

A Constraint Programming Model with Time Uncertainty for Cooperative Flight Departures

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Abstract— The lack of a proper integration of strategic ATM decision support tools with tactical ATC interventions usually generates a negative impact in the Reference Business Trajectory adherence, and in consequence affecting the potential of TBO framework. In this paper it is presented a new mechanism to reduce the amount of ATC interventions at tactical level while preserving ATFM planned operations. The PARTAKE project fosters adherence of air space user's trajectory preferences enhancing Trajectory Based Operation (TBO) concepts by identifying tight interdependencies between trajectories and introducing a new mechanism to improve aircraft separation at the hot spots. The underlying philosophy is to capitalize present freedom degrees between layered Air Traffic Management (ATM) planning tools, when sequencing departures at airports by considering the benefits of small time stamp changes in the assigned CTOT departures.

Keyword- Air traffic management; Trajectory Based Operations; Decision Support Tool; Constraint Programming

I. INTRODUCTION

Air transport is an integral part of transport infrastructure and a significant sector of the economy predicted next decades with steady growth. Therefore, the identification of operational and managing policies for better performance of existing airspace procedures is important in order to cut European Air Traffic Management (ATM) costs, increase capacity and operational safety and decrease the environmental impact. The intention of this innovative approach is to design a competitive ATM system, supporting up to a certain extent the Airspace User (AU's) demands at the right time (i.e. departure slots), at the right cost (i.e. suitable level of Air Traffic Control (ATC) service) at the right place (i.e. AU's preferred trajectories) and at the right service quality (i.e. safety) without extra investments, just by removing the ATM non-added-value operations that indirectly impact on present ATM capacity.

By empowering the concept of Trajectory-based operations (TBO) as a flexible synchronization mechanism towards an efficient and competitive ATM service a precise description of an aircraft path in space and time can be retrieved. Under this TBO approach, airspace users should fly precise 4-dimensional trajectories (4DTs), previously agreed upon with the network manager.

The presented research is conducted in the context of the PARTAKE Exploratory Research project supported by the European Union's Horizon 2020 research and innovation program. Europe has some of the busiest airspace in the world, managed by a network covering 11.5 million km² of airspace [13]. The Network Manager Operations Centre receives, processes and distributes up to 35,000 flight plans a day [2]. PARTAKE aims to improve the present demand/capacity balance in ATM by introducing small ground delays in the programmed departure that will not affect the planned traffic since the slot time window assigned to each aircraft will be preserved. The major challenge PARTAKE is facing is to achieve ATC Minimum Tactical Interventions: Reference Business Trajectories (RBT) provide an excellent source of information to identify long time in advance situations in which 2 or more aircraft could require ATC directives to maintain the required separation minima. PARTAKE proposes mitigation methods able to taking into account the preferences of Airlines and Airports.

This work presents a set of tools to determine small adjustments within the $[-5,10]$ interval around the Calculated Take-Off Time (CTOT), along with bounded modifications on the flight duration, as the control actions to be taken by considering the RBT and the impact on potential ATC interventions. These actions will be calculated to mitigate the potential tight trajectory interdependencies that can emerge after inserting the traffic ready to depart. The use of Constraint Programming (CP) [5] is proposed to calculate those feasible departure configurations. CP is an emergent software technology for declarative description and effective solving of large, particularly combinatorial, problems especially involving scheduling, resource allocation, placement and planning.

The paper is organized as follows: Section 0 briefly introduces the mapping and filtering mechanism implemented to detect the trajectory interdependencies between airborne flights and the new traffic; Section III describes the CP model proposed to mitigate the detected collective micro-regions that would require ATC interventions; Section IV proposes a model to deal with time uncertainty in the RBT; Section V discusses the experimental results achieved so far; finally, some conclusions and open questions for further research are discussed in Section VI.

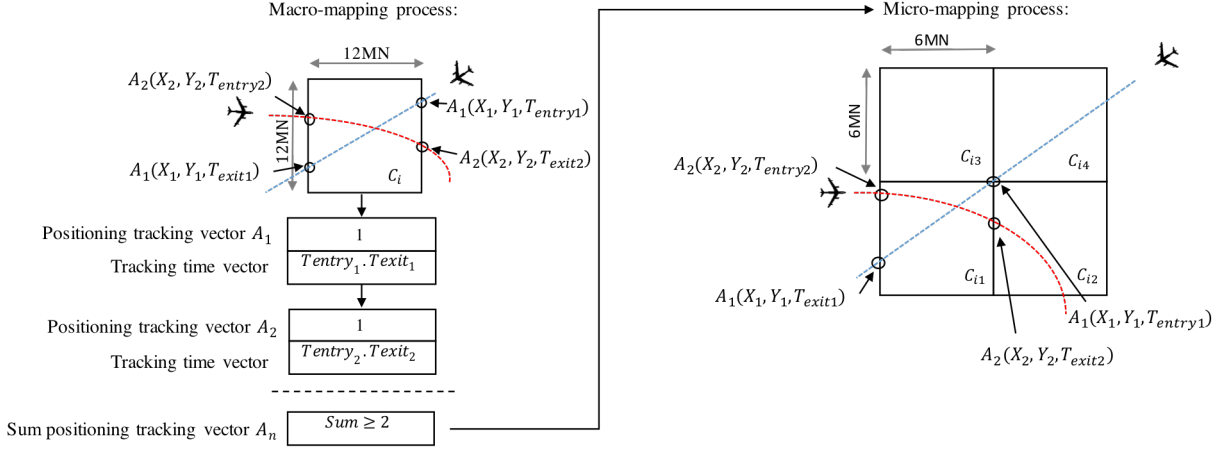


Figure 1. Macro and micro mapping process

II. DETECTION MECHANIM

The detection of tight trajectory interdependencies is realized in three constituent processes which will be presented in the following. The output of the detection of tight trajectory interdependencies enables the resolution of these interdependencies using CP.

