# Generating Diverse Reroutes for Tactical Constraint Avoidance

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Abstract- Decision support capabilities that provide flightspecific reroutes around constraints can enable more flexible and agile management of the airspace. For this benefit to be realized, automation must reliably provide operationally-acceptable alternatives to traffic managers. This paper proposes an approach for generating a small number of diverse, feasible solutions for further evaluation by traffic managers. Using a variation on Dijkstra's shortest path algorithm, reroutes are designed for one or more flights, where multi-flight problems promote the active design of reroute flows. A multi-objective genetic algorithm is employed to evaluate trade-offs between multiple criteria of operational acceptability, removing the need to pre-define relative metric weightings. Finally, a combination of Principal Components Analysis and Spectral Clustering is used to identify distinct groups of solutions and representative reroutes that capture different trade-offs between metrics of operational acceptability. Results are generated for a historical convective weather event and evaluated for their characterization of the trade-space.

Keywords- Traffic flow management; decision support; dynamic rerouting; weather avoidance

#### I. INTRODUCTION

In today's operation, if a flight's route is blocked by convective weather or other severe constraints, a traffic manager must manually identify, coordinate, and communicate an alternate route. Developing individualized solutions is timeconsuming; if a high number of flights are expected to be impacted, large-scale traffic management actions (e.g., airspace flow restrictions, adherence to published routes, etc.) are issued instead. To capture all targeted flights, these large-scale actions must be issued well in advance of the predicted constraints. Given the uncertainty in both traffic and constraint forecasts, these restrictions may ineffectually or even unnecessarily impact flights. To make the best use of available capacity, a more dynamic and agile traffic management response is needed.

Decision support capabilities that reliably provide traffic managers with viable alternatives can attain the benefits sought. Reliability is key for successful deployment – traffic managers must trust that the automation will identify alternatives for most, if not all flights. Furthermore, these alternatives must be operationally acceptable, providing traffic managers with real options. However, as automation may not fully capture the operational environment, returning multiple, distinct solutions increases the likelihood that at least one option is viable.

The challenge is that operational acceptability is determined by numerous factors which are often difficult to express and vary based on the overall environment. Published routes are precoordinated to ensure their acceptability; however, these routes provide few options for avoiding large-scale constraints. Given the dynamic nature of the constraints, research has focused on generating larger option spaces, trading increased reliability of returning a solution for potential loss in operational acceptability of the solution.

Route-based approaches extend the option space to include historically-flown routes, as these solutions are more likely to be operationally acceptable and are fast to construct and evaluate. However, the challenge of finding constraint-free solutions using this approach persists [1]. Extending the option space to include alternatives constructed by modifying established routes can work well within restricted airspaces, such as the terminal area [2, 3], but these are not readily extensible to all airports or airborne flights.

In contrast, "free space" approaches provide the greatest degree of flexibility as they are not constrained to conform to airspace structures (i.e., fixes, waypoints, routes). Generally, approaches in this category overlay a grid of nodes with connections defined between adjacent points. Weather or other airspace constraints can be captured by removing affected nodes or penalizing impacted arcs [4]. One advantage of these approaches is that efficient graph-based optimization algorithms (e.g., Dijkstra's Shortest Path (DSP) Algorithm [5]) can be used to generate the least-cost reroute, where factors other than distance can be used to compute arc costs [6, 7]. However, graph-based optimization approaches are limited to direct evaluation of arc costs; path-dependent evaluation criteria must be measured post-optimization. Iterative post-optimization of shortest path reroutes can improve the viability of these reroutes [8].

An intermediate approach proposed in our previous work [9] defines a network constructed from historically-flown fix-pair segments. The network creation was inspired by terrestrial routing research [10], where any existing roadway can be considered as a possible segment of a reroute. Extending this concept to air traffic rerouting implies that any previously-flown segment can be included in a reroute, even if it is not normally used by traffic travelling between the origin and destination pair of the flight.

Formal optimization approaches can be applied to these networks, but enhancements are required to increase the operational acceptability of the reroutes generated. In Reference [9], a K-shortest path algorithm is used to generate multiple (top K) paths through the network. Returning multiple paths overcomes the limitations associated with generating only a single option, as the reroute is constructed based on a limited set of criteria. However, we found only modest improvements were realized as there was little variation among the reroutes returned.

