Balancing Reliability, Efficiency and Equity in Airport Scheduling Interventions

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***Abstract*—In the absence of opportunities for capacity expansion or operational enhancements, air traffic congestion mitigation may require scheduling interventions aimed to control the extent of over-capacity scheduling at busy airports. While existing approaches have focused on minimizing the overall impact of scheduling interventions across the airlines, this paper designs, optimizes, and assesses a novel approach for airport scheduling interventions that incorporates inter-airline equity objectives. It relies on a lexicographic modeling architecture based on efficiency (i.e., meeting airline scheduling preferences), equity (i.e., balancing scheduling adjustments fairly among the airlines), and reliability (i.e., mitigating airport congestion) objectives, subject to scheduling and network connectivity constraints. Theoretical and computational results show that ignoring inter-airline equity can lead to highly inequitable outcomes, but that our modeling approach achieves inter-airline equity at no, or small, losses in efficiency.**

Keywords-airport demand management; efficiency-equity trade-off; integer programming

I. Introduction

Many airports and air traffic management systems worldwide have had to accommodate growing volumes of operations with limited infrastructure capacity and operating capabilities. To cope with this challenge, busy airports outside the United States operate under a demand management regime known as schedule coordination, which imposes strict limits on the number of flights that can be scheduled per hour (or any other unit of time), and distribute a corresponding number of slots across the different airlines through an administrative procedure [1]. In contrast, US airports have been subject to weak scheduling constraints since the phase-out of the High Density Rule, effective in 2007. This has created large imbalances between schedules and capacity at the busiest airports, resulting in severe congestion, whose nationwide impact was estimated at over $30 billion for the year 2007 [2]. This has motivated the imposition of "flight caps" at a few of the busiest airports, including New York's John F. Kennedy International Airport (JFK), Newark International Airport (EWR), and LaGuardia Airport (LGA). Given forecasts of significant air traffic growth over the next 20 years [3], such demand management practices are expected to remain a prominent lever to ensure adequate levels of service at the busiest airports worldwide.

The topic of airport demand management has attracted significant attention in the economics and operations research literatures [4]. Proposed mechanisms fall into two broad categories: (i) market-based approaches based on congestion pricing [5, 6, 7] and slot auctions [8, 9], and (ii) administrative approaches relying on non-monetary *scheduling interventions* that propose adjustments to airlines' preferred schedules of flights in order to limit, or control, peak-hour scheduling levels at busy airports. While market-based mechanisms could potentially enhance social welfare, all existing demand management practices are based on the administrative approach, both at schedule-coordinated airports operating under the aegis of the International Air Transport Association (IATA) and at US airports operating under the "flight caps" regime.

Recent research has showed the potential to improve current scheduling intervention practices under the administrative approach to achieve adequate levels of congestion while minimizing deviations from airlines' scheduling preferences. Several papers have aimed to enhance the procedure at the IATA schedule-coordinated airports to match airlines' scheduling requests as closely as possible [10, 11, 12]. In the US context, marginal reductions in allocated capacity across the airlines could reduce delays significantly, improve airline profitability and enhance passenger welfare [13, 14]. This motivated recent models of intra-day scheduling interventions, which optimized congestion-mitigating adjustments in airline timetabling of flights [15, 16]. On the negative side, all these existing approaches are focused exclusively on overall scheduling levels at the airports, without considering explicitly the impact of the interventions on the different airlines. In turn, they may displace a disproportionate number of flights from one airline (or a small number of airlines).

This paper provides an original approach that optimizes scheduling interventions at busy airports in a way that achieves reliability (i.e., on-time performance) objectives, minimizes interference with airlines' competitive scheduling and, for the first time, balances the impact of such interventions equitably among the airlines. This approach takes as inputs, (i) capacity estimates at an airport under consideration, obtained from historical records of operations, and (ii) a schedule of flights requested by the airlines to a central decision-maker (e.g., administratively appointed schedule coordinators at slot-controlled airports, the Federal Aviation Administration (FAA) in the United States). It then produces a schedule of flights to reduce anticipated delays at the considered airport, while minimizing the *displacement* from the schedule requested by the airlines, and incorporating novel inter-airline equity objectives.