To identify tight trajectory interdependencies, the entire European Airspace is classified into so called collective microregions. Based on the TBO concept the enroute trajectories are initially projected on a discrete grid by flight level covering the European Airspace (longitude -20 and 30 degrees and latitude of 0 to 80 degrees). The trajectories and relevant flight information must be supported by computational efficient algorithms and databases.

A. Macro-mapping process

One objective when developing the search algorithm to detect tight trajectory interdependencies is to solve the scalability problem and to design a computational efficient algorithm. Therefore, the airspace is first divided into macrocells with a size of 12NM (22,224 km). The position tracking is stored as a vector. Each position in the vector takes a binary value where the presence in a cell is represented by 1 and absence by 0 (see Figure 1). The entry and exit times of an aircraft into a cell are registered and stored in a vector.

B. Micro-mapping process

After the initial mapping, the macrocells with an occupancy rate equal or greater than two are partitioned for the identification of collective microregions, that is the set of cells showing potential concurrent events. The microcells represent square cells of 6NM that are in use by at least two aircraft simultaneously [7]. The size 6NM (11.112 km) has been chosen with respect to the safety distance two aircrafts always have to respect. For collective microregions, entry times and exit times are used to determine the size of the overlap or clearance between aircraft pairs. As it can be seen in Figure 1, the process is identical to the previous presented macro-mapping process considering smaller cells. To ensure the reliability of the

collective microregion identification, four areas located on the boundaries of surrounding cells, macro- and micro-mapping processes are applied in order to detect any concurrence event between trajectories neighbor cells.

C. Filtering process

Finally, the detected concurrence events are filtered for each pair of aircraft. The outcome after the filter are the tightest potential concurrence events for each pair of aircraft (see Figure 3). This process shall filter those trajectories either losing the required clearance (separation minima) or in a risk of losing it after mitigation measures are applied. The filtering process is based on the collective micro-regions detected in the mapping tool. For these collective micro-regions, entry (t_a^e) and exit (t_a^s) times are used to determine the temporal looseness, referring to the size of the overlap or clearance between aircraft pairs. The calculation of the temporal looseness H between time windows of aircrafts a_1 and a_2 in a collective micro-region is expressed as follows:

$$H = \min(t_{a_1}^s, t_{a_2}^s) - \max(t_{a_1}^e, t_{a_2}^e)$$

Concurrence events exist when H is positive, indicating an overlap, whereas events with a negative H value have a clearance time, indicating a potential concurrence event. Based on the calculation of H (the size of an overlap or clearance), it is possible to identify the tightest concurrence events, or potentially concurrence events, for each pair of aircraft.

III. MITIGATION PROCESS

The conflict mitigation process is modeled as Constraint Programming (CP) model [5]. CP is a powerful paradigm for representing and solving a wide range of combinatorial problems. In the last few decades it has attracted much attention among researchers due to its flexibility and its potential for solving hard combinatorial problems in areas such as scheduling, planning, timetabling and routing. CP combines strong theoretical foundations (e.g. techniques originated in different areas such as Mathematics, Artificial Intelligence, and Operations Research) with a wide range of application in the

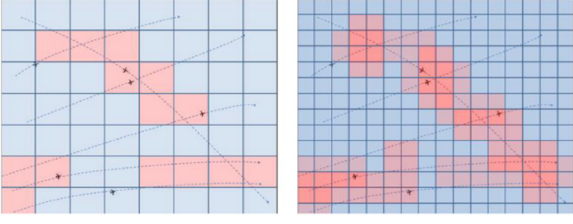


Figure 3. Detection of collective microregions

areas of modelling heterogeneous optimization and satisfaction problems. Moreover, the nature of CP provides other important advantages such as fast program development, economic program maintenance and efficient runtime performance. Problems are expressed in terms of three entities: variables, their corresponding domains, and constraints relating them.

The presented approach recognizes the synchronization problem as a scheduling problem, similar to some extend to the well-known Job Shop Scheduling Problem (JSSP). Roughly, this problem consists in allocating the proper resources to the list of jobs facing an optimization goal to minimize some temporal, productivity or efficiency cost function. By setting the analogy to the JSSP, the available cells as portions of the airspace can be considered as the existing resource and the aircraft as the jobs that are performed requiring the resource.

A. Tight trajectory interdependencies resolution

The mapping and filtering processes generate a representation of all the conflicts that must be removed by the optimization model. In a first CP model version, the tight trajectory resolution is modeled using one control action: shifting the entire trajectory by a delay to be applied on the CTOT. As Figure 2 illustrates, the CTOT of aircraft 2 is shifted ahead of its original schedule and the CTOT of aircraft 3 is delayed in order to guarantee that all three aircraft arrive to the cells in conflict at different time windows.