This paper builds upon previous research, employing a combination of algorithms that can generate a small number of distinct, operationally-acceptable solutions. The solution process begins with a Problem Set, a group of flights which are encountering a common constraint (typically weather) and should logically be considered for coordinated rerouting. Using a variant of DSP, termed DSP-M, a larger and more diverse set of candidate reroutes are generated for each flight from the fix-pair segment network. Advisory Sets, which define the coordinated set of reroutes for the multi-flight problem, are constructed using a Multi-Objective Genetic Algorithm (MOGA).

The MOGA evaluates each Advisory Set against a number of metrics which characterize different aspects of operational acceptability. The MOGA returns the trade-space of good solutions without requiring that the relative importance of each metric be pre-specified. A combination of Principal Components Analysis (PCA) and clustering reduces the number of Advisory Sets while maintaining the desired diversity. Figure 1 depicts the proposed solution approach.



Figure 1. Flow Diagram of Proposed Approach

This paper begins with a brief overview of the operationalacceptability metrics that define the solution trade-space. Section III describes the process of generating flight-specific reroutes using DSP-M and the coordinated reroute Advisory Sets using MOGA. Section IV introduces PCA and discusses how the principal components can be used to identify a reduced number of diverse Advisory Sets. Section V illustrates the proposed approach using a historical convective weather example involving nine (9) flights. The analysis shows that first, DSP-M can reliably generate reroutes for tactical constraint avoidance and that PCA can capture critical trade-offs within a multi-metric space. The reroutes corresponding to three selected Advisory Sets are visualized, highlighting the ability to naturally identify coordinated reroute flows for multi-flight problem. This section finishes with a discussion on the extensibility of the approach and continuing work. Section VI summarizes the contributions of the paper.

## II. EVALUATING OPERATIONAL ACCEPTABILITY

The operational acceptability of a reroute is determined by a number of factors, where the relative importance of each factor may vary based on the current operational environment. Drawing on previous work, this paper evaluates reroutes based on ten metrics, which are grouped into five categories for convenience. The metrics are outlined qualitatively here due to space limitations, but full detail can be found in the references [1, 9, 11, 12]. We readily acknowledge that the metrics considered are by no means exhaustive, but posit that the groupings characterize the set of important considerations and trade-offs and as such, the value of the proposed approach can be demonstrated regardless of the specific metric calculations used.

# A. Measures of Design Acceptability

Design acceptability metrics characterize the geometric properties of the route with respect to airspace geometry and nominal operating procedures. In this paper, we define two metrics to characterize design acceptability; distance and flow conformance.

The distance metric penalizes the additional distance added by a reroute, as compared to the original route, and rewards reductions. The penalty/reward is proportional to the original route's distance but a higher penalty is assessed for active flights (within 20 minutes of departure or airborne) regardless of an increase or decrease in distance as the flight has already been fueled.

Flow conformance calculates the relative (directional) usage of route segments, and is weighted by the normalized segment length. This enables reroutes to briefly use non-traditional segments for constraint avoidance, yet still captures the preference for routing in alignment with nominal airspace operations.

# B. Measures of Management Acceptability

Metrics of management acceptability capture challenges associated with issuing a reroute. In this paper, we define three metrics of management acceptability: coordination, return to original route, number of segments.

The coordination metric penalizes reroutes that transit through different facilities (as compared to the original route) as these reroutes require traffic managers to obtain approval prior to rerouting the flight. Similarly, if a reroute doesn't return to the original route prior to arrival at the destination airport, additional coordination is required to ensure conformance with the airport's arrival configuration. The final metric, number of segments, penalizes reroutes that are overly complex in that they have an excessive number of maneuvers and are therefore more difficult to coordinate and execute.

#### C. Measures of Constraint Avoidance

En route constraints arise when there is a reduction or loss in airspace capacity. Frequently, convective weather is the driving constraint for a reroute; however, flights planning to fly through congested sectors or restricted airspace regions may necessitate that a reroute be issued.

As weather and the associated forecast uncertainty is the most frequent constraint, we define two metrics: route blockage and blockage probability. The route blockage metric penalizes reroutes that intersect predicted weather areas with discounted penalties for incursions further in the future, to account for forecast uncertainty.

Blockage probability, on the other hand, measures the relative likelihood of constraint incursions as compared to the original route. The goal of this metric is to encourage reroutes that reduce the likelihood of using constrained areas, even if the existence or severity of the constraints is uncertain. We note that the blockage probability metric can be readily extended to capture predicted interactions with non-weather constraints.