In the aviation context, this builds upon recent developments in Air Traffic Flow Management (ATFM). ATFM aims to optimize the flows of aircraft at airports or through air traffic control sectors over the day of operations to reduce local imbalances between demand and capacity. Whereas early ATFM developments were exclusively based on efficiency objectives (minimizing total congestion costs), recent studies have incorporated inter-airline equity considerations, aiming to make the ATFM outcomes more acceptable to each airline [17, 18, 19, 20, 21]. This paper aims to integrate similar objectives into the optimization of scheduling interventions. However, unlike ATFM, no standard of equity has been accepted in the industry with respect to scheduling interventions. Moreover, scheduling interventions may result in flights being rescheduled later or earlier than their preferred times requested by the airlines. This contrasts with the situation in ATFM where flights cannot be moved earlier than their scheduled time. Thus, the ATFM schemes of ration-by-schedule and schedule compression do not have any direct analogs in the context of scheduling interventions. It is thus necessary to propose new metrics of inter-airline equity and to develop new modeling frameworks for scheduling interventions.

From a modeling standpoint, this paper builds upon the *Integrated Capacity Utilization and Scheduling Model (ICUSM)* from [16] that optimizes such interventions through temporal shifts in demand (i.e., changes in intra-day flight timetabling), and no reduction in overall demand (i.e., no change in the set of flights scheduled in the day). It extends it in a way that balances scheduling adjustments equitably among the airlines. Specifically, this paper makes the following contributions:

- *Quantifying and optimizing the trade space between performance attributes for scheduling interventions*. We identify efficiency (i.e., meeting airline scheduling preferences), equity (i.e., balancing scheduling adjustments fairly among the airlines), and reliability (i.e., mitigating airport congestion) as three performance attributes. We develop quantitative indicators for each of them, using a unified framework of scheduling interventions. We then formulate a tractable lexicographic architecture to characterize and optimize the trade space between efficiency, equity, and reliability in airport scheduling interventions.
- *Identify conditions under which efficiency and equity can be jointly maximized*. We summarize results showing that, in the absence of network connections and in the case where all flights are equally costly (or equally inconvenient) to reschedule, efficiency and equity can be jointly maximized when the imbalances between demand and capacity are limited to non-consecutive periods in the day, or when the schedules of flights of the different airlines exhibit the same intra-day patterns. We then describe instances where the schedules of flights, network connections, or unequal flight valuations can give rise to a trade-off between efficiency and equity.
- *Generating and solving real-world full scale computational scenarios at the John F. Kennedy Airport (JFK)*. We show that, under a wide range of realistic and hypothetical scheduling conditions, the consideration of efficiency-based objectives exclusively in airport scheduling interventions may lead to highly inequitable outcomes, but that inter-airline equity can be achieved at no (or minimal) efficiency losses. This suggests that existing approaches for scheduling interventions can be extended to include inter-airline equity considerations.

In Section II, we formulate our model of scheduling interventions with inter-airline equity considerations. Section III summarizes some theoretical results that identify some scheduling conditions under which efficiency and equity may be jointly maximized, and, conversely, the conditions under which a trade-off between these two objectives may arise. In Section IV, we show computational results from a case study at JFK Airport. Section V concludes.

II. Model Development

We formulate our model's inputs, decision variables, constraints, and objectives of reliability, efficiency, and inter-airline equity. We then provide the solution architecture used to solve this multi-criteria decision-making problem.

A. Inputs

We denote by Π the airport where the scheduling interventions are considered. We consider the following sets:

$$\begin{aligned} \mathcal{T} = \{1, ..., T\} &= \text{set of time periods, indexed by } t \\ \mathcal{F} = \{1, ..., S\} &= \text{set of flights, indexed by } i \text{ or } j \\ \mathcal{F}^{\text{arr}}/\mathcal{F}^{\text{dep}} &= \text{set of flights arriving at/departing from } \Pi \\ \mathcal{C} \in \mathcal{F} \times \mathcal{F} &= \text{set of connecting flight pairs } (i, j) \\ \mathcal{A} &= \text{set of airlines, indexed by } \{1, ..., A\} \\ \mathcal{F}_a &= \text{set of flights from airline } a \text{ at airport } \Pi \end{aligned}$$

A connection refers to any pair of flights between which a minimum and/or a maximum time must be maintained to enable an aircraft or passengers to connect. Note that the set of flights considered in the model may include flights that are not scheduled to land or take off at the airport Π where the scheduling interventions are applied, i.e., $\mathcal{F}^{\text{arr}} \cup \mathcal{F}^{\text{dep}}$ may be a strict subset of $\mathcal{F}$. This arises from the need to maintain feasible connections in a network of airports.

