at least 2 for en route facilities, with significant differences across centers. These results were found by using the number of aircraft in the sector at the time each OE occurred as the measure for traffic. Hansen and Zhang [1] have shown that the value of the exponent is around 1.7 for daily operations at all TRACON facilities.

Secondly, the impact of weather on the occurrence of OEs is important. Many authors have included weather effects through an overall subjective metric called traffic complexity, which has been shown to be a significant factor contributing to OEs [3]. Rodgers and Nye [5] found that once the number of operations was accounted for, air traffic complexity was a significant predictor of the total number of operational errors. Air traffic complexity is partially a subjective measure, but is a function of the variety of operations, airspace limitations, and

weather. Weather variables such as wind, visibility, and temperature can represent a portion of traffic complexity that could give rise to operational errors.

Finally, another factor of interest is the effect of policy changes on OE reporting. At TRACON facilities, no automated tool is currently universally used to detect losses of separation like the Operational Error Detection Patch is used in the en route environment [6]. As a result, some errors will go unreported. The specific policy change that we investigated is the adoption of the Separation Conformance as a metric for OE severity. In June 25, 2007, an FAA order was sent out that specified a new measure of OE severity that would replace the OE Severity Index as the official measure. A key component of the new safety measure was the introduction of proximity events which would no longer be classified as operational errors, although they are still violations of separation minimums. Because they are no longer considered operational errors, controllers should not be penalized for reporting them as they would for a normal OE. Thus one would expect the number of reported errors in this category to increase.

B. OE Severity Metrics

The OE severity index had a range from 0-100 and was based on many weighted factors, such as horizontal separation, closure rates, and converging / diverging paths. The separation conformance metric is much simpler, and relies only on horizontal and vertical separation retained at the closest point of proximity. Depending on the relative percentage of vertical and horizontal separation retained, the OE is classified into four categories, A, B, C and PE (proximity event), with A being the most severe type and a PE being a very minor breach of separation minimums.

Figure 1, shown below, illustrates the categories of separation conformance as a function of the horizontal and vertical separation retained at the point of minimum separation. Only the percentages of the minimum separation requirements matter, so the metric is the same regardless of the magnitude of the separation required in each direction.

III. TRACON ANALYSIS

The OE data used for this study is the daily count of errors at each of the largest 16 stand-alone TRACON facilities from October 1, 2004 to June 28, 2009 (see Table 1, below), resulting in a total of 27,710 TRACON-days. A total of 1,798 operational errors were observed over this time period, which is roughly a rate of 0.06 OEs/TRACON-day, or 1 OE/day across all TRACONs studied here. For purposes of comparison between the time period before and after the adoption of the separation conformance metric, all OEs will be classified as A, B, C, and PEs using the separation conformance metric even if the OEs occurred before the measure went into effect.

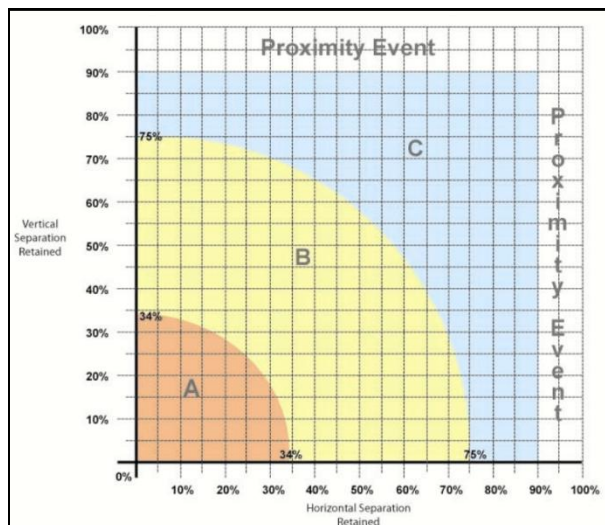


Figure 1. Separation conformance severity metric (Source: FAA)

A. TRACON Model

Poisson regression is a common model that is used to study count data, and can be used for sparse data like we observe with operational errors. One observation in our model will be defined as the number of OEs at a particular TRACON on a particular day. Thus, each facility will have many different observations for each day in our time period of study, which we will assume are independent of each other. This type of model will allow us to capture longitudinal changes in OE occurrence as well as cross-sectional variation across facilities.