The final metric in this category measures whether a reroute increases or decreases overall sector congestion, as compared to the original route, where increases are penalized and decreases are rewarded. Closed sectors can be viewed as having no capacity and reroutes using this airspace can be penalized or even removed from consideration.

#### D. Metrics of Flight Operator Acceptability

Flight Operator acceptability metrics represent typical considerations of airline operators when assessing the impact of a reroute. The airline schedule disruption metric captures the non-linear impact that delay can have on an airline's operation. Although individual flight operators have different operational plans, it is reasonable to assume that longer delays have a more significant impact. Similarly, the impact on the schedule depends on the flight's departure time, where delays incurred earlier in the day are likely to disrupt the schedule more than those incurred in the evening.

#### E. Flights in Flow Metric

The final metric, flights in flow, is distinguished from the above metrics as it applies to the coordination between reroutes within the Advisory Set. Generating flights with common consecutive segments is valuable to traffic managers as it reduces the coordination effort required for rerouting subsequent flights. In addition, generating reroute flows creates structure in the airspace, a benefit when operating in an off-nominal environment.

This paper proposes the flights in flow metric to reward Advisory Sets which contain reroutes that share consecutive common segments. Note that no artificial constraints are imposed - the algorithm does not specify which flights will form flows nor does it constrain where flows begin and end. Instead, the metric computes the Longest Common Segment (LCS) between each pair of reroutes in the Advisory Set. The LCS does not measure physical distance, as we are interested routes that use the exact same segments, as opposed to nearby routes; instead it calculates the number of consecutive shared fixes between two path. The LCS is the length of the longest sequence.

Recall that candidate reroutes are generated for each flight using the DSP-M algorithm. For a given flight f, we define the candidate reroute transiting through the specified intermediate node  $m \text{ as } \hat{p}_m^f$ . If we define  $L_{fi,fj}$  to be the LCS between  $\hat{p}_m^{fi}$ and  $\hat{p}_{\bar{m}}^{fj}$ , then the similarity of the Advisory Set is defined as

$$L = \sum_{j \neq i} \sum_{i} L_{fi,fj} \,\forall i, j \in \mathbf{F}$$

The where F refers to the set of flights defined in the Problem Set. Note, the notation  $\overline{m}$  in  $\hat{p}_{\overline{m}}^{\overline{j}\overline{j}}$  simply implies that the intermediate node *m* need not be the same as in  $\hat{p}_m^{fi}$ .

#### F. Evaluating Reroute Acceptability

A common approach for evaluating a solution against multiple metrics is to compute the weighted sum, resulting in a single measure of reroute acceptability. This approach requires the set of weights, which capture the relative importance of each metric in the overall decision, be pre-specified. These prespecified weights would need to be validated for such an approach to be used within real operations.

The alternate approach proposed in this paper is to compute the Pareto Set of solutions. The Pareto set is defined as the set of non-dominated solutions, where one solution is said to



#### Figure 2. Description of Pareto Front

'dominate' another if at least one of the metrics is better than the corresponding metric from a second solution, and if none of the other metrics is worse than those of the second solution. Put another way, the Pareto set includes all the "best" solutions, in that any solution not in the Pareto set is inferior to any solution in the set.

Figure 2 depicts a simple example of a Pareto Set, representing the trade-off between metric A and metric B. Note that in this example, we seek to minimize metric A but maximize metric B. The "Utopia Point" (which is hypothetical and not a potential solution) refers to the desired performance direction, namely lowest values of metric A and highest values of metric B. Viewing Figure 1, the four solutions in the Pareto Set are represented as colored circles while the other dominated solutions are shown as smaller, un-filled circles.

This section defined 10 metrics, each of which will create a dimension in the Pareto Set. The first nine metrics defined in Sections II.A-II.D can be evaluated against a flight's individual reroute, where the goal is to minimize the value returned. The Pareto Set evaluates the total score for each of these metrics

across all solutions in the Advisory Set. The corresponding Advisory Set score for each metric is the sum of the scores for the individual flight reroutes. Again, the goal is to minimize these values. The final metric, flights in flow, is evaluated for the entire Advisory Set. For this metric, we seek solutions that maximize value.