We also introduce the following parameters:

$$\mathcal{S}_{it}^{\text{arr}} = \begin{cases} 1 & \text{if } i \text{ is scheduled to land no earlier than } t \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{S}_{it}^{\text{dep}} = \begin{cases} 1 & \text{if } i \text{ is scheduled to take off no earlier than } t \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} t_{ij}^{\min} &= \text{minimum connection time between } i \text{ and } j \\ t_{ij}^{\max} &= \text{maximum connection time between } i \text{ and } j \\ \lambda_t^{\text{arr}} &= \text{limit on the number of arrivals at } \Pi \text{ during } t \\ \lambda_t^{\text{dep}} &= \text{limit on the number of departures at } \Pi \text{ during } t \end{aligned}$$

v_i = "valuation" of flight i

Note, first, that the limits on the number of scheduled arrivals and departures λ_t^{arr} and λ_t^{dep} can be adjusted to reflect various levels of scheduling interventions. At schedule-coordinated airports, this corresponds to the values of the declared capacity. Alternatively, they can also be determined through the integration of on-time performance targets using the procedure developed in [16].

The "flight valuations" v_i aim to reflect airlines' preferences regarding which flights to reschedule, and which ones to maintain at requested times. Flights with lower valuations can be thought of as less "costly" to reschedule, or as the flights that exhibit more timetabling flexibility. Note that this setting captures the current paradigm under which "a flight is a flight" with $v_i = 1$ for all i. More broadly, this setting captures potential extensions of existing mechanisms, such as non-monetary processes that would allow the airlines to indicate their preferences through ranking or credit allocation, or auction-based mechanism where airlines would submit a bid for each flight i. While the design of such mechanisms is beyond the scope of this paper, our modeling approach incorporates inter-airline equity objectives in instances with either identical or differentiated flight valuations.

B. Variables

The model determines which flights to reschedule to later or earlier times to minimize the displacement from the airlines' preferred schedule of flights (as mentioned in the introduction, no flight request is rejected by the model). This is modeled with the following decision variables:

$$w_{it}^{\text{arr}} = \begin{cases} 1 & \text{if } i \text{ is rescheduled to land no earlier than } t \\ 0 & \text{otherwise} \end{cases}$$

$$w_{it}^{\text{dep}} = \begin{cases} 1 & \text{if } i \text{ is rescheduled to take off no earlier than } t \\ 0 & \text{otherwise} \end{cases}$$

u_i = displacement of i (in 15-minute periods)

Note that the variables w_{it}^{arr} and w_{it}^{dep} take the form, for each flight i, (1,...,1,0,...,0). By convention, we assume that $w_{i,T+1}^{\text{arr}} = w_{i,T+1}^{\text{dep}} = 0, \forall i \in \mathcal{F}$.

a) Constraints: The optimization is subject to scheduling, network connectivity and schedule limits constraints.

$$w_{it}^{\text{arr}} \geq w_{i,t+1}^{\text{arr}} \quad \forall i \in \mathcal{F}, \forall t \in \mathcal{T} \quad (1)$$

$$w_{it}^{\text{dep}} \geq w_{i,t+1}^{\text{dep}} \quad \forall i \in \mathcal{F}, \forall t \in \mathcal{T} \quad (2)$$

$$w_{i1}^{\text{arr}} = 1 \quad \forall i \in \mathcal{F} \quad (3)$$

$$w_{i1}^{\text{dep}} = 1 \quad \forall i \in \mathcal{F} \quad (4)$$

$$\sum_{t \in \mathcal{T}} (w_{it}^{\text{arr}} - S_{it}^{\text{arr}}) = u_i \quad \forall i \in \mathcal{F} \quad (5)$$

$$\sum_{t \in \mathcal{T}} \left(w_{it}^{\text{dep}} - S_{it}^{\text{dep}}\right) = u_i \quad \forall i \in \mathcal{F} \quad (6)$$

$$\sum_{t \in \mathcal{T}} \left(w_{jt}^{\text{dep}} - w_{it}^{\text{arr}}\right) \geq t_{ij}^{\min} \quad \forall (i,j) \in \mathcal{C} \quad (7)$$

$$\sum_{t \in \mathcal{T}} \left(w_{jt}^{\text{dep}} - w_{it}^{\text{arr}}\right) \leq t_{ij}^{\max} \quad \forall (i,j) \in \mathcal{C} \quad (8)$$