TABLE I. TRACON FACILITIES

TRACON Facility	Primary City
N90	New York
D10	Dallas
A80	Atlanta
C90	Chicago
PCT	Washington D.C.
SCT	Los Angeles
D21	Detroit
I90	Houston
NCT	San Francisco
D01	Denver
L30	Las Vegas
P50	Phoenix
S46	Seattle
M98	Minneapolis
S56	Salt Lake City
A90	Boston

For our model, we assume that the occurrence of daily operational errors at any TRACON follows the Poisson distribution. The probability of observing a specific number of OEs for a given facility and date is shown by the following equation:

$$P(y_i = Y_i) = \frac{e^{-\lambda_i} \lambda_i^{Y_i}}{Y_i!} \quad (1)$$

where: y_i = OE number at date and facility i

Y_i = observed OE number at date and facility i

λ_i = average number of OEs expected at date and facility i

We will model the expected number of OEs, λ_i , with a logarithm link function of our explanatory variables:

$$\ln(\lambda_i) = \alpha + \sum_j \beta_j x_{ji} \quad (2)$$

where: x_{ji} = explanatory variable j for date / facility i

β_j = coefficient for explanatory variable j

α = the model intercept

A list of our explanatory variables is given below in Table 2. Previous research has investigated the quantity and type of operations and how they influence the occurrence of operational errors. We use two traffic measures to model these metrics: the natural logarithm of the daily traffic and the percentage of daily traffic that is listed as itinerant. The natural log of traffic is used so the coefficient obtained will represent the elasticity between OE rate and traffic. Itinerant operations are flights that are departing or arriving to an airport within the TRACON facility being observed, rather than being a through flight, which originates and exits the observed TRACON without landing.

TABLE II. EXPLANATORY VARIABLES

Explanatory Variable	TRACON Model – Variable Description
Log_Traffic	Natural logarithm of daily operations
Percent_Itinerant	Percent of operations that are itinerant
IMC	Percent of the time daily operations are in IMC conditions
TempF	Average daily temperature (deg F)
WindSpd	Average daily wind speed (knots)
Vis	Average daily visibility (miles)
N90_Audit	Dummy for N90 during 2005 audit
Old_Rule	Dummy for dates prior to June 25, 2007
Spring	Dummy for March, April, May
Summer	Dummy for June, July, August
Fall	Dummy for Sept., October, November

The percent of itinerant operations is used because we assume that controlling non-itinerant traffic is fundamentally different than controlling aircraft arriving or departing from airports within the area. The percentage of flights that are designated as itinerant is a proxy for the complexity of traffic

in the sector, due to the complicated trajectories of departing and arriving flights. The source of the traffic data is the OPSNET database within ASPM.

Another contributor to traffic complexity is the weather in the sector on the day the operational error occurred, which we obtained from ASPM as well. It would be difficult to quantify the weather over the airspace of the entire TRACON, so the largest airport within the facility's airspace was used as a proxy for the entire region. For example, the weather at JFK was used to represent the New York TRACON and the weather at SFO was used to represent the weather at the Northern California TRACON. Although this is a somewhat crude measurement, the airspace around TRACONS are small with low flight levels, so the airport weather measurements will be used as a first approximation for TRACON weather.

The weather variables obtained were hourly measures of temperature, wind speed, visibility, and whether the operations were in IMC or VMC. Because hourly weather observations are obtained, each metric was averaged across a day for each facility, weighted by the hourly operations at the airport of interest. Weighing the weather measurements by the hourly operations will give us average measures that are representative of the weather an average flight departing or arriving at the airport of interest will experience. We use this as a proxy for conditions throughout the TRACON.

The dummy variable N90_Audit was set equal to 1 if the observation was at the New York TRACON during the time period of the 45 day audit in 2005. The reason this is included is that the audit revealed a very large number of unreported OEs during this time period that is not representative of the same reporting behavior at other facilities or at other times. Seasonal dummy variables were included to capture the variation across seasons. The Old_Rule dummy is a measure of policy changes in the system. We set this variable equal to 1 for all time periods prior to June 25, 2007, when the separation conformance measure went into effect.

B. TRACON Results

Three models were run for counts of different groups of OEs. The first model used the counts of all OEs, including proximity events. The second model did not include the proximity events (only A, B, and C errors), and the third model uses only the two most severe error types, A and Bs. The results from all three models are presented in Table 3 below.

Table 3 reveals that the Log_Traffic coefficient is less than 2 and highly significant for all three models. It ranges from 1.31 for all OEs up to 1.48 for A & B errors. This suggests that the incidence of more severe errors is more sensitive to traffic than the less severe errors. The second thing to notice is that the Old_Rule coefficient is negative and significant for all three cases, indicating that all types of OEs have increased after the Separation Conformance metric went into effect. As might be expected, the effect is weaker when only severe OEs are considered, but is still highly significant even in this case. The seasonal dummy variables do not reveal any obvious trends, as most of the variables are not statistically significant.

