ANALYSIS OF SATURATION AT THE AIRPORT-AIRSPACE INTEGRATED OPERATIONS

A case study regarding delay indicators and their predictability

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Abstract-This paper develops a functional analysis of the operations that represent the aircraft flow through the airportairspace system. In this analysis, we use a dynamic spatial boundary associated with the Extended Terminal Maneuvering Area (E-TMA) concept, so inbound and outbound timestamps can be considered. The aircraft flow is characterized by several temporal milestones related to the Airport Collaborative Decision Making (A-CDM) method, which allows us to study the successive hierarchical tasks. By considering the accumulated delay across the different processes and its evolution, different metrics are proposed to evaluate the system's state and its ability to ensure an appropriate aircraft flow in terms of timesaturation. The objective is to establish a taxonomy that classifies the system's capacity to "receive and transmit" the expected aircraft flows. Finally, the relationships among the factors that influence the aircraft flow are evaluated to create a probabilistic graphical model, using a Bayesian Network approach. This model predicts outbound delays given the probability of having different values at the causal control variables. The methodology is developed through a case study at Adolfo Suárez Madrid-Barajas Airport (LEMD): a collection of 1,500 turnaround operations (registered at the peak month of 2015) is used to statistically determine the aircraft flow characteristics. The contribution of the paper is twofold: it presents a new methodological approach to evaluate the system state at the rotation stage and it also provides insights on the interdependencies between factors influencing performance.

Keywords- aircraft processes, system saturation, performance indicators; delay propagation; Bayesian Networks

I. INTRODUCTION

Air transport depends on a complex network architecture, where several facilities, processes and agents are interrelated and interact with each other [1]. In this large-scale and dynamic system, airports represent the interconnection nodes that facilitate aircraft distribution through the network and transport modal changes for passengers [2]. José Manuel Cordero García and Lucía Meler García CRIDA A.I.E. (Reference Center for Research,

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Potential incidents, failures and delays (due to service disruptions, unexpected events or capacity constraints) may propagate throughout the different stages of the system, making the network vulnerable [3]. This situation has led to system-wide congestion problems and has worsened due to the strong growth in the number of airport operations during the last decades [4].

The economic cost of congestion in this interconnected and sometimes overscheduled network of airports and aircraft is enormous: direct costs due to flight delays in Europe reached $\notin 1,250$ million during 2010, according to [5]. In the United States, during 2007, the directly or indirectly costs originated by delays were around \$40,700 million [6]. Furthermore, delays have also a substantial impact on the schedule adherence of airports and airlines, passenger experience, customer satisfaction and system reliability [7] [8].

A significant portion of delay generation occurs at airports, where aircraft connectivity acts as a key driver for delay propagation [9]. During 2015, in the EUROCONTROL Statistical Reference Area, the share of reactionary delay (due to the late arrival of the aircraft on its previous leg) was 44% of total delay minutes (4.6 minutes of the 10.4 average delay per flight) and airline related delays (a category that includes crew connections) accounted for another 29% of delay minutes [10]. Moreover, 33% of all delayed flights in the United States in 2015 where due to reactionary delays (40% of total delay minutes), while airline delay was the cause for another 28% of delayed flights and 32% of total delay minutes [11]. "Rotation" (flight cycle through the airport and its surrounding airspace, from inbound to outbound processes) is therefore the stage that has the greatest influence on punctuality within the entire air transport network and accumulates its impact over the day [3]. Hence, this paper focuses on the rotation stage and analyzes the aircraft flow through the airport operational environment, which is the dominant mechanism by which delays propagate through the air transport network [12].

The evolution of a flight can be described as a flow of events or processes [13]. Each of these events occurs before the next, and if any of them gets delayed, this may result in subsequent processes also being delayed (unless certain buffers or "slacks" are added into the times allocated to the completion of certain events). In order to analyze the evolution of the aircraft flow and the potential delays in the successive phases, this paper follows a "milestone approach" by assigning completion times to each event. This view, in line with the Airport Collaborative Decision Making (A-CDM) method [14] allows us to understand the operational performance and the potential saturation of the system. Saturation is here understood as the capacity of the airport-airspace system to "receive and transmit" aircraft flows in an appropriate time.