$$\sum_{i \in \mathcal{F}^{\text{arr}}} \left(w_{it}^{\text{arr}} - w_{i,t+1}^{\text{arr}}\right) \leq \lambda_t^{\text{arr}} \quad \forall t \in \mathcal{T} \quad (9)$$

$$\sum_{i \in \mathcal{F}^{\text{dep}}} \left(w_{it}^{\text{dep}} - w_{i,t+1}^{\text{dep}}\right) \leq \lambda_t^{\text{dep}} \quad \forall t \in \mathcal{T} \quad (10)$$

Constraints (1) and (2) ensure that w^{arr} and w^{dep} are non-increasing in t, consistent with their definition. Constraints (3) and (4) ensure that no flight is eliminated. Constraints (5) and (6) define flight displacement as the difference between rescheduled and original scheduled times, and ensure that the scheduled block-times are left unchanged. Constraints (7) and (8) maintain connection times within the specified ranges. Constraints (9) and (10) ensure that the number of scheduled arrivals and departures do not exceed their respective allowable limits λ_t^{arr} and λ_t^{dep}.

C. Objectives

We consider the following three performance attributes of scheduling interventions: efficiency, inter-airline equity, and reliability.

a) Efficiency: This refers to the ability to meet airline scheduling preferences. Since no flight is eliminated, efficiency is measured by the displacement from the schedule of flights requested by the airlines. We consider two efficiency objectives: (i) *min-max efficiency*, defined as the largest displacement sustained by any flight and denoted by δ, and (ii) *weighted efficiency*, defined as the weighted sum of schedule displacements sustained by all flights, and denoted by Δ.

$$\delta = \max_{i \in \mathcal{F}} |u_i| \Longrightarrow \min \delta \quad (11)$$

$$\Delta = \sum_{i \in \mathcal{F}} v_i |u_i| \Longrightarrow \min \Delta \quad (12)$$

b) Inter-airline Equity: This refers to the ability to balance schedule displacement fairly among the airlines. We describe each airline's disutility as the weighted average of per-flight displacements, denoted by σ_a. Perfect equity is achieved when the weighted sum of displacements borne by any airline is proportional to its number of flights scheduled at airport Π, i.e., when the weighted average of per-flight displacements is the same for all airlines. In order to maximize inter-airline equity, we minimize airline disutilities lexicographically, i.e., we first minimize the largest airline disutility (i.e., the largest weighted average per-flight displacements borne by any airline), then the second-largest, etc. This extends the min-max equity formulation proposed in [22, 23].

$$\sigma_a = \frac{1}{|\mathcal{F}_a|} \sum_{i \in \mathcal{F}_a} v_i |u_i|, \forall a \in \mathcal{A} \Longrightarrow \text{lex} \min \sigma \quad (13)$$

We denote the largest airline disutility by Φ:

$$\Phi = \max_{a \in \mathcal{A}} \sigma_a \quad (14)$$

c) Reliability: This refers to the ability to mitigate airport congestion. Airport delays can be quantified as a function of flight schedules and airport capacity by means of a queuing model. In this paper, we use the model developed in [24], which captures the variations in the number of arrivals and departures scheduled over the course of the day and their impact on the utilization of airport capacity by air traffic controllers. Since airport capacity patterns are fixed, this essentially provides a relationship between the number of flights scheduled (determined by the limits λ_t^{arr} and λ_t^{dep}) and the resulting expected arrival and departure delays over the course of the day.

D. Lexicographic Modeling Approach

We characterize the trade space between efficiency, equity, and reliability by determining its Pareto frontier, i.e., the set of solutions such that no other feasible solution could improve at least one of the three objectives without worsening at least one of others. This representation of the trade space is flexible enough to be used by system managers and policy makers to select the most appropriate level of compromise between these objectives. To this end, we develop a lexicographic optimization approach that (i) fixes scheduling limits (hence, reliability targets); (ii) maximizes efficiency under schedule limits; and (iii) maximizes equity under schedule limits and efficiency targets.

First, we fix the schedule limits λ_t^{arr} and λ_t^{dep} with respect to reliability objectives. We then aim to find the "best" schedule (in terms of efficiency and equity) that meets these constraints.