# III. GENERATING CANDIDATE ADVISORY SETS

This section describes the approach for generating reroutes and subsequent Advisory Sets that provide a small number of diverse solutions for further evaluation by traffic managers. First, reroute candidates are defined for each flight using a variation of the DSP algorithm. The MOGA evaluates how combinations of these reroutes produce Advisory Sets that characterize useful trade-offs between the multiple metrics generated. For example, one set might produce the lowest additional flying time across flights, while another would group most of the flights along a common route segment to promote operational acceptability.

#### A. Network Optimization

The first step in the proposed approach generates a candidate pool of reroutes for each flight. The flights are generated as paths through a network constructed from fix-pairs [9]. This section briefly describes the steps required to create the network and generate flight-specific reroutes using the DSP-M algorithm.

# 1) Constructing the Network

The original route of flight f is deconstructed into the route string – the set of fixes from the current position to the destination airport. The deviation point,  $dev^{f}$ , is the first fix on the original route from which the reroute can diverge and will correspond to the departure airport if the flight is not yet active. Similarly, the rejoin point,  $rej^{f}$ , is the final fix on the original route at which the reroute can reconnect, nominally at the destination airport.

Using the method proposed in [9], the geographic boundary of the network is calculated as an ellipse encompassing the original route between the deviation and rejoin points. The reroute network for flight f, denoted as  $N^f$ , is defined as the subset of fix-pair segments drawn from a database of historically-flown reroutes (denoted as FF) contained within the ellipse. The fix-pair segments or arcs are defined as

$$A^{f} = \{a_{ij}\} \quad s.t.i, j \in N^{f}, i, j \in FF.$$

The original route is included in the network to ensure connectivity and provide increased flexibility for diverging from and reconnecting to the original route.

## 2) Measuring Costs in a Network

Each arc in the network  $A^f$  has an assigned  $\cot c_{ij}^f \ge 0$ , which may be different for each flight using arc  $a_{ij}$ . The arc cost can represent a weighted sum of multiple measures; however, all measures must be defined solely based on the arc and not the path that may include the arc (as many different paths can contain the same arc in a different sequence). Based on previous analyses, we define three arc cost measures: normalized great-circle distance, flow conformance and weather avoidance [9], which are assumed to be weighted equally in the computation of arc costs.

3) Generating Reroute Candidates

The candidate set of reroutes for each flight is generated using a variation of the DSP algorithm, referred to here as DSP-M, where the "M" represents the inclusion of a specified intermediate node through which the shortest path is constrained to pass. Although DSP-M does not generate shortest paths, it produces a more diverse set of candidates, potentially resulting in solutions with higher operational acceptability.

For every flight *f*, the sub-network  $M^f \subseteq N^f$ ; however, for computational efficiency, it is beneficial to further constrain this relationship. To encourage reroute flows within the multi-flight context, the selection of  $M^f$  can be constrained to identify nodes common to multiple networks. As such, we define that for every node  $m \in M^f$  there exists some pre-specified number of flight-specific networks such that  $N^f \ni m$ .

DSP-M generates candidate reroutes using the following approach. Given a node  $m \in M^f$ , the shortest path from  $dev^f$  to m is computed using DSP. DSP also computes the shortest path from m to  $rej^f$ . The two paths are concatenated and the repetitive m is removed. The process repeats until all nodes  $m \in M^f$  have been evaluated.

#### B. Multi-Objective Genetic Algorithm

Each flight can have as many as  $|\mathbf{M}^f|$  paths, potentially resulting in  $\prod_f |\mathbf{M}^f|$  unique Advisory Sets. Given the exponential growth of the design space, enumeration of all possible sets is prohibitive. Instead, a MOGA is employed to conduct an intelligent search and identify optimal trade-offs between the operational acceptability metrics.

#### 1) Algorithm Overview

A Genetic Algorithm (GA) is a heuristic optimization approach that emulates biological evolution [13]. Each individual or chromosome corresponds to a candidate Advisory Set and the genes in the chromosome specify which DSP-M generated reroute is selected for each flight. The fitness of each individual (Advisory Set) is calculated using the operational acceptability metrics described in Section II. For the first nine metrics, the MOGA computes the sum of each metric as evaluated against the individual reroutes specified in the chromosome. The final metric, flights in flow, is evaluated using the definition provided in Section II.E.