In the analysis we use a dynamic spatial boundary associated with the Extended Terminal Maneuvering Area (E-TMA) concept, which allows us to consider inbound and outbound timestamps. This management boundary (airport centric limit of 200-500 NM) has already been implemented at multiple airports, with a horizon that varies from around 190 NM for Stockholm to 250 NM for Rome and 350 NM for Heathrow [15]. The E-TMA (and not just the basic on-ground turnaround path in the airport that connects inbound and outbound flights) is selected in order to integrate delay propagation in the airport system with global delays in the air traffic network. This approach reflects the interaction between airport and airspace integrated processes. In time, we restrict actions to a tactical phase (day of operations) in order to consider the primary and initial inefficiencies at the system.

The main objectives of the study are: (a) to analyze the aircraft flow of processes, in order to define metrics and indicators that enable airport operators to assess the system state (in terms of time-saturation); and (b) to generate a practical probabilistic model that predicts the outbound delay given different explanatory variables.

II. BACKGROUND

This paper revises three main topics: the airport-airspace integrated flow of an aircraft, the propagation of delays through the E-TMA processes and the evaluation of the system efficiency in terms of time-saturation.

A review of the literature about airport-airspace integration illustrates that several prior studies have deal with the importance of connectivity at airports [16] [17] [18] [19] [20]. This paper revises the linkage between inbound and outbound flights by assessing the aircraft operational flow (turnaround integration in the air transport network). This approach is in line with past studies [13] [21] [22] and with the SESAR's "Airport Transit View" concept [23]. Our main contribution in this field is the construction of a business process model that shape the airport-airspace integration, by extending the spatial scope to the E-TMA boundaries. The statistical characterisation of the different processes enables us to understand the particularities of the rotation stage.

Regarding delay propagation through the air transport system, a large number of studies deal with the complexity of the network [24] [25] [4] and the potential impact of delays on the system's reliability [26] [27] [28] [29]. Delay propagation is a global process fostered by relationships inside the network: disruptions in one part of the system can propagate to many others [7] [9]. Therefore, network analysis provides a global view of the propagation process [30]. Nevertheless, a significant portion of these propagations (44% in 2015 according to [10]) occurs at airports (i.e. the nodes of the system): "rotation" (delayed flight cycles) is the stage that has the greatest impact on punctuality within the entire air transport network. Delay propagation affecting internal E-TMA and airport processes has received little attention [31] [21]. The contribution of our study relies on adjusting the spatial scope of the problem to the rotation stage and the potential effects of its congestion.

The inherent complexity of the delay propagation problem and the inherent challenges in predicting system behavior explain the use of different modelling techniques: queuing theory [4] [32], stochastic delay distributions [33], propagation trees [8] [34] [35], periodic patterns [36], chain effect analysis [37], random forest algorithms [12], statistical approaches [38], non-linear physics [39], phase changes [40] and dynamic analysis [9]. In this paper, delay propagation patterns and influence variables are characterized using a Bayesian Network (BN) approach, including stochastic parameters to reflect the inherent uncertainty of the performance of the aircraft flow at the E-TMA. Several studies [41] [42] demonstrate the utility of BNs as a methodology for modelling the diffusion of events and incidents from a node-level to a system-level (interdependence of multiple factors). Moreover, [43] [44] confirmed that BNs can explain how subsystem-level causes propagate to provoke system-level effects, specifically focusing on how delays at an origin airport propagate to create delays at a destination airport.

Regarding time performance metrics and efficiency indexes, [45] established the foundations with regard to punctuality and predictability indicators in aviation. The generally accepted key performance indicator (KPI) for operational air transport performance is 'punctuality', which can be defined as the proportion of flights delayed by more than fifteen minutes compared to the published schedule [7]. The fifteen-minute threshold for defining arrival and departure delay has historically been common to both Europe and the US [46] [47]. SESAR's Performance Targets [48] significantly refined this approach to delay measurement, by developing new parameters, indicators and targets. References [30] [46] showed that, although delay propagation remains a significant and costly operational challenge to ATM (Air Traffic Management), there is a significant absence of metrics that specifically measure this problem. Findings settled by [30] [49] developed a framework for complexity and new metrics as regards ATM.

Moreover, [50] analyzed the applicability of a series of network connectivity and concentration indexes, in order to typify complex airline network configurations. Other studies have proposed performance indicators for airports operations [51], delay model calibration [52] and delay propagation networks [40]. For the purposes of this paper, different metrics are formulated to evaluate the system's state and its ability to ensure an appropriate aircraft flow in terms of time-saturation. Therefore, our main contribution in this field is a taxonomy that classifies the system's capacity to "receive and transmit" the expected aircraft flows.