Second, we determine the schedule of flights that maximizes efficiency, subject to scheduling constraints, network connectivity constraints, and schedule limits constraints. We formulate the efficiency-maximizing problem by lexicographically maximizing, first, min-max efficiency δ, and, second, weighted efficiency Δ. This is motivated by the objective of avoiding large flight displacements, and consistent with the literature on this topic [15, 16]. This is expressed in Problems **P1** and **P2** described below:

*a) **P1**:* We minimize min-max efficiency metric δ, subject to scheduling, network connectivity and schedule limits constraints. We denote by δ^* its optimal value.

$$\begin{aligned} \min \quad & \delta \\ \text{s.t.} \quad & \text{Constraints (1) to (10)} \end{aligned}$$

*b) **P2**:* We minimize weighted efficiency metric Δ, subject to scheduling, network connectivity and schedule limits constraints, and subject to the constraint that no flight may be displaced by more than δ^*. We denote by Δ^* its optimal value.

$$\begin{aligned} \min \quad & \Delta \\ \text{s.t.} \quad & \text{Constraints (1) to (10)} \\ & \text{Min-max efficiency: } |u_i| \leq \delta^*, \forall i \in \mathcal{F} \end{aligned}$$

Third, we maximize inter-airline equity, subject to scheduling constraints, network connectivity constraints, schedule limits constraints, and efficiency targets. This is formulated in the class of problems **P3**(ρ) described below:

*c) **P3**(ρ):* We fix efficiency targets, and we lexicographically minimize airline disutilities, subject to scheduling, network connectivity, schedule limits, and efficiency constraints. We characterize the trade space between efficiency and equity by varying the efficiency target. Specifically, we impose that min-max efficiency must be optimal (i.e., no flight may be rescheduled by more than δ^*) and we denote by $\rho \in [0, \infty)$ the relative loss in weighted efficiency that is allowed (i.e., the weighted displacement must not exceed $(1+\rho)\Delta^*$). When $\rho = \infty$, we only maximize equity (without any weighted efficiency consideration). When $\rho = 0$, we maximize equity, under optimal min-max and optimal weighted efficiency.

$$\begin{aligned} \text{lex}\min \quad & \sigma \\ \text{s.t.} \quad & \text{Constraints (1) to (10)} \\ & \text{Min-max efficiency: } |u_i| \leq \delta^*, \forall i \in \mathcal{F} \\ & \text{Weighted efficiency: } \sum_{i \in \mathcal{F}} v_i \, |u_i| \leq (1+\rho)\, \Delta^* \end{aligned}$$

Problems **P1**, **P2**, and **P3**(ρ) together determine the Pareto frontier of the trade space between efficiency, equity, and reliability. First, variations in the schedule limits λ_t^{arr} and λ_t^{dep} quantify the trade-off between the costs of scheduling interventions (in terms of inefficiency and inequity) and delay reductions. Second, for any schedule limits λ_t^{arr} and λ_t^{dep}, varying the parameter ρ quantifies the potential trade-off between weighted efficiency and inter-airline equity (under optimal min-max efficiency).

We denote by $\sigma^*(\rho)$ the equity-maximizing vector of airline per-flight displacements, as a function of ρ, and $\Phi^*(\rho) = \max_{a \in \mathcal{A}} \sigma_a^*(\rho)$. We denote by Δ^{eq} the smallest equity-maximizing value of Δ, and by ρ^* the minimum loss in weighted efficiency required to attain optimal equity (i.e., $\Delta^{\text{eq}} = (1+\rho^*)\Delta^*$). With these notations, the "price of efficiency" and the "price of equity" will be characterized by $P_{\text{eff}} = \frac{\Phi^*(0) - \Phi^*(\infty)}{\Phi^*(\infty)}$, and by $P_{\text{eq}} = \frac{\Delta^{\text{eq}} - \Delta^*}{\Delta^*} = \rho^*$, respectively. They correspond to the relative loss in one objective when the other one is optimized.