Based on their individual fitness, pairs of individuals are selected to populate the successive generation. A pair of 'parent' solutions swap portions of their chromosomes (i.e. sections of their design vectors) via a 'cross-over' operation to generate 'offspring' for the subsequent generation. The GA introduces random changes to individual genes (single parameter values) during a 'mutation' operation to maintain parameter diversity and search the design space beyond local minima. This process continues until the specified termination criteria is met, nominally a fixed number of generations.

#### 2) Creating the Pareto Set

For multi-objective genetic algorithms—which evaluate multiple criteria to define the fitness of individuals—we instead consider the set of non-dominated solutions. To obtain the Pareto Set of solutions, we use the non-dominated sorting genetic algorithm II (NSGA-II) [14]. The NSGA-II selects parents in a generation based on their non-dominance rank and crowding distance. The non-dominance rank of a solution roughly evaluates how "close" to the Pareto front the solution is. Specifically, if we assign a non-dominance rank of 1 to the global Pareto front, then the set of non-dominated solutions after removing the global Pareto front has non-dominance rank of 2, and so on. The algorithm then sorts solutions within each nondominance rank based on their crowding distance, or the average Euclidian distance from each solution to its nearest neighbors with the same non-dominance rank. Solutions farther from their nearest neighbors are more desirable for selection in order to promote more uniformly spaced fronts.

The set of non-dominated solutions consists of Advisory Sets that provide the best trade-offs between the multiple metrics considered. The MOGA populates the Pareto front with solutions from every generation, if qualified. Therefore, the result may contain more solutions than the population of a single generation.

#### IV. GENERATING REPRESENTATIVE ADVISORY SETS

The Pareto Set identified by the MOGA is likely to contain many more solutions than can be evaluated and therefore clustering approaches are typically used to identify a smaller number of distinct solutions [15, 16, 17]. However, for largedimensional trade-spaces, direct evaluation is likely to obscure the key trade-offs sought. As such, this paper proposes to use PCA to identify correlations within the trade-space, reducing the dimensionality of the problem. Spectral clustering is performed on the reduced dataset and representative solutions that characterize critical trade-offs that persist can be readily identified for further evaluation.

#### A. Principal Components Analysis

PCA is a mathematical approach for identifying the rotation matrix of the axes such that the primary axis (first Principal Component, PC1) captures the maximum variation within the Pareto Set. Each subsequent axis is orthogonal and aligns with the next highest direction of variation. Each solution in the Pareto Set corresponds to a point in the objective space, defined by its value for each metric, and therefore, solutions can be expressed as linear combinations of metric values. The correlation matrix, which characterizes the relative co-variation between these metrics, can be readily computed.

Using matrix algebra, the correlation matrix is then transformed into its diagonal components, or eigenvalues. The corresponding eigenvectors provide new directions for the coordinate axes. The directions are ordered by the magnitude of the eigenvalues, where the largest eigenvalue defines the first principal component.

The number of principal components is the same as the original dimension (10, in this case). However, as the variation captured by later components is often quite small, these dimensions can be ignored with little loss in representation. As a general rule, only components with corresponding eigenvalues greater than one are needed to adequately represent the trade-space.

#### B. Clustering the Pareto Set

Clustering can now be done using the remaining principal components. Specifically, a modified spectral clustering algorithm [18] iteratively partitions a group of solutions into two until the stopping criteria are reached. This algorithm does not require that a number of desired groups be determined a priori. Rather, it employs scale factors for the number of nearest neighbors considered to define the mean and variance of a given cluster, which can be tuned to produce a reasonable number of clusters.

#### C. Selecting Representative Solutions

Within each cluster, a single Advisory Set is selected to represent that corresponding portion of the trade-space. As such, it is desirable that the Advisory Set contains the reroutes most common among these solutions. To evaluate the relative importance of each flight's reroute to the cluster, we calculate the central design for each cluster using the approach developed in [19].

The central design of a cluster is defined by the most common reroutes selected for each flight. Specifically, if we consider  $C^k$  to be the set of design vectors in cluster k, then we can compute the probability mass function for the values of each design attribute m, denoted as  $f_m(C^k)$ . Using the probability mass function, we can then define the most common value for each design attribute as well as its frequency of occurrence for the specified attribute across the cluster of design vectors.

$$v_m^k = \arg \max(f_m(\mathcal{C}^k)) \ \forall \ m$$
 3

$$f_m^k = \frac{\max(f_m(C^k))}{|C^k|} \forall m$$

As the design vector  $v_m^k$  may not correspond to an existing Pareto Set solution, we seek the closest Pareto solution to this vector. The distance between a Pareto solution and the central design measures whether two attributes have the same value. As such, the most prominent attributes of the cluster, (in our example reroute candidates) will be part of the representative solution.