III. METHODOLOGY & DISCUSSION

The analysis is divided into two steps:

Firstly, we develop a theoretical appraisal of the aircraft operation within the E-TMA, characterizing the processes and structuring the different time-milestones. This provides us with a conceptual framework (a business process model) for the practical analysis of the rotation flow.

The second part of the analysis is developed through a case study at Adolfo Suárez Madrid-Barajas (LEMD) Airport. We assess the system state and saturation by evaluating time efficiency performance (through the processes previously recorded in the first section), and also by defining metrics, indexes and performance indicators that represent the delay behavior. After that, a probabilistic model is assembled considering the interactions among the different delay explanatory variables. This model offers information about the system state, by predicting outbound delays. The method is applied to a practical case study to validate its contribution.

Fig. 1 shows the logic behind the analysis.



Figure 1. High level methodology for the analysis

A. Model for airport-airspace integrated operations (rotation stage)

The aircraft rotation stage is usually the critical node for the air transport network: incoming aircraft continue on the subsequent legs of their planned itineraries and crew members and passengers may connect to other flights or other transport modes [9].

The aim of the study is the description of the "visit" of an aircraft to the E-TMA, as an extension of the SESAR's "Airport Transit View" concept. This "visit" consists basically of three separate sections [23]:

- The final approach and inbound ground section of the inbound flight.
- The turnaround process section in which the inbound and the outbound flights are linked.
- The outbound ground section and the initial climb segment of the outbound flight.

Developing the conceptual structure of the aircraft flow within the E-TMA requires input from various sources and consists of four main steps [53] [54]:

1. The first step is a review of relevant literature and existing aircraft flow models [23] [55].

2. Next, a hierarchical task analysis is developed [56]. This appraisal follows a top-down approach that incorporates several sources of information in order to give a detailed understanding of the processes:

a) Analysis of operations manuals [57] [58], standards and procedures [59] [60] [61] [62] [63].

b) Observations at Adolfo Suárez Madrid-Barajas Airport (LEMD) during 2015.

c) Structured communications with relevant stakeholders (Table I).

3. The previous steps leads to an initial process model.

4. Finally, the initial model is refined and validated with the help of subject-matter experts (Table I).

TABLE I. LIST OF INFORMANTS, INTERVIEWEES AND CONTRIBUTORS

Organization	Stakeholder
AENA - Spanish Airport Authority and Airport Manager.	Airport operator
IBERIA - Member of International Airlines Group (IAG).	Airline
ENAIRE - Spanish Air Navigation Service Provider.	Air Navigation Services Provider (ANSP)
IBERIA Airport Services.	Ground Handling Agent
DGAC – Spanish General Directorate of Civil Aviation. This is a public body answerable to the Ministry of Public Works.	Policy maker - Regulator
AESA - Spanish Aviation Safety and Security Agency.	Policy maker - Supervisor

Fig. 2 depicts the methodology for creating a Business Process Model (BPM) of the aircraft flow at the E-TMA (rotation stage).



Figure 2. Methodology for creating the BPM of the aircraft flow

We employ Unified Modelling Language (UML) to graphically represent the BPM. UML is a visual modelling language that enables a pattern of a system to be created [64]. The designed conceptual structure for the airport-airspace integrated operations is basically a UML sequence diagram (Fig. 3). This model is now confronted to the operational milestones defined by the A-CDM methodology. A-CDM aims at improving the overall efficiency of airport operations by optimizing the use of resources and improving the predictability of events. It focuses especially on aircraft turnaround and pre-departure sequencing processes [14]. The milestones approach main goal is to achieve common situational awareness by tracking the progress of a flight from the initial planning to the take off. It describes the progress of a flight from the initial planning to the take off by defining "timestamps" to enable close monitoring of significant events [14]. Fig. 4 and Table II show the set of selected milestones along the progress of the flight at the A-CDM concept.