TABLE III. TRACON MODEL REGRESSION ESTIMATES

Variable	All OEs Model			A,B&C Model			A&B Model		
	Estimate		Std. Error	Estimate		Std. Error	Estimate		Std. Error
Intercept	-15.9	**	1.13	-16.7	**	1.29	-16.3	**	1.95
Log_Traffic	1.31	**	0.06	1.35	**	0.07	1.48	**	0.10
Percent_Itinerant	2.54	**	0.93	2.66	*	1.05	0.90		1.54
IMC	0.80	**	0.11	0.73	**	0.12	0.20		0.21
TempF	0.0015		0.002	0.011	**	0.002	0.009	*	0.004
WindSpd	0.056	**	0.006	0.058	**	0.007	0.04	**	0.01
Vis	-0.12	**	0.02	-0.11	**	0.02	-0.16	**	0.03
N90_Audit	3.81	**	0.11	3.76	**	0.12	3.13	**	0.22
Old_Rule	-0.57	**	0.05	-0.43	**	0.06	-0.27	**	0.09
Spring	-0.12		0.07	-0.07		0.08	0.22		0.14
Summer	-0.32	**	0.09	-0.16		0.11	0.12		0.18
Fall	-0.06		0.08	0.02	**	0.09	0.32	*	0.15

** Significant at 1% level

* Significant at 5% level

The weather variables that are significant in each model include temperature, wind speed, and visibility. Increasing wind speed and decreasing visibility are both likely to increase the number of OEs by creating a more complicated airspace to navigate. Temperature, which can be an indicator of overall good weather, has a positive sign, which indicates higher temperature increases the occurrence of all types of OEs. The IMC variable is positive, as expected, but only significant for the models including the least severe types of OEs.

All three traffic coefficients are lower than we expected based on intuition and previous work. One consideration that we left out of the first set of models was the distinction between the consolidated TRACON facilities and non-consolidated ones. Consolidated TRACONs effectively function as a group of smaller TRACONs that are located in the same building. The operational characteristics of these facilities differ enough from the smaller TRACONs to suggest that this model may not be capturing the true effect of traffic on the occurrence of OEs.

To illustrate this concept, imagine that the number of OEs at an individual facility is proportional to the square of its

traffic. Thus a doubling of the traffic at any facility should increase the number of OEs at that facility by a factor of four. Assume we have two identical facilities, with the same traffic and number of OEs. If we combine these two facilities into one, we will now have doubled the traffic, but only doubled the number of OEs, which is not consistent with our assumption that the number of OEs rises with the square of traffic. Thus, if the consolidated TRACONs are actually behaving like the sum of two or more smaller TRACONs, any quadratic or other non-linear behavior at the facility level would be masked by linearly combining the traffic and OEs at each sub-TRACON facility.

To categorize the TRACONs into consolidated and stand-alone facilities, we used the definition of a consolidated TRACON from ASPM. These facilities provide approach control for two or more large hub airports where no single airport accounts for more than 60 percent of the total TRACON traffic count. This metric fits for four different facilities: Southern California (SCT), Northern California (NCT), New York (N90), and Potomac (PCT) TRACONs. To correct for this difference across facility types, we included a dummy variable for the consolidated TRACONs, and recalculated the coefficient estimates. The results are shown below in Table 4.

TABLE IV. TRACON MODEL REGRESSION ESTIMATES

Variable	All OEs Model			A,B&C Model			A&B Model		
	Estimate		Std. Error	Estimate		Std. Error	Estimate		Std. Error
Intercept	-17.0	**	1.05	-17.8	**	1.23	-17.5	**	1.90
Log_Traffic	2.04	**	0.08	2.00	**	0.10	1.95	**	0.16
Percent_Itinerant	-1.75	*	0.88	-1.00		1.04	-1.40		1.55
IMC	0.90	**	0.11	0.82	**	0.13	0.27		0.21
TempF	0.0011		0.002	0.008	**	0.002	0.006		0.004
WindSpd	0.055	**	0.006	0.057	**	0.007	0.04	**	0.01
Vis	-0.13	**	0.02	-0.12	**	0.02	-0.17	**	0.03
N90_Audit	3.83	**	0.11	3.77	**	0.12	3.13	**	0.22
Old_Rule	-0.64	**	0.05	-0.49	**	0.06	-0.31	**	0.09
Spring	-0.11		0.07	-0.06		0.08	0.23		0.14
Summer	-0.29	**	0.09	-0.14		0.11	0.13		0.18
Fall	-0.05		0.08	0.02		0.09	0.32	*	0.15
Consolidated	-0.98	**	0.08	-0.85	**	0.09	-0.60	**	0.15