To compute categorical distance, we use the weighted Jaccard distance [20], where the weights are defined by the frequency of the most common value. If each solution in  $C^k$  is described by a design vector,  $x^i$ , consisting of *m* design attributes, the Jaccard distance,  $D_{i,v}^k$  is defined as shown in Equation 5.

$$D_{i,v}^{k} = 1 - \frac{\sum_{m} f_{m}^{k} * (x_{m}^{i} \wedge v_{m}^{k})}{\sum_{m} f_{m}^{k} * (x_{m}^{i} \vee x_{m}^{j})}$$
5

Here,  $\wedge$  represents the logical 'and', and  $\vee$  represents the logical 'or'. The representative solution for the cluster corresponds to the design with the lowest Jaccard distance to the central design.

#### V. RESULTS AND ANALYSIS

A historical example used to evaluate the proposed approach involves nine flights requiring reroutes around convective weather, as illustrated in the snapshot shown in Figure 3.



Figure 3. Snapshot of Convective Weather Blocking Routes for Nine Flights on 20 April, 2009 at 20:30

Using DSP-M, reroute candidates were generated for each of the nine flights, showing the reliability of the approach. Table 1 lists the origin and destination of each flight and the number of candidates generated. The final column in Table 1 lists the color used to display each flight's reroute, in the figures at the end of this section.

 Table 1. Flight Origin, Destination, Number of Options

 and Reroute Color

Flight	Origin	Destination	#	Color
А	Westchester County (HPN)	Raleigh Durham International (RDU)	64	Orange Red
В	LaGuardia (LGA)	Raleigh Durham International (RDU)	62	Olive Green
С	Reagan National (DCA)	Charleston International (CHS)	17	Royal Blue
D	Norfolk International (ORF)	Orlando International (MCO)	84	Dark Blue
Е	Raleigh Durham International (RDU)	Reagan National (DCA)	90	Coral Pink
F	Charlotte Douglas International (CLT)	LaGuardia (LGA)	275	Lime Green
G	Hartsfield Jackson Atlanta International (ATL)	LaGuardia (LGA)	272	Gray
Н	Baltimore Washington International (BWI)	Norfolk International (ORF)	15	Magenta
Ι	Hartsfield Jackson Atlanta International (ATL)	LaGuardia (LGA)	322	Maroon

Eigenvalue 20 40 60 8	D	Loading Matrix		
6.4699			Prin1	Prin2
1.8148		Distance	0.96160	0.23517
0.5832		Flow Conformance	0.71743	0.20593
0.4308		Coordination	0.79033	-0.07804
0.3303		Return to Route	0.98075	-0.14735
0.2303		Sector Congestion	0.81036	0.16285
0.0981		Route Blockage	0.55824	-0.79727
0.0300		Blockage Probability	0.91255	-0.38330
0.0109		# of Segments	0.91647	0.08850
0.0017	I	Schedule Disruption	0.30781	0.92100
		Flights In Flow	0.84160	0.15517

# Figure 4. Principal Components: left (a) displays eigenvalues and cumulative variation, and right (b) displays eigenvectors of first two principal directions

Viewing Table 1 we note that the number of candidates varies by flight which has implications for the diversity in Pareto Set solutions. However, this candidate pool still results in 1.8 x 10^17 possible Advisory Sets. Due to the size of the design space and to ensure that the 10-D trade-space is adequately populated, the MOGA returns 5000 Pareto-optimal solutions.

# A. Application of Principal Components Analysis

The Principal Components of the Pareto Set are shown in Figure 4. Figure 4a (left) displays, in descending order, the ten eigenvalues of the correlation matrix which indicates the variation captured in each corresponding direction. The line to the right of the bars indicates the cumulative variation captured by each subsequent principal component, revealing that over 80% of the variation is captured by the first two components. As subsequent components have eigenvalues less than one, the remainder of this analysis focuses on these first two principal components, referred to as PC1 and PC2, respectively.