Figure 3. UML for the BPM of the aircraft flow (airport-airspace integrated operations)



Figure 4. Milestones reflecting the progress of the flight (A-CDM concept) [14]

 TABLE II.
 SELECTED MILESTONES ALONG THE PROGRESS OF THE FLIGHT AT THE A-CDM CONCEPT [14]

Number	Milestone
M1	EOBT (Estimated Off-Block Time)-3hrs
M2	EOBT (Estimated Off-Block Time)-2hrs
M3	Take Off from Outstation
M4	Local Radar Update
M5	Final Approach
M6	ALDT (Actual Landing Time)
M7	AIBT (Actual In-Block Time)
M8	Actual Ground Handling Starts
M9	TOBT (Target Off-Block Time)
M10	TSAT (Target Start-Up Approval Time)
M11	Boarding Start
M12	ARDT (Aircraft Ready Time)
M13	ASRT (Actual Start-Up Request Time)
M14	ASAT (Actual Start-Up Approval Time)
M15	AOBT (Actual Off-Block Time)
M16	ATOT (Actual Take Off Time)

By combining the BPM and the milestone approach, Fig. 5 shows a conceptual diagram for the E-TMA (airport-airspace stage). This diagram allows us to:

- Determine significant events in order to track the progress of the flight (arrival, landing, taxi-in, turnaround, taxi-out and departure) and the distribution of these key events as milestones.
- Ensure linkage between arriving and departing flights.
- Assess time efficiency performance, which is measured for each milestone or between two milestones.
- Enable early decision making when there are disruptions to an event.

B. Evaluation of the system level of saturation

The analysis of the system level of saturation is developed through a case study at Adolfo Suárez Madrid-Barajas Airport (LEMD). The methodology can be nevertheless applied to other airports, by adjusting the model to the infrastructure characteristics, the operational situation and the available data.

Fig. 6 shows the structure of LEMD, with four runways (36L-18R, 36R-18L, 32L-14R, 32R-14L), two terminal areas (T123 and T4T4S) and 163 parking spaces [65].



Figure 5. Combination of the BMP for the airport-airspace integrated operations and the A-CDM concept)



Figure 6. Madrid Airport (LEMD) functional structure [65]

LEMD is a large airport in terms of passengers and aircraft movements (50,420,583 passengers and 378,150 aircraft movements in 2015, according to [66]). Therefore, there were sufficient operations (and a continuous demand) during the observation period: the first week of July, which was the peak month in terms of traffic (2,997,408 passengers and 25,516 aircraft movements, according to [66]).

The operational preferential configuration at LEMD is called north configuration, with arrivals from runways 32L/32R and departures from runways 36L/36R. The non-preferential configuration (south) presents arrivals from runways 18L/18R and departures from runways 14L/14R. Night flights (between 23:00 and 07:00 local time) use 32R (arrivals) – 36L (departures) for north configuration and 18L (arrivals) – 14L (departures) for south configuration [65].

Regarding registered delays at LEMD, as a departure airport it ranked number 13 among the top 20 delay affected departure airports in Europe during 2015, with 10.1 minutes of average delay per departure (an increase of 19% since 2014), 25.7 minutes of average delay per delayed departure and 39.3% of delayed departures [10]. As an arrival airport, LEMD ranked number 10 among the top 20 delay affected arrival airports in Europe during 2015, with 9.6 minutes of average delay per flight (an increase of 16% since 2014), 27.3 minutes of average delay per delayed arrival and 35.3% of delayed arrivals [10].

A collection of 1,500 turnaround operations at LEMD is used to describe the aircraft flow characteristics, through a statistical analysis of the processes. The size of the sample allows us not only for post analysis but also for developing reliable predictions and for studying interdependencies between the turnaround processes and schedule adherence. The observation period corresponds to the first week of July 2015. Data include information about processes milestones, delays, route origin and destination, runway and stand use, airline and aircraft characteristics (type and registration number). This allows us to link the inbound and outbound flights, assessing their "turnaround" operation. Although the dataset is rather out of date, we mainly use it to calibrate the model and trace the airport-airspace integrated operations.

Fig. 7 shows the demand profile for the 1st of July 2015 (baseline day scenario) against the practical capacity of the airport. Fig. 8 depicts the accumulated hourly delay for arrival and departure operations against the demand profile. Departure delay is defined by the sum of arrival upstream delay and the

aggregated delay at the rotation stage. Delays can be positive or negative, as they are defined in relation to scheduled times. Finally, Fig. 9 shows the hourly number of turnaround operations against the airport total capacity and the hourly accumulated final departure delay.



These figures show that arrival delay increases and accumulates its impact over the day, due to the network effect. But departure delay does not follow this pattern, which implies that the airport-airspace system is somehow capable of absorbing a fraction of the arrival delay across the rotation stage. We analyze this aptitude by studying the different processes that were previously identified with the BPM and the milestone approach. It is the evaluation of the system level of saturation. Time efficiency performance is measured for each milestone (when scheduled and actual timestamps are available) or between two milestones (to assess the process length). Table III illustrates the steps that are appraised (given the available data for the case study). It contains the definition of the step, its importance and influence over the analysis and the statistical data, which allows characterization and modeling of processes (by fitting the duration or the starting time accuracy of the process to a statistical distribution).