** Significant at 1% level

* Significant at 5% level

Another interesting change in this model is the sign of the Percent_Itinerant variable, which is now negative. The change is likely due to lower average percentage of operations that are itinerant for traffic at consolidated TRACONs compared with the stand-alone TRACONs. The lower percentage of itinerant operations at consolidated TRACON facilities is another reason to treat them separately from the stand-alone facilities. The Old_Rule remains negative and significant for all three models.

Interestingly, the traffic coefficient is very close to 2 for each of the models, and is highly significant. This suggests that the occurrence of operational errors of all severity levels roughly increases with the square of traffic, all else equal. The Consolidated dummy variable is negative and highly significant for all three models, suggesting that these facilities have fewer OEs than the other facilities, all else equal. This is consistent with our argument about linearly combining facilities where OEs increase with the square of traffic.

C. TRACON Model Fit

Two common measures of goodness-of-fit for Poisson regression models are the deviance and the Pearson Chi-Square statistics. These statistics are used to provide a validation of the Poisson regression assumption that the occurrence of OEs follows a Poisson distribution and thus has equal mean and variance. If the deviance and the Pearson statistics divided by the degrees of freedom in the model are both close to 1, then the Poisson model assumption is generally accepted. If these statistics are greater than 1, this is an indication that the model is over-dispersed (e.g. the variance is actually greater than the mean) and the Poisson model is not valid. Typically in these situations a more general model, such as the Negative Binomial is used.

The other case, where the statistics divided by the degrees of freedom are less than 1, indicating under-dispersion, is not as commonly seen and as a result fewer methods have been

developed to deal with these situations. The deviance statistics divided by the number of degrees of freedom for our models range from 0.330 for the All OE model to 0.137 for the A&B model. The results for the Pearson statistics are 1.33 for the All OE model and 1.07 for the A&B model. The low deviance numbers suggest a poor fit due to under-dispersion but the Pearson numbers suggests a good fit. Thus our results rule out over-dispersion but are ambiguous with regard to under-dispersion.

Boyle and Flowerdew [7] have shown that using Poisson regression on very sparse data sets can lead to low deviance values due to the lack of asymptotic convergence of the deviance statistic to the Chi-Square distribution. A simulation method has been developed to determine if the low deviance is a proper indicator of lack of fit due to under-dispersion or is simply a result of very sparse data [8]. The simulation begins with using the fitted values from the original model as the means of a set of Poisson random variables that represent the true distribution of OE occurrences. These Poisson random variables are then used to create a new set of observed values by taking a random draw from each Poisson random variable for each observation. For each new set of observations, we run the same model and calculate the new deviance. If the model is a proper fit for the data, then the mean of these simulated deviances will be close to the original deviance.

For the first model using all OEs, the simulated deviance mean is 8309 with a standard deviation of about 200. The actual deviance of 9147 is somewhat larger than the simulated mean, thus indicating mild under-dispersion. The A,B & C model has a very similar distribution to the All OEs model, indicating some under-dispersion as well. The model for A&B OEs has a true deviance of 3797 with a simulated mean and standard deviation of 3720 and 185, respectively. The similarity between the simulated mean and the actual deviance suggests that the low deviance value arose simply due to highly sparse data and is not an indication of under-dispersion. Under-dispersion in the first two models, since it is fairly mild, does

not necessarily invalidate our parameter estimates, but further work is needed to address this issue.

D. TRACON Prediction

The final models were used to predict the number of OEs at each facility over the time period studied. For each facility we wanted to test the null hypothesis that the observed data were produced by a distribution defined by the results from our model. If we reject the null, we can conclude that there are differences between the facilities that affect the incidence of OEs and are not accounted for in the model. A common method of evaluating this goodness-of-fit is to use Pearson's Chi-Square statistic, shown by the following equation:

$$TS = \sum_{i=1}^k \frac{(O_i - E_i)^2}{E_i} \quad (3)$$

Where: k = number of categories

O_i = number of observations in category i

E_i = expected number in category i

The test statistic is distributed Chi-Squared for large sample sizes. Also, the expected number of counts in each category should be larger than 5. However, the Chi-Square distribution assumption can be invalid when any category has a much larger observed count than expected count, which we have in many of our facility predictions. Rather than use Pearson's statistic, we will use the G-Test for goodness of fit. The G-Test statistic is based on likelihood-ratio and is approximately Chi-Square distributed with k-1 degrees of freedom. The equation for calculation of the G-Statistic is shown below.