Figure 4b (right) lists the correlation of each original metric to these first two directions. PC1, which accounts for 64% of the variation in the Pareto Set, has a strong correlation with all but 2 metrics of operational acceptability. These metrics, Route Blockage and Schedule Disruption, are instead correlated with the second direction, PC2, which accounts for an additional 18% of the variation.

To understand these correlations intuitively, we refer back to the metric categories described in Section II. The first category, design acceptability, is defined by two metrics, distance and flow conformance, where low values of each indicate higher acceptability. The correlation between these metrics and PC1 is 96% and 71%, respectively, implying that solutions with low PC1 values have higher design acceptability. Similarly, the metrics defining management acceptability, namely coordination, return to route and number of segments have correlations of 79%, 98% and 91%, respectively, with PC1. Again, as low values in the original metrics indicate better management acceptability, solutions with low values of PC1 will exhibit better management acceptability.

The flights in flow metric is also highly correlated with PC1; however, the positive correlation here is misleading. Recall from Section II.E, that the flights in flow metric computes the number of consecutive common segments between reroutes in the Advisory Set. As such, Advisory Sets with *high* values in PC1 will have more reroutes organized into



Figure 5. Principal Components of Pareto Set Solutions

flows. This relationship indicates the first critical trade-off for this example.

The final two metrics with high correlations to PC1 are sector congestion (81%) and blockage probability (91%). Recall that PCA characterizes the variations exhibited by solutions in the Pareto Set, which is problem-specific. In this example, little variation in blockage probability is exhibited by any of the candidates listed in Table 1 and the small variations that exist are due to differences in the underlying reroutes in the Advisory Set, and thus can be correlated with the other metrics, for example distance.

For sector congestion, the lack of variation is due to the selection of reroutes included in the Pareto Set of solutions. Although the original candidates show variations in performance with respect to sector congestion, the Advisory Sets defined by the MOGA include only reroutes that exhibit the lowest (best) sector congestion. By capturing these relationships explicitly,



Figure 6. Clustering using Principal Components

PCA readily identifies the critical trade-offs that exist in the set of candidates returned.

The remaining two metrics, route blockage and schedule disruption are inversely correlated with each other and highly correlated with PC2. The inverse relationship implies that Advisory Sets which perform better in schedule disruption (lower values) will be represented by solutions with low values in PC2, while solutions that perform better in route blockage (lower values) will be represented by solutions with high values in PC2. This inverse relationship identifies the second critical trade-off in the Pareto Set for this example.

Figure 5 displays the Pareto Set of solutions as points in the two-dimensional space defined by PC1 and PC2. Figure 5 is annotated to include the relationship between the principal components and the original ten metrics. In Figure 5, we see four distinct clusters of solutions. These clusters clearly capture the correlations between the original metrics represented in PC1, but which would likely have been obscured in a higher dimension space.

#### B. Comparison of Pareto Set Clustering

To illustrate the benefits of reducing trade-space dimensionality, we employ the clustering procedure described in Section IV. Figure 6 shows the clusters and representative solution generated when clustering the solutions based on their similarity in the two principal components. Thirty-nine (39) clusters are generated and are distinguished by color in Figure 6. Within each cluster, the representative solution is shown as a rectangle. Viewing Figure 6, we see that the clusters and representative solutions are well distributed throughout the points as viewed in the PC1 verses PC2 space.

To evaluate the benefits of clustering using PCA, the Pareto Set was also clustered by comparing values of all ten metrics. Using the same approach, 35 clusters were generated and a representative solution was identified for each cluster. Figure 7 displays both sets of representative solutions; the representatives identified from clustering the principal components are shown as "green rectangles" and the representatives identified from directly clustering the ten metrics are shown as "pink x's".

Figure 7 compares these solutions within the critical tradespace defined by distance (as a representative for all correlated design and management acceptability metrics), schedule disruption, route blockage and flights in flow. To capture all four dimensions on a single plot, categories of route blockage (displayed across the top of Figure 6) show the associated distance verses schedule disruption trade-offs and flights in flow is shown by marker size, where larger (higher) values indicate more Advisory Sets that create more flows.

Examining Figure 7, we see the general trend between the trade-offs defined by PCA, namely that distance varies inversely with flights in flow and that route blockage varies inversely with schedule disruption. However, this trend is not universal, and solutions exist that balance these objectives. Specifically, the middle panel in Figure 6 captures solutions that have different levels of schedule disruption for the same route blockage cost. Almost all representatives in this category were generated by the PCA-defined clusters; directly clustering the 10 metrics fails to capture this critical region in the trade-space.