Let A be the set of aircrafts, C the set of cells belonging to one collective microregion and $c_a = \langle c, a \rangle$ the pairing between the aircraft a using a given cell c at the microregions. The pairings $c_a \in C_A$ are defined as:

$$C_A = \{ \langle c, a \rangle \mid \forall c \in C, \forall a \in A \}$$

Finally, the time occupancy of the cell c by aircraft a is defined by the two parameters:

$$\begin{aligned} c_a^{te} &\equiv \text{entry time} \\ c_a^{ts} &\equiv \text{exit time} \end{aligned} \quad (1)$$

1) Decision variables

To ensure that the departure adjustment of the aircraft remain in the defined timeframe of $[-5, 10]$ minutes, the integer decision variable δ_a is defined as the delay applied to the CTOT of aircraft a :

$$\delta_a \in [-\delta_{min}, \delta_{max}],$$

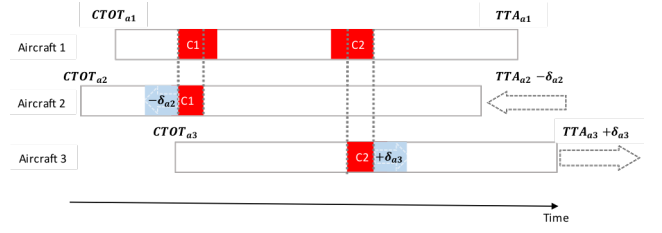


Figure 2. Resolution of tight trajectory interdependencies (C=Conflict)

where $\delta_{min} = 5$ and $\delta_{max} = 10$ minutes, sets the domain for the delay decision variable.

The use of a cell by an aircraft is modeled by means of interval decision variables. Interval decision variables represent time periods whose duration and position in time are unknown in the optimization problem. The interval is characterized by a start value, an end value and a size. Addressing this concept as a scheduling problem, the interval is the time during which something happens (e.g. an activity is carried out). In this case, it is the occupancy of the cell c by an aircraft a is modeled by the interval decision variable:

$$P_{ca} = [s_{ca}, e_{ca}), \quad \forall c_a \in C_A \quad (2)$$

and the size:

$$sz(P_{ca}) = e_{ca} - s_{ca} (= c_a^{ts} - c_a^{te})$$

where s_{ca} and e_{ca} are the interval start and end time respectively.

Since the shifting applied to the trajectory to avoid the proximate events is determined by the delay δ_a and no speed adjustment are accepted, the domain of the interval variable can be defined as (see also (1)):

$$P_{ca} \in [c_a^{te} - \delta_{min}, c_a^{ts} + \delta_{max}], \forall c_a \in C_A \quad (3)$$

As illustrated in Figure 2, the time occupancy of the cell that is involved in a concurrent event remains constant. The aircraft takeoff time instants are shifted according to the delay δ that is applied to avoid the concurrent event in the cell.

Each of the cells can be occupied by one aircraft at a time, so the aircrafts going through the cell must be sequenced accordingly. The decisions on the use of conflicting cells are modeled by sequence variables, which are defined as:

$$F_c = \{P_{ca} \mid c_a \in C_A\}, \quad \forall c \in C \quad (4)$$

with the permutation π of the sequence variable F_c as the function

$$\pi: F_c \rightarrow [1, m]$$

where $m = |F_c|$ is the number of aircrafts going through the cell c . The elements of the sequence meet the following conditions:

$$P_{c_{ai}} \neq P_{c_{aj}} \Rightarrow \pi(P_{c_{ai}}) = \pi(P_{c_{aj}}), \forall P_{c_{ai}}, P_{c_{aj}} \in F_c$$

2) Constraints

Two constraints are identified in order to define the space of feasible solutions. The first constraint aims to model the shifting of every interval variable according to the applied delay:

$$s(P_{c_a}) = c_a^{te} + \delta_a, \forall c_a \in C_A \quad (5)$$

where the function $s(\cdot)$ is defined as the interval start time (aircraft entry to cell c):

$$s(P_{c_a}) = s_{c_a} \quad (6)$$

The second constraint is the *no overlap* constraint that imposes a set of interval variables to not overlap each other in time. In this case, all aircraft in a cell c with proximate events should have no overlap:

$$\forall P_{c_i}, P_{c_j} \in F_c \quad (7)$$

$$NO(F_c) \Leftrightarrow \pi(P_{c_i}) < \pi(P_{c_j}) \Rightarrow e(P_{c_i}) \leq s(P_{c_j})$$

where the function $e(\cdot)$ are defined as the interval end time (aircraft exit from cell c):

$$e(P_{c_a}) = e_{c_a} \quad (8)$$

and the no overlap is guaranteed for the proximate event P_{c_i} at a position prior to any P_{c_j} by constraining its exit time to be lower or equal to the entry time of the subsequent proximate events P_{c_j} .

3) Optimization goal

The objective function was chosen to enhance adherence with a synchronization mechanism, though flexible, does not preserve the TTA at destination airport. Therefore, it aims to minimize the differences between actual takeoff times and the planned or CTOTs.

The optimization goal of the solution is to minimize the total aircraft delays, and it is formulated as follows:

$$\sum_{a=1}^n |\delta_a| \quad (9)$$

where a refers to the aircraft and δ_a is the delay applied. The whole optimization model is listed here:

A set of aircrafts

C set of cells at a collective microregion

$$C_A = \{ \langle c, a \rangle \mid \forall c \in C, \forall a \in A \}$$

$$\text{d.v. } \delta_a \in [-\delta_{min}, \delta_{max}], \forall a \in A$$

$$\text{d.v. } P_{c_a} \in [c_a^{te} - \delta_{min}, c_a^{ts} + \delta_{max}], \forall c_a \in C_A$$

$$\text{d.v. } F_c = \{ P_{c_a} \mid c_a \in C_A \}, \forall c \in C$$

$$\text{minimize } \sum_{a=1}^n |\delta_a|$$

subject to {

$$s(P_{c_a}) = c_a^{te} + \delta_a, \forall c_a \in C_A$$

$$\forall P_{c_i}, P_{c_j} \in F_c$$

$$NO(F_c) \Leftrightarrow \pi(P_{c_i}) < \pi(P_{c_j}) \Rightarrow e(P_{c_i}) \leq s(P_{c_j})$$