Measure		Statistical data
		(mean and standard deviation)
AIBT-SIBT. Arrival delay	It represents the upstream arrival delay (reactionary delay), by assessing if the In-Block operation (timestamp) is developed as scheduled, with delay or in advance.	$\mu = 9.8 \min (\sigma = 28.9 \min)$
ALDT-AIBT	It represents the Taxi-In process length.	$\mu = 8.8 \min (\sigma = 16.7 \min)$
Actual turnaround time (AIBT- AOBT) against Scheduled turnaround time (SIBT-SOBT). Turnaround delay.	It represents a measure for evaluating if the turnaround operation at the airport is developed as scheduled, with delay or better than expected (absorbing delay). It allows assessing the relationship among the arrival delay and the departure delay for the different operations, i.e. the ability of the airport to absorb the arrival delay.	$\mu = 4.7 \min (\sigma = 27.2 \min)$
AIBT-AOBT	It represents the turnaround process duration.	$\mu = 151.9 \min (\sigma = 168.6 \min)$
ASRT-ASAT	It allows assessing the difference in time between the aircraft operator request for start- up and the actual start-up approval permission by the Air Traffic Controller (ATC).	$\mu = -1.5 \min (\sigma = 6.7 \min)$
ASAT-AOBT	It allows assessing the difference in time between the actual start-up approval permission by the Air Traffic Controller (ATC) and the Off-Block operation.	$\mu = -1.3 \min (\sigma = 40 \min)$
TSAT-ASAT	It allows to understand if there is some delay between the target time for start-up and the actual one.	$\mu = 0.9 \min (\sigma = 33.1 \min)$
AOBT-SOBT	It represents a measure for evaluating if the Off-Block operation (timestamp) is developed as scheduled, with delay or better than expected (absorbing delay).	$\mu = 14.7 \min (\sigma = 6.7 \min)$
AOBT-ATOT	It represents the Taxi-Out process duration.	$\mu = 15.8 \min (\sigma = 25.5 \min)$
Actual Taxi-Out duration		
(AOBT-ATOT) against Scheduled Taxi-Out duration (SOBT-STOT), Taxi-Out delay	It represents a measure for evaluating if the Taxi-Out operation at the airport is developed as scheduled, with delay or better than expected (absorbing delay).	$\mu = 9.9 \min (\sigma = 29.2 \min)$
Departure delay	Departure delay = Arrival delay + Turnaround delay + Taxi-Out delay	$\mu = 18.5 \min (\sigma = 24.8 \min)$

TABLE III. LIST OF INFORMANTS, INTERVIEWEES AND CONTRIBUTORS

We then assess the system time efficiency performance and its level of saturation by evaluating three mutually exclusive stages: arrival (including Taxi-In), turnaround and Taxi-Out.

Fig. 10 shows the delay pattern for the total delay (sum of the accumulated delay in each of the stages) through the first 3 days of July 2015, while Fig. 11 shows the delay pattern at each stage.



Figure 10. Total delay for each operation

Fig. 12 represents the accumulated hourly delay for each of these stages (arrival, turnaround, Taxi-Out and total) against the number of operations at the observed hour. It can be seen that there is not a clear relationship between delay and the amount of operations for the partial stages. Nevertheless, this relationship does appear when assessing total delay. There is also a correlation between arrival delay and turnaround delay: the turnaround step is partially absorbing the arrival delay.



Figure 11. Delay pattern through the first days of July 2015



Figure 12. Accumulated hourly delay at each stage against the hourly number of operations

Then, we analyze the turnaround stage, as it is found to be fundamental when assessing the system ability to absorb delays. Fig. 13 illustrates the histogram of the actual turnaround time. Although the sample is rather heterogeneous, almost 70% of turnaround operations last for less than 120 min. 36% of operations are in the range between 30 and 60 min. This allows revising the system operational behavior and the demand characteristics.



Figure 13. Histogram for the turnaround stage duration

Delay at the turnaround stage can adequately be represented by a Normal distribution ($\mu = 4.7$ min, $\sigma = 27.2$ min), being expressed with the following probability density function:

$$f_{NORMAL}(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\left(\frac{x-\mu}{2\sigma}\right)^2}$$
(1)

The fitting of delay turnaround data to a Normal distribution is achieved with a chi-square test at 95% significance level (confidence interval) and with 7 degrees of freedom (Fig. 14). This procedure was already found efficient in [21]. Fitting delay to a statistical distribution may help in the definition of an operational buffer or "slack", with the final objective of absorbing arrival delay.