Figure 7. Comparison of Representative Solutions Across Critical Trade-Space Metrics. (Marker size corresponds to Flights in Flow values)

#### C. Comparison of Representative Advisory Sets

The gray circles in Figure 7 correspond to three Advisory Sets from different regions of this trade-space. These three solutions were selected to illustrate how the reroutes within these Advisory Sets characterize the associated trade-offs. Table 1 lists the color corresponding to each flight's reroute.

Figure 8 shows the reroutes for Advisory Set 1, representing a solution that performs well in distance and route blockage, moderately in schedule disruption, but poorly in flights in flow. Viewing Figure 8, we see that the reroutes move around the weather shown in Figure 3 (corresponding to the airspace south of the south of the Washington-areas airports). As flights-inflow is not a priority for reroutes in this areas of the trade-space, the reroutes do not generate flows.

Figure 9 shows the reroutes for Advisory Set 2, drawn from the critical trade-space identified in Figure 7. Viewing the reroutes in Figure 7, we see that a number of reroutes overlap for portions of their new route, forming the flows prioritized by solutions in this region of the trade-space. The most prominent flows are located in the NY-area region (top right of the figure), which is a particularly important area for adding structure.

Figure 10, displays the reroutes for Advisory Set 3, which prioritizes schedule disruption above all else. As expected, the reroutes are fairly direct; however, the high route blockage costs indicate that they do not avoid the primary constraint of the problem.

#### D. Discussion

The results generated by the proposed approach provide the reliability sought while also characterizing the critical trade-



Figure 8. Reroutes in Advisory Set 1



Figure 9. Reroutes in Advisory Set 2

space available to traffic managers for consideration. The above analysis highlights the value of PCA in reducing the size of the trade-space and identifying the critical trade-offs that exist. The resulting clusters show the benefit of reducing the design space in this manner, namely that critical trade-offs are better represented than through directly clustering all metrics, where the latter approach can overlook regions of the trade-space containing diverse solutions.

However, both approaches still produce too many clusters for a traffic manager to evaluate and further reduction is needed. Although this is an area of continuing research, a few promising directions have been identified. First, PCA can be directly



Figure 10. Reroutes in Advisory Set 3

included within the MOGA, as suggested by Reference [21]. This approach would limit the size of the Pareto Set produced by the MOGA and could significantly reduce the computation effort required; however, additional analysis is required to ensure that the solutions generated would provide diverse and viable options to traffic managers.

An alternate approach is to use PCA in order to identify persistent correlations between the multiple metrics considered, potentially identifying a set of relative weightings that can be used to rank solutions. As opposed to a single static prioritization between metrics, this approach could propose multiple weightings, resulting in multiple solutions being returned. Furthermore, the selection of potential weightings could be influenced by problem-specific parameters. For example, if congestion isn't a major consideration, then weights that emphasize congestion-avoiding solutions could be replaced by weights which vary the importance of other metrics.

In either case, additional testing on multiple examples is required. Although this problem is representative of a convective weather situation, flights were limited to those that can be captured on a fix-pair segment network, an assumption that would need to be relaxed for more general applications. Furthermore, the computation time associated with the MOGA is relatively fast but not fast enough for real-time. However, MOGA computation time is directly related to the size of the Pareto Set sought – insights gained through additional analysis can identify appropriate methods for reducing the Pareto Set and the associated computation requirements of the MOGA.

# VI. CONCLUSIONS

This paper describes an approach for generating a set of weather/constraint avoidance reroutes for tactical traffic flow management applications. In this approach, multiple flights are considered in a coordinated way, and several feasible sets of reroutes (Advisory Sets) are produced that offer meaningful tradeoffs among important performance metrics. This will provide traffic managers with multiple, distinct options for resolving constraints, making it more likely that an operationally acceptable solution can be found quickly. The approach leverages multi-objective optimization, principal component analysis, and spectral clustering to characterize and search huge design spaces, and to isolate the critical design trade-offs that must be considered.

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#### **ACKNOWLEDGEMENTS**

The authors would like to thank all who helped with this work for their valuable insights, especially Daniel Greenbaum, Dave Chaloux and Steven Zobell from the MITRE Corporation, whose contributions were integral to this research.

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