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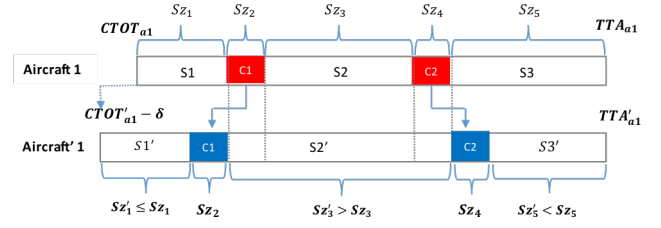


Figure 4. Resolution of tight trajectory interdependencies with speed change (C=Conflict; S=Segment; Sz=Size)

This model was applied to successfully solve an over-stressed realistic scenario. The scenario was composed of a set of 4010 real 4D trajectories in the European airspace for a time window of 2 h, showing more than 65.000 proximate events. Nevertheless, the modified trajectories do not meet the TTA, since no speed adjustment possibility is included in this model. Next section extends the model in order to improve the RBT adherence of the modified trajectories.

B. Tight trajectory interdependencies resolution with speed adjustments

TTA adherence is a main objective to enhance capacity at arrival airports. Clearly, the TTA cannot be preserved by shifting the CTOT and therefore, the full trajectory. The TTA in ATM has a small margin of [-1,1] minute. Therefore, its compliance is of high importance. To meet these conditions, the model described in section 3.1 has been extended by introducing the concept of segments for describing the full trajectory from departure (CTOT) until the arrival time to the destination (TTA). The Figure 4 illustrates this concept. For instance, aircraft 1 in the figure is divided into five segments: C1 and C2 represent the concurrence events while S1, S2 and S3 are the segments between the concurrence events. In the modified trajectory, the segment S1' is shifted according to the applied delay on the CTOT to avoid the first concurrence event while S3' is shortened in time by speed change in order to preserve the TTA within the margin. The intermediate segment S2' is extended in time by flying with reduced speed to avoid concurrence event C2.

The speed adjustments are realized under the condition that the segment between proximate events are of a certain minimum duration. That allows to introduce a speed change that is efficient in the sense of fuel consumption and in the effect on the resolution of the conflict while trying to preserve the TTA.

New data structures are included to model the trajectory segments for speed adjustments. Let \hat{g}_i^a be a segment of the aircraft a trajectory. Therefore, the RBT can be noted as:

$$RBT_a = \{ \hat{g}_i^a \}, \quad i = 1..p(a)$$

where $p(\cdot)$ is the number of segments required for describing the trajectory. For instance, the Figure 4 shows the trajectory segments of aircraft 1, represented as $RBT_a = \{ S1, C1, S2, C2, S3 \}$ with

$$s(\hat{g}_i^a) = \text{start time of } \hat{g}_i^a,$$

$$e(\hat{g}_i^a) = \text{end time of } \hat{g}_i^a$$

where the functions $s(\cdot)$ and $e(\cdot)$ yield the start and end times of the corresponding RBT segments. See (6) and (8) for the function definition.

Finally, the concept of segment elasticity $l(\hat{g}_i^a)$ is introduced to denote the allowed speed variation as a percentage of the \hat{g}_i^a segment duration $sz(\hat{g}_i^a)$.

1) Additional decision variables

In this new CP model approach, the duration of the entire flight becomes an unknown itself, since CTOT can be delayed while keeping the intend to preserve the TTA.

A decision interval variable G_a is introduced for representing the entire flight:

$$G_a = [s_a, e_a)$$

where s_a will be the takeoff time and e_a the arrival time in the solution.

Secondly, the interval variables representing the segments of the G_a solution trajectory are modeled. Let g_i^a be the interval variable:

$$g_i^a = [s(g_i^a), e(g_i^a))$$

and the size of the g_i^a segment is

$$sz(g_i^a) = e(g_i^a) - s(g_i^a)$$

The domain of the g_i^a segment can be defined as:

$$sz(g_i^a) \in [sz(\hat{g}_i^a) - l(\hat{g}_i^a), sz(\hat{g}_i^a) + l(\hat{g}_i^a)] \quad (10)$$

Note that in this model version, interval duration can differ from RBT segment duration, since some elasticity is enabled by the bounded speed changes, whereas the domain for the interval start and end time cannot be specified, since their values at the solution are a combination of the takeoff delay and the bounded speed adjustments.

Finally, a sequence variable T_a is introduced to set the relationship between the trajectory segments g_i^a and the entire trajectory G_a :

$$T_a = \{g_i^a \mid \forall a \in A, i \in 1..p(a)\}$$

$$\pi: T_a \rightarrow [1, n] \quad (11)$$

$$g_i^a \neq g_j^a \Rightarrow \pi(g_i^a) \neq \pi(g_j^a), \forall g_i^a, g_j^a \in T_a$$

2) Additional Constraints for speed change

The duration of the flight is determined by the constraint of the takeoff time and the time to arrival.

$$s(G_a) = CTOT_a \pm \delta_a \quad (12)$$

$$e(G_a) \in [TTA_a - 1, TTA_a + 1] \quad (13)$$

The relationship between the flight interval variable and its segments is modeled by the following *span* condition:

$$span(G_a, \{g_i^a\}), \forall a \in A, \forall g_i^a \in T_a$$

This constraint sets the following time relationship among the interval variables:

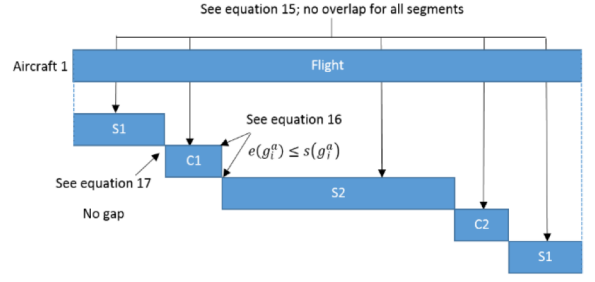


Figure 5. Representation of decision variable flight and RBT segments

$$\begin{cases} s(G_a) = \min_{i \in [1, p(a)]} (\{s(g_i^a)\}) \\ e(G_a) = \max_{i \in [1, p(a)]} (\{e(g_i^a)\}) \end{cases} \quad (14)$$

The constraint *span* states that the interval flight spans over all present intervals from the set segments. That is, interval flight G_a starts together with the first present segment interval and ends together with the last one.