Figure 1 illustrates our approach to maximizing weighted efficiency and inter-airline equity, for given schedule limits λ_t^{arr} and λ_t^{dep}, and the optimal value of min-max efficiency δ^*. Specifically, it shows hypothetical variations in three airlines' disutilities (σ_1, σ_2, and σ_3) as a function of the weighted efficiency target $\Delta = (1+\rho)\Delta^*$. By construction, the region on the left side of Δ^* is infeasible, i.e., the weighted schedule displacement must be at least Δ^*. Moreover, the largest airline disutility Φ is a non-increasing function of the value of weighted efficiency Δ (i.e., of ρ). Note that the other airlines' utilities (here, σ_2 and σ_3) may increase or decrease as Φ is reduced. As the largest airline disutility Φ attains its optimal value, the second-largest disutility may still be larger than its optimal value. In this case, further increases in ρ may yield further improvements in the lexicographic minimization of airline disutilities. Optimal equity is attained when the largest, second largest, third largest, etc., airline disutilities have all reached their optimal values (i.e., the values that

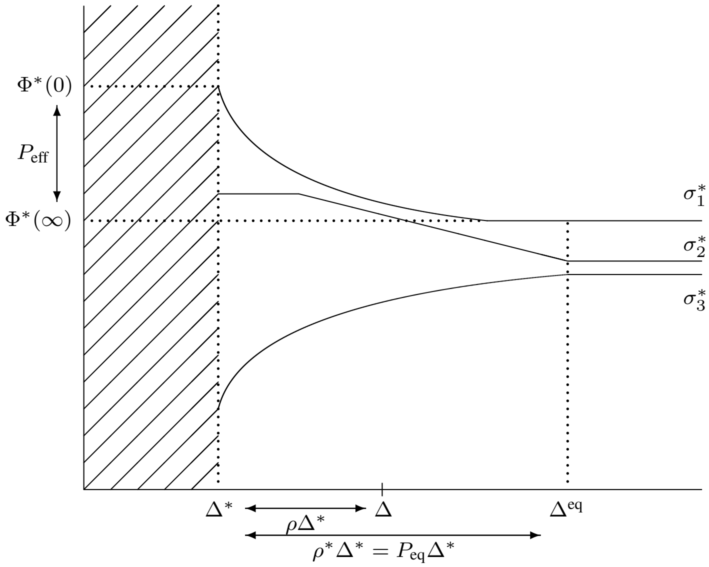


Fig. 1: Schematic trade space between efficiency and equity

would be obtained without any efficiency consideration, or with $\rho = \infty$). This representation shows the price of efficiency and the price of equity as the relative difference between $\Phi^*(\infty)$ and $\Phi(0)$ and between Δ^* and Δ^{eq}, respectively. Note that Figure 1 shows an instance where the order of airline disutilities remains identical for all values of ρ (i.e., in this case, $\sigma_1^*(\rho) > \sigma_2^*(\rho) > \sigma_3^*(\rho), \forall \rho \geq 0$), but this need not be the case (i.e., the curves may intersect).

III. Efficiency/Equity Trade-off

In this section, we summarize some theoretical results that identify some conditions on the scheduling inputs provided by the airlines under which efficiency and equity can be jointly optimized and, conversely, some conditions where a trade-off arises between the two objectives. Further details on these results can be found in [25].

We consider the case where: (i) no network connections need to be maintained ($\mathcal{C} = \emptyset$) and (ii) all flights are equally valued ($v_i = 1, \forall i \in \mathcal{F}$). We denote by D_t^{arr} and D_t^{dep} the number of arrivals and departures, respectively, scheduled during period t before the scheduling interventions, i.e., $D_t^{\text{arr}} = \sum_{i \in \mathcal{F}} \left(S_{it}^{\text{arr}} - S_{i,t+1}^{\text{arr}}\right)$ and $D_t^{\text{dep}} = \sum_{i \in \mathcal{F}} \left(S_{it}^{\text{dep}} - S_{i,t+1}^{\text{dep}}\right)$. Similarly, we denote by D_{at}^{arr} and D_{at}^{dep} the number of arrivals and departures, respectively, scheduled by airline a during period t,so $\sum_{a \in \mathcal{A}} D_{at}^{\text{arr}} = D_t^{\text{arr}}$ and $\sum_{a \in \mathcal{A}} D_{at}^{\text{dep}} = D_t^{\text{dep}}$ for all $t \in \mathcal{T}$.