Figure 14. Turnaround delay fitting to a Normal distribution

The delay behavior at each stage and the system level of saturation is evaluated by the definition of several indexes and performance indicators.

1) K_i : delay evolution indicator (applied to the total and to each of the three partial stages)

With this indicator, we seek to measure the system operation during a flight turnaround, considering the system behavior over the last flights. To calculate the value of this indicator, we use two parameters: the average delay over the last 20 operations and the delay generated for that operation.

$K_{i} = \frac{(Delay for flight i) - (Average delay over the last 20 flights)}{|(Average delay over the last 20 flights)|} (\%)$ (2)

The average delay of the last 20 flights can be calculated as a basic average (Fig. 15) or by weighing the operations to assign a greater impact on the latest (moving average). Data obtained from the average delay over the last 20 flights also provides a diagram of delay concentration throughout the day (an application of the Gini index [67]).

The evolution of this indicator (Fig. 16) evaluates the system level of saturation. It enables us to judge the efficiency of the actions that have been taken to solve the congestion problems at the different elements involved in the turnaround process. For this purpose, it will be necessary to study both the time it takes to improve the situation and also the level of suitability of the actions (percentage of K_i variation obtained).



Figure 15. Average total delay over the last 20 flights (01/07/2015)



Figure 16. Ki (total) evolution throughout the day (01/07/2015)

2) Hourly delay index based on the aircraft type (an application of the Herfindahl-Hirschman Index [68])

Each type of aircraft, according to its wake-turbulence category (H, M, L) [69], requires different operational procedures (with different levels of complexity) that may have an impact on the final delay. To calculate the value of this indicator, we use two parameters: the weight of each aircraft wake-turbulence category (β_i) and the average delay of the aircraft operation at the system (D_i). We have two ways of defining the index:

$$\alpha_1 (hourly) = \frac{\sum (\beta i * Di)}{N^2 ops} \left(\frac{min}{op}\right)$$
(3)

$$\alpha_{2}(hourly) = \frac{\sum(\beta i * Di)}{N^{\circ} ops * < hourly average delay >} (\%)$$
(4)

This index allows measuring the concentration of delays within each hour, referring to the type of operations served. The aircraft classification could be changed to reflect the influence of different variables: the engine category (turbofan, turboprop), the aircraft size (narrow body, wide body) or the flight type (regional, national, international).

3) Hourly performance indicator (total operations)

This indicator evaluates the level of saturation that presents each hour in reference to the overall situation of the airport throughout the day. By using the average daily total delay at the airport, a global situation can be established through a gradation (e.g.: absorbing delays, normal operation, congested, saturated). That allows us to assess the saturation level at each hour compared to the daily average.

$$\delta_i = \frac{Average \ delay \ (hour \ i)}{Daily \ average \ delay} \ (\%) \tag{5}$$

If δ_i is between 0% and 100%, the system is generating less delay at this hour than in the average day (the airport is less congested or has been operating in a better way). Values greater than 100%, will indicate a congestion level at that time greater than the average, or a lower performance of operations.

This indicator can be used at an "a posteriori" analysis, by considering the overall day operations as the timeframe (denominator); or as a "real-time" evaluator, by considering the average delay of the previous hours as the denominator. The latter option allows us to appraise how the airport situation is evolving across time in terms of saturation.

4) Hourly performance indicator (contribution of each stage)

We have defined three stages within the total performance of the system: arrival, turnaround and Taxi-Out. This indicator allows us to assess the saturation level that presents each of these processes throughout the day.

$$\theta_{ij} = \frac{Average \ delay \ of \ the \ stage \ i \ at \ hour \ j}{Average \ delay \ for \ hour \ j} \ (\%) \tag{6}$$

This indicator provides the percentage of delay that each of the three elements introduces at each hour. θ allows us to understand the evolution of the level of saturation at each stage throughout the day (Fig. 17) and also to evaluate the principal contributor to the total delay at each hour (Fig. 18). As the previous metric, this indicator can be used at an "a posteriori" analysis or as a "real-time" evaluator.