Additionally, the following three constraints are set to order the trajectory segments:

1. The *no overlap* constraint to ensure that interval variables do not overlap each other.

$$NO(G_a) \Leftrightarrow \pi(g_i^a) < \pi(g_j^a) \Rightarrow e(g_i^a) \leq s(g_j^a) \quad (15)$$

2. The constraint that one segment has to start before the next:

$$e(g_i^a) \leq s(g_j^a), \forall i, j: i < j \quad (16)$$

3. The constraint that ensure that the start of segment j results after the end of segment i .

$$e(g_i^a) = s(g_j^a), \forall i, j: j = i + 1 \quad (17)$$

The graphical representation of this three constraints is shown in Figure 5. Aircraft 1 has a flight duration and the projection of the segments onto the flight duration with the three conditions is shown.

Finally, the P_{c_a} interval variable, that is used in combination with the sequence variable F_c to remove the concurrence events at cell c , must be linked with the concurrence segments of the trajectory T_a (e.g. C1 and C2 in Figure 5), since they are representing the same time windows.

This is accomplished by the following constraint:

$$\begin{cases} s(g_i^a) = s(P_{c_a}) \\ e(g_i^a) = e(P_{c_a}) \end{cases} \Leftrightarrow \begin{cases} s(\hat{g}_i^a) = c_a^{t_e} \\ e(\hat{g}_i^a) = c_a^{t_s} \end{cases}, \forall c_a \in c_A$$

3) Objective function

The constraint in (13) binds to the TTA attainment, but it might happen that no solution is found because time adjustment is bounded so it is possible that the required delays δ_a cannot be compensated by the speed adjustments. For this reason, the TTA constraint is relaxed.

The following logical function is added:

$$L(G_a) = \begin{cases} 1, & e(G_a) \notin [TTA_a - 1, TTA_a + 1] \\ 0, & \text{otherwise} \end{cases}$$

With this function, the number of TTA violations can be counted for introducing its minimization as an objective that can be combined with the objective function stated in (9) to minimize the total delay of the aircraft takeoffs. The following equation weights both objectives to get the optimization goal:

$$\min w_1 \sum_{a=1}^n |\delta_a| + w_2 \sum_{a=1}^n L(G_a) \quad (18)$$

The extended optimization model is listed here:

A set of aircrafts

C set of cells at a collective microregion

$$C_A = \{ \langle c, a \rangle \mid \forall c \in C, \forall a \in A \}$$

$$RBT_a = \{ \hat{g}_i^a \mid \forall a \in A, i = 1..p(a) \}$$

$$\text{d.v. } \delta_a \in [-\delta_{min}, \delta_{max}], \forall a \in A$$

$$\text{d.v. } P_{c_a} \in [c_a^{te} - \delta_{min}, c_a^{ts} + \delta_{max}], \forall c_a \in C_a$$

$$\text{d.v. } F_c = \{ P_{c_a} \mid c_a \in C_A \}, \forall c \in C$$

$$\text{d.v. } G_a, \forall a \in A$$

$$\text{d.v. } g_i^a, \forall a \in A, \forall i \in 1..p(a):$$

$$sz(g_i^a) \in [sz(\hat{g}_i^a) - l(\hat{g}_i^a), sz(\hat{g}_i^a) + l(\hat{g}_i^a)]$$

$$\text{d.v. } T_a = \{ g_i^a \}, \forall a \in A$$

minimize

$$w_1 \sum_{a=1}^n |\delta_a| + w_2 \sum_{a=1}^n L(G_a)$$

subject to {

$$s(g_i^a) = CTOT_a \pm \delta_a \quad \forall a \in A$$

$$\forall P_{c_i}, P_{c_j} \in F_c$$

$$NO(F_c) \Leftrightarrow \pi(P_{c_i}) < \pi(P_{c_j}) \Rightarrow e(P_{c_i}) \leq s(P_{c_j})$$

$$span(G_a, \{g_i^a\}), \forall a \in A, \forall g_i^a \in T_a$$

$$\forall a \in A, \forall i, j \in 1..p(a)$$

$$NO(G_a) \Leftrightarrow \pi(g_i^a) < \pi(g_j^a) \Rightarrow e(g_i^a) \leq s(g_j^a)$$

$$s(g_j^a)$$

$$e(g_i^a) \leq s(g_j^a) : i \leq j$$

$$e(g_i^a) = s(g_j^a) : j = i + 1$$

$$\left\{ \begin{array}{l} s(g_i^a) = s(P_{c_a}) \\ e(g_i^a) = e(P_{c_a}) \end{array} \Leftrightarrow \begin{array}{l} s(\hat{g}_i^a) = c_a^{te} \\ e(\hat{g}_i^a) = c_a^{ts} \end{array}, \forall c_a \in C_A \right.$$

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IV. DEALING WITH UNCERTAINTY

Considering the ATM sources of uncertainty (parameters of aircraft models in trajectory prediction, weather, failure of ATC systems, cancellation of flights, etc.) and the degree of safety required in PARTAKE, a rolling horizon model scope should be considered to handle reasonable amounts of uncertainty.