$$G = 2 \sum_{i=1}^k O_i \ln \left(\frac{O_i}{E_i} \right) \quad (4)$$

The number of categories for some of our facilities is very small (2 or 3), so we will use simulation to determine the exact p-value of the G-Test. This method is common when the asymptotic behavior of the test statistic is in question. The method followed is to assume that the null hypothesis is true and draw a new set of observed data using the predicted results as the true distribution. A new G-statistic is calculated and compared to the original G-statistic. This process is repeated 10,000 times for each facility, and the p-value for the G-Test is the percentage of times the simulated G-statistic is greater than the original G-statistic. The simulation results for each facility are shown in Table 5.

The p-values in the table above correspond to the null hypothesis that the model accurately predicts the distribution of the observed data. Thus, a good model fit will have a large p-value in this table, because we will not be able to reject the null hypothesis at a high level of significance. Lack of rejection of the null is not the same as accepting the null, and thus we must be careful when interpreting these values as acceptance of a good model fit. For the very large p-values shown above, however, these at least suggest that the model fit is adequate.

Notice that the model fit for the severe errors (A & B model) is much better than for all the errors together (All OE Model). Perhaps many of the less severe errors are caused by facility-specific effects not included in our model. On the other hand, the model appears to accurately represent the random process generating severe OEs at most facilities.

IV. CONCLUSIONS

Operational errors in terminal radar approach control (TRACON) facilities were modeled using Poisson regression. The daily OE count at each facility was used as the dependent variable while operational and weather measures were used as the independent variables. The rate of daily OE occurrences at TRACON facilities was found to increase with the square of daily traffic, which is consistent with previous research and general intuition. It was also found that consolidated TRACON facilities behave effectively as a sum of several other, smaller TRACON facilities in terms of how the number of OEs is influenced by traffic. The rate of occurrence of all types of OEs at TRACONs has been found to increase after the introduction of the separation conformance severity metric.

Possible under-dispersion exists for the models of the two least severe OE types for the TRACON facilities. The A & B model has a very low deviance likely because of very sparse data. Our Poisson regression model accurately captures the random process generating the most severe types of OEs. That is, the most severe error types are the easiest to predict.

V. FUTURE WORK

Future work could include using more information about each operational error than we included. Although our goal was to disaggregate our data as much as possible, we still have daily counts of operational errors, traffic, and average weather variables. Using the exact conditions in the sector at the time the OEs occurred could be more representative of the true causes of these rare events.

Additional models could be used, such as zero-inflated Poisson regression, to more accurately model the large amount of zeros in the data set. Our model could also be modified to somehow consider the exact way the consolidated TRACONs are acting like the sum of a number of smaller TRACONs. Detailed operational characteristics of these large consolidated facilities would be needed to perform this analysis, however. Also, other effects of policy changes could be investigated if different time periods were used. For example, the implementation of the Air Traffic Safety Action Program or the Traffic Analysis Review Program will both affect the way OEs are reported in the TRACON environment.

TABLE V. TRACON FACILITY PREDICTIONS

TRACON Facility	G-Test P-Values			Associated Airport(s)
	All OEs Model	A,B, & C Model	A & B Model	
N90	0.001	0.014	0.003	LGA / JFK
D10	0.000	0.000	0.961*	DFW
A80	0.000	0.000	0.019	ATL
C90	0.000	0.000	0.419*	ORD
PCT	0.000	0.000	0.000	IAD / BWI
SCT	0.000	0.000	0.092*	LAX / SAN
D21	0.000	0.000	0.195*	DTW
I90	0.000	0.000	1.000*	IAH
NCT	0.005	0.029	0.808*	SFO / OAK
D01	0.009	0.750*	0.408*	DEN
L30	0.002	0.041	0.776*	LAS
P50	0.008	0.008	0.037	PHX
S46	0.037	0.324*	0.602*	SEA
M98	0.001	0.011	0.381*	MSP
S56	0.150*	0.056*	0.440*	SLC
A90	0.000	0.000	0.002	BOS
All Facilities	0.000	0.000	0.000	

* Not significant at 5% level

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