5) Global performance indicator (an application of the Lerner Index [70])

This indicator seeks to measure delay concentration throughout the day, assigning greater importance to the hours when the system has been more congested. This allows evaluating if the actions taken during these periods of saturation have been correct in terms of reducing delay. To calculate the value of this indicator, we use three parameters: γ (the influence coefficient of each hour), <AD_i> (average delay at hour i), <ADt> (average delay over the day). To assign an objective weight to each hour, γ is defined as follows:

$$\gamma_i = \frac{N^{\circ} of operations at hour i}{N^{\circ} of daily operations}$$
(7)

Then, the global index η is defined as:

$$\eta_1 = \sum_{1}^{24} \gamma i * < AD_i > (min)$$
(8)

$$\eta_2 = \frac{\sum_{1}^{24} \gamma i * < AD_i >}{< ADt >} (\%) \tag{9}$$

This indicator compares the system performance at a given hour against the daily operation. It also assesses if the operational procedures are reducing the level of saturation: if γ is high (i.e. this hour represents a lot of traffic from the daily total), but <ADi> is low (i.e. the system is generating little delay), it is a symptom that the actions to reduce delay are adequate (Fig. 19).

Moreover, if we compare the total delay obtained without the weights with the total delay obtained by this method, the result illustrates how fractional the generation of delays has been throughout the day.



Figure 17. Evolution of the turnaround stage saturation throughout the day (01/07/2015)



(01/07/2015)

The definition of these indexes may allow policy makers or airport operators to establish a taxonomy that classifies the system's capacity to "receive and transmit" the expected aircraft flows. It also enables us to make time projections of future delays throughout the day.



Figure 19. γ against hourly average delays throughout the day (01/07/2015)

C. Predicting departure delay (a causality analysis)

Finally, the study is completed with a causality analysis through a Bayesian Network approach, which aims to understand the interdependencies between factors influencing performance and delay.

Bayesian networks (BNs) are graphical probabilistic models used for reasoning under uncertainty [71]. A BN is a directed acyclic graph (DAG), in which each node denotes a random variable, and each arc denotes a direct dependence between variables (nodes that are not connected symbolize variables that are conditionally independent of each other) [72]. The DAG that results from the construction of a BN is quantified through a series of conditional probabilities based on data or information available on the system or problem and defines a factorization of a joint probability distribution over the variables represented in the DAG [73] [74]. The factorization is represented by the directed links in the DAG [75]. That is, each node is associated with a probability function that takes, as input, a particular set of values for the node's parent variables, and gives (as output) the probability (or probability distribution, if applicable) of the variable represented by the node [73]. Therefore, the BN model structure (nodes and arcs) encodes conditional dependence relationships between the random variables. Each random variable is associated with a set of local probability distributions (parameters in the Conditional Probability Tables (CPT)). Probability information in a BN is specified via these local distributions [74]. Therefore, a BN is a pair (G, P), where G is a directed acyclic graph (DAG) defined on a set of nodes X (the random variables), and P = {p ($x_1 \mid \pi_1$), ..., p ($x_n \mid \pi_n$)} is a set of n conditional probability densities (CPD), one for each variable. It is the set of parents of node Xi in G. The set P defines the associated joint probability density of all nodes as (the chain rule for BN) [71] [72]:

$$p(\mathbf{x}) = p(x_1, \dots, x_n) = \prod_{i=1}^n p(x_i | \pi(x_i))$$
(10)

The graph G contains all the qualitative information about the relationships between the variables, no matter which probability values are assigned to them. Additionally, the probabilities in P contain quantitative information, i.e., they complement the qualitative properties revealed by the graphical structure [73] [74].

A BN can be constructed either manually, based on knowledge and experience acquired from previous studies and literature, or automatically from data [72]. In this study, the selection of variables (Table IV) is constrained by the availably of data. We use the elements (timestamps, aircraft data and airport information/configuration) that have been analyzed through the study. The first step is to generate the correlation matrix for the variables involved, to assess the correlation among pairs. Subsequently a data-driven process was applied to build the BN, following a Bayesian Search Algorithm [76] [72]. The final architecture presented in Fig. 20 was determined by applying this algorithm (including variable discretization and validation) and refining it with previous knowledge. Therefore, our model is built applying a combination of datadriven process with practical adjustments, in order to obtain a model reflecting reality. We develop a statistical significance test on pairs of nodes connected by an arc in the BN: associations between the nodes were statistically significant at level 0.05 (p-value test).