The flight management system (FMS) provides the primary navigation, flight planning, and optimized route determination and enroute guidance for the aircraft. The guidance function of the FMS ensures that a flight within TBO concept will flight its RBT. But some deviations may occur both in time and space as a consequence of the mentioned causes. The time uncertainty is

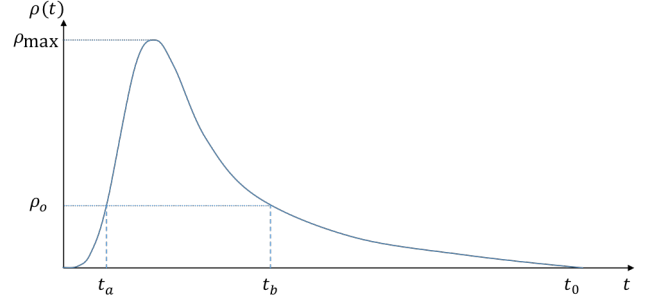


Figure 6. Evolution of $\rho(t)$ while FMS corrects the time diversion

defined as the inability to exactly determine in which instant of time an aircraft will overfly a certain fixed spot in the space.

The space uncertainty, which would be defined in the same way, shall not be taken into account since with the space discretization (mapping) done by the conflict detection, is considered to be solved: PARTAKE considers the effects on potential conflicts due lateral deviations from RBT to be absorbed by the size of the cells (6 NM). That is, any deviation below 1 mile will result in the same conflicts detected at the mapping process.

However, time differences between the enroute flights with respect to their corresponding RBT can be detected when the CTOT of the flights ready to depart is to be calculated. These differences may affect to the time window occupancy of the cells in conflict as defined in (1), so that the action to be taken for mitigating the potential conflicts can be affected too.

More formally, let be $\gamma(t) \in \mathbb{R}^3$ the RBT and $\bar{\gamma}(t) \in \mathbb{R}^3$ the actual flown trajectory. Then, under the TBO concept we will expect that $\|\gamma(t) - \bar{\gamma}(t)\|_2 \approx 0$ at least in most cases. Should be observed in any of the verifications of the actual state of airborne flights, which is conducted every 5, that a determined aircraft is not where it ought to be, ρ will be defined as the time difference existent between the actual position and the predefined position in the RBT. That is, if $\|\gamma(t) - \bar{\gamma}(t)\|_2 \neq 0$ is observed, then $\rho \in \mathbb{R}$ will be defined satisfying:

$$\|\gamma(t) - \bar{\gamma}(t + \rho)\|_2 = 0 \quad (19)$$

This work focuses on those deviations that may occur in time as a consequence of the effect of the wind. Two possibilities are under consideration when a ρ is detected:

1. The FMS guidance functionality has not yet acted because ρ is less than the alert value set on it.
2. The FMS is correcting ρ changing some aircraft flight parameters.

Therefore, ρ is defined as a time function $\rho(t)$ that describes the time diversion between the RBT and the actual flown trajectory as a function of time. As shown in Figure 6, it is assumed that $\rho(t)$ at the beginning will increase depending on the wind until the maximum diversion allowed by the FMS is reached. Then the FMS corrections will force $\rho(t)$ to decrease until the condition $\|\gamma(t) - \bar{\gamma}(t)\|_2 \approx 0$ is reached again. When a ρ_0 is observed, the most conservative period $[t_a, t_0]$ is selected for considering the duration of the time diversion.

A. Estimation of the time uncertainty

Four dimensional navigation is based on reaching a three dimensional waypoint at a required time of arrival by changing the aircraft's flight profile. In this context several variables and parameters are involved in the concept of changing a flight profile. According to [6], airspeed is considered as the most important control variable in order to achieve a waypoint at a given time of arrival. The time that an aircraft spends moving from an initial point $x_1 = (lat, lon)$ to a final point $x_2 = (lat, lon)$ can be represented as follows:

$$t = \frac{1}{\beta} \int_{x_1}^{x_2} \frac{\partial x}{V_a(z)} \quad (20)$$

where V_a is the true airspeed of the aircraft at a given altitude, z is the altitude and β is a conversion factor from *knots* to *feet/seconds*. The along-track wind effect has to be taken into account for time computation. The wind produces an important change of the airspeed of the aircraft and this could affect the estimation of the time of arrival. The effect of the along-track wind over the aircraft airspeed in cruise level is associated with two elements [6]:

1. The direction of the wind with respect to the aircraft. Depending of the relationship between the heading of the along-track wind and the aircraft, it is called tailwind or headwind.
2. The magnitude of the along-track wind which represents the constant velocity of the wind at a given altitude (cruise altitude).

The along-track component of wind in the horizontal plane can be represented as follows (equation parameters are defined in Table 1):

$$w(z) = V_w(z) \cdot \cos|B(z) - H_w(z) \pm \delta| \quad (21)$$

By combining (20) and (21), the resultant equation used to compute the time of an aircraft's flight between an initial point and a final point with constant velocity, is as follows:

$$t = \frac{1}{\beta} \int_{x_1}^{x_2} \frac{\partial x}{V_a(z) \pm V_w(z) \cdot \cos|B(z) - H_w(z) \pm \delta|} \quad (22)$$

By integrating (22), the result is the time estimation equation with wind effect:

$$t = \frac{1}{\beta} \frac{[x_2 - x_1]}{V_a(z) \pm V_w(z) \cdot \cos|B(z) - H_w(z) \pm \delta|} \quad (23)$$

TABLE 1 PARAMETERS FOR ESTIMATING WIND EFFECTS

Constant	Details
V_w	Speed of the wind at a given altitude
H_w	Heading of the wind at a given altitude
δ	Factor to correct the magnetic north to true north
B	Bearing of the aircraft's track

Equation (23) can be used to estimate the minimum time in which an aircraft can fly from the initial point to the final point at true airspeed. Therefore, the time to the next collective micro-region of the delayed flight can be estimated from its current

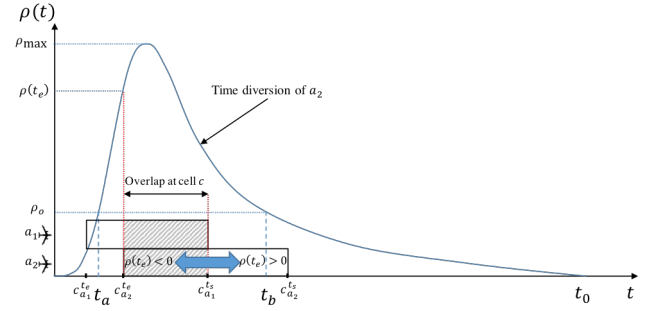


Figure 7. Effects of time diversion on conflict window

observed position and true airspeed, determining whether the time diversion is affecting the future concurrence events or not. Furthermore, using (23), the available weather data and taking into account factors such as aircraft particular characteristics and the maximum operational values allowed by the FMS system a model for $\rho(t)$ can be postulated [1][11].

When the following condition holds:

$$\|\gamma(t) - \bar{\gamma}(t)\|_2 \neq 0, t \in [t_a, t_0] \quad (24)$$

For every occupancy time window overlapping the period $[t_a, t_0]$, the new entry time to the micro-region can be recalculated from the estimated $\rho(t_e)$ as illustrated in Figure 7. The following modification is introduced in the CP model replacing (1):

$$\begin{aligned} c_a^{te} + \rho(t_e) &\equiv \text{entry time} \\ c_a^{ts} + \rho(t_e) &\equiv \text{exit time} \end{aligned} \quad (25)$$

V. RESULTS

The model was applied to an over-stressed realistic scenario. The scenario was composed of a set of 4010 real 4D trajectories in the European airspace for a time window of 2 h. In these experiments, we assumed RBT without uncertainties. In this context, the trajectories were discretized at each second, and each position was specified in terms of geographic coordinates and a time stamp. This scenario was designed and analyzed in the STREAM project [12], a EUROCONTROL SESAR WP-E project. The CP model has been implemented with the ILOG Optimization Suite [4] and the following results were obtained.

A. Macro and Micro Mapping

The detection of the concurrence events in this paper is based on the algorithms and results presented at [8][9] and [10]. The aforementioned scenario is analyzed in these works, leading to the detection of the collective micro-regions that have been used in this work to find the optimal adjustments on CTOT and speed changes to reduce proximate events and, therefore, ATC interventions.

In Figure 8 (a) the enroute traffic through the collective micro-regions is shown. The cells with potential concurrence events are detected based on the RBT trajectories of those aircrafts ready to depart, but still on ground, according to their CTOT. Therefore, enroute trajectories are conflict free at the given time instant.

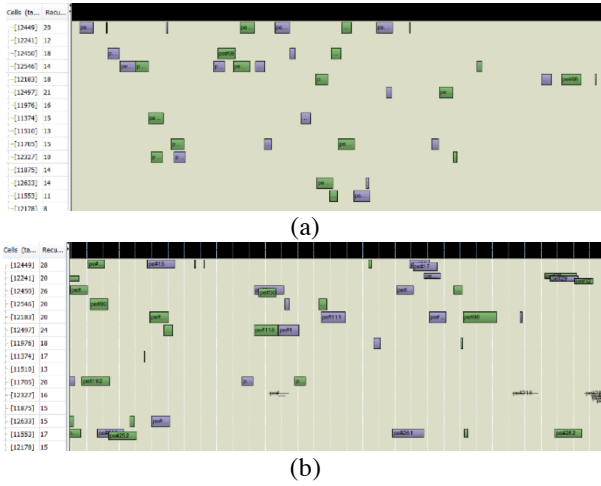


Figure 9. Gantt diagrams showing the traffic through the cells with potential concurrence events. Diagram (a) shows the conflict free en-route traffic and (b) shows the emerging conflict after inserting the departing traffic for the same time period

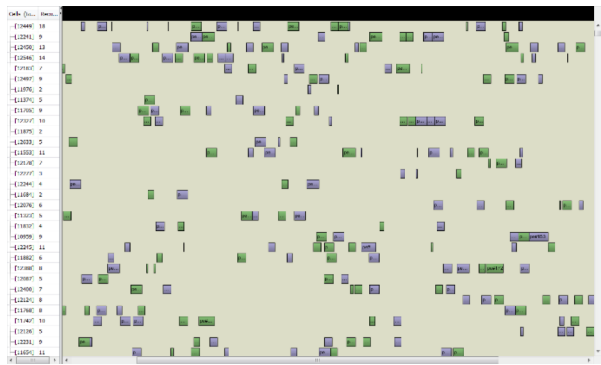


Figure 8. The diagram shows the conflict free solution after applying small adjustments on CTOT and segment speed.

The Figure 8 (b) shows the situation found when the grounded aircrafts depart according to their RBT CTOT. As it can be seen, for instance, at cells 12241, 12449 and 12450 among others, concurrent events will appear between several aircraft if they depart according to their CTOT. In this case, aircraft regulations could be issued by ATM or, later on, ATC interventions would be needed to remove the proximate events caused by the inserted traffic.

B. Trajectory adjustments

The proposed CP model is used to determine the proper adjustments on the CTOT and aircraft trajectories to remove the potential concurrence events.