TABLE IV. LIST OF VARIABLES REPRESENTED IN THE MODEL (VARIABLES)

Node	Meaning
1	Arrival delay for the operation
2	Turnaround delay for the operation
3	Taxi-Out delay for the operation
4	Total delay for the operation
5	Average total delay of the previous 20 flights
6	Delay indicator (Ki) for the operation
7	ALDT (Actual Landing Time)
8	Arrival Runway
9	Departure Runway
10	Route origin (national, UE Schengen, international)
11	Route destination (national, UE Schengen, international)
12	Terminal Area (T123, T4T4S)
13	Wake-turbulence category (H, M, L)
14	Aircraft size (narrow body, wide body)
15	Taxi-In process duration
16	Taxi-Out process duration (scheduled)
17	Taxi-Out process duration (actual)
18	SOBT-SIBT (scheduled turnaround)
19	AOBT-AIBT (actual turnaround)
20	ATOT (Actual Take Off Time)
21	TOAT (Taxi-Out Approval Time)
22	AOBT-SOBT (delay in the Off-Block process)
23	ASRT-AOBT
24	ASRT-ASAT
25	TSAT-ASAT

Due to the conditional dependence relationship of the variables within the BN, it is possible to derive posterior probability from prior probability (forward analysis) as well as implementing backward reasoning to evaluate the influence of the variables for a target result. Therefore, two main scenarios reflect the utility of the model:

- *Scenario 1* (forward/inter-causal scenario). The model predicts departure delay (output-child node) by setting the probability of having certain configuration, i.e. by setting one or more parent-input nodes.
- *Scenario 2* (backward inference). The model delivers a particular configuration in the parent nodes by setting the delay node to a target value. It allows understanding who are the main contributors to delay (if delay is settled to a high positive value) or finding the configuration that optimize operations (if delay is settled to a negative value).

A sub-sample of 90% of the observations was selected to build the model structure and to estimate parameters (a test sample to establish the model's ability to explain delay propagation). The remaining 10% of the data was set aside to test the accuracy of the predictions made by the model (a sample to test the model's predictive capacity). The scenarios tested provided promising results regarding the model's ability to reduce uncertainty (by explaining system performance and predicting delay propagation). The test error ranged from 20% - 35%, and the average value was 27%.



Figure 20. BN model to understand the interdependencies between factors influencing performance and delay^a [76]. a. The thickness of an arc represents the strength of influence between two directly connected nodes.

IV. CONCLUSIONS

This paper develops a functional analysis of the operations that represent the aircraft flow through the airport-airspace system. In this analysis, we use a dynamic spatial boundary associated with the E-TMA concept, so a linkage between inbound and outbound flights can be proposed. The aircraft flow is characterized by several temporal milestones related to the A-CDM method and structured by a hieratical task analysis, providing a BPM for the rotation stage.

The application of the methodology to a case study of 1,500 turnarounds (registered at the peak month of 2015) at Madrid Airport showed that arrival delay increases and accumulates its impact over the day, due to network effects. But departure delay does not follow this pattern, which implies that the airport-airspace system is somehow capable of absorbing a fraction of the arrival delay across the rotation stage. We analyze this aptitude by studying and characterizing the different processes that were previously identified with the BPM and the milestone approach. This evaluation of the system level of saturation is completed by the definition of different indexes and performance indicators.

Finally, the relationships among the factors that influence the aircraft flow are evaluated to create a probabilistic graphical model, using a BN approach. This model predicts outbound delays given the probability of having different values at the causal control variables. Moreover, by setting a target to the output delay, the model provides the optimal configuration for the input nodes. The proposed methodology has several applications:

- Achieve a comprehensive understanding of operations.
- Detect possible incidents or irregularities that may occur during processes.
- Define the different operational actions that may correct the inefficiencies identified.
- Investigate the impact of changes in tactical decisions and policies on the management and propagation of delays in the E-TMA system.
- The propagation model and the proposed indicators may be used to ensure that all agents collaborate in reducing delays, guaranteeing some target levels of efficiency.
- Using "forward" analysis it is possible to estimate the final departure delay (settlement of buffer time and optimal rotation times).
- Using "backward" analysis it is possible to identify the main contributors (causes) to a final delay (locate inefficiencies).

Future work needs to focus on improving the accuracy of the model (more complete testing data and methodological improvements), and to assess whether the model is suitable for use in other airports. We also need to analyze potential response strategies (reduce delays, mitigate inefficiencies and optimize operations), and apply the propagation model to other types of incidents (not just delays).

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