As Figure 9 illustrates, all the potential concurrence events are removed by applying a combination of bounded delays on CTOT and/or speed adjustments, leading to a conflict free scenario. The bounded adjustments impose the actual takeoff time to be within the $[-5,10]$ minutes of the aircraft CTOT as shown in (12) and the speed adjustments to be less than 10% of the RBT proposed by the airline (10). The ILOG CP solver was limited to 180 seconds to get the best suboptimal solution. All

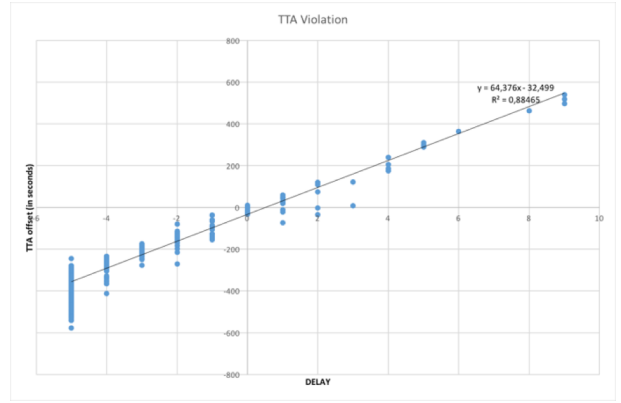


Figure 10. Correlation between TTA violation and the delays applied to the aircraft takeoff times.



Figure 11. A/C not meeting their TTA with respect to the applied CTOT delay.

the experiments were performed on a Window 10 computer with an Intel Core I7 CPU 2,30 GHz and 16GB RAM.

C. Solution analysis

Since the adjustments on CTOT and speed changes are bounded, the TTA fulfilment cannot be ensured. As stated at (18), the TTA requirement was relaxed, and its fulfilment was included in the optimization goal. The used weights were $w_1 = 10\%$ and $w_2 = 90\%$, so giving priority to the TTA preservation.

The Figure 10 shows the correlation between the actual time of arrival (ATA) compared to the TTA with respect to the applied CTOT delay. As it can be observed, in most of the cases the bounded speed adjustments are not enough to recover the effect of the applied delays. In Figure 11 it is shown the absolute numbers of aircrafts not able to meet their TTA with respect to the applied delay. There are two main reasons explaining this results.

The first observable one is that most of the aircrafts are moved ahead of their CTOT. This is a consequence of the solver search strategy [14], since time to get the suboptimal solution was limited to 180 seconds. This strategy is the default one and first takes the smallest values in the decision variable domains. In this case, this value is -5 minutes for the δ_a delay. Further research is required to define search strategies leading to better solutions.

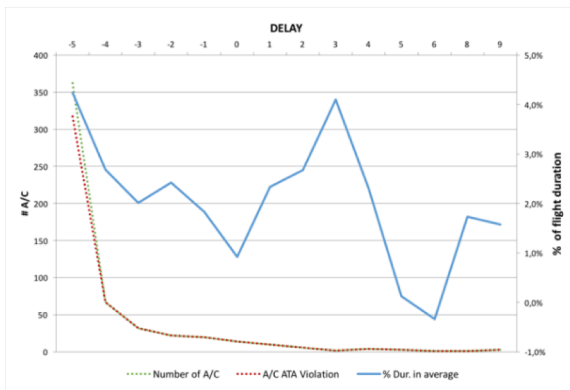


Figure 12.: blue line represents the average modification of flight duration with respect to the applied delay; dotted lines represent the number of A/C with the respective delay and the number of A/C not meeting their TTA.

The second reason can be explained from the curves at Figure 12. The number of aircrafts not meeting their TTA is tightly related to the applied delay, as it can be observed from dotted curves. However, the average modification on flight duration is not related to the applied delay. The margins enabled by the bounded speed adjustments are not enough for compensating the applied delays. This fact could be overcome only if, first, solutions with lower absolute delays can be found (better search strategy) and, second, if the aircraft trajectory allows a bigger absolute elasticity. The latest does not depend on the solution method, but on the duration of the flight and on the number and relative position of the proximate events where it is involved.

VI. CONCLUSIONS

In this work a CP model is presented for solving the concurrence events that might happen when the departure traffic is inserted into the enroute traffic. The model has been proved in a realistic and overstressed scenario and it has been able to find suboptimal solutions in a timeframe of 180 seconds for all the performed experiments. The model constraints ensure that all the proximate events are resolved by introducing small time adjustment both on the CTOT and relevant TTO's while maximizing the adherence to the RBT's. Although the model is not able to ensure that the ATM concept of preserving the TTA in a strict time frame is met, the CP solver can find solutions that remove all the conflicts reducing the number of potential ATC interventions. The concept of preserving the TTA has been relaxed and the objective function penalizes the TTA violation. The reason for this is the limit of the trajectory elasticity, since speed adjustments are bounded to a percentage of the total RBT duration. Furthermore, the quality of the solution found so far is directly linked to the solver search strategy. In this work, default parameters for searching have been used, leading to a solution where the smallest domain values at the delay variable are tested first. The search starts with -5 minutes of adjustment on the CTOT and, due to time restriction for finding a solution, possible better solutions cannot be explored by the solver. In consequence, the obtained total delay requires extra effort for recovering the TTA and, since the trajectory elasticity is limited, no acceptable speed change can be found to meet the TTA. Further research is required to define search strategies favoring

the selection of adjustments close to zero in first term. This way speed adjustment efforts are expected to be smaller.

A modeling approach for dealing with uncertainty in RBT time stamps has been also proposed. Currently, the model is under validation by using RBT and radar data obtained from DDR2 [3].

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