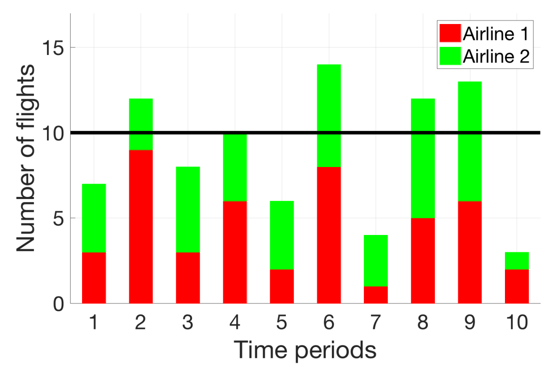
Proposition 1 shows that efficiency and equity can be jointly maximized if the number of flight arrivals and the number of flight departures scheduled over any set of three consecutive time periods is lower than the total number of arrivals and departures, respectively, that can be scheduled over the same three periods.

Proposition 1: If $\sum_{l=t-1}^{t+1} D_l^{\text{arr}} \leq \sum_{l=t-1}^{t+1} \lambda_l^{\text{arr}}, \forall t \in \mathcal{T}$ and $\sum_{l=t-1}^{t+1} D_l^{\text{dep}} \leq \sum_{l=t-1}^{t+1} \lambda_l^{\text{dep}}, \forall t \in \mathcal{T}$, then there exists a solution that simultaneously maximizes efficiency and inter-airline equity.

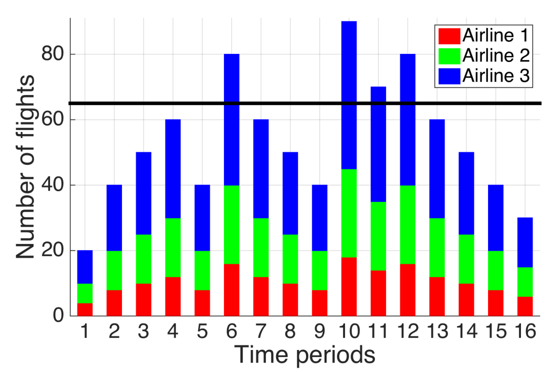
Proposition 2 shows that efficiency and equity can be jointly maximized if each airline's share of flights is identical across all periods. Specifically, we assume that the number of arrivals (resp. departures) scheduled by each airline a during each period t is the product of an airline-related factor α_a^{arr} (resp. α_a^{dep}) and a period-related factor β_t^{arr} (resp. β_t^{dep}). In that case, there is significant flexibility in terms of the airlines whose flights should be rescheduled, which enables equity-maximization at no efficiency loss.

Proposition 2: If there exist integers $\left(\alpha_a^{\text{arr}}\right)_{a \in \mathcal{A}}$, $\left(\beta_t^{\text{arr}}\right)_{t \in \mathcal{T}}$, $\left(\alpha_a^{\text{dep}}\right)_{a \in \mathcal{A}}$ and $\left(\beta_t^{\text{dep}}\right)_{t \in \mathcal{T}}$ such that $D_{at}^{\text{arr}} = \alpha_a^{\text{arr}} \beta_t^{\text{arr}}$ and $D_{at}^{\text{dep}} = \alpha_a^{\text{dep}} \beta_t^{\text{dep}}$ for all $a \in \mathcal{A}, t \in \mathcal{T}$, then there exists a solution that simultaneously maximizes efficiency and inter-airline equity.

The conditions of these two propositions are illustrated in Figure 2 (for the schedule of arrivals or the schedule of departures). Under the conditions of Proposition 1 (Figure 2a), the imbalances between demand and capacity are small enough so no time period is such that some flights get displaced *to* that period and some other flights get displaced *from* that period. Under the conditions of Proposition 2 (Figure 2b), the schedules of flights of the airlines exhibit the same intra-day variations. Even though these conditions are usually not exactly satisfied in practice, our computational experiments



(a) Proposition 1



(b) Proposition 2

Fig. 2: Example of scheduling conditions of Propositions 1 and 2

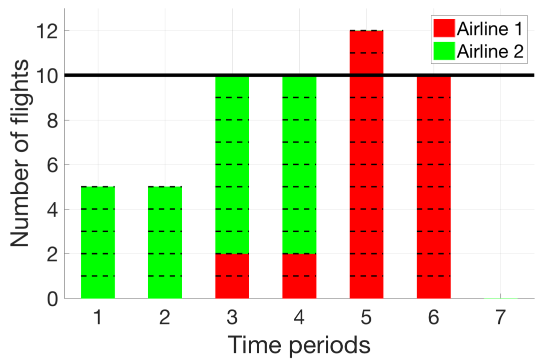
reported in Section IV show that the insights derived in these two cases can be relevant and applicable in practical settings.

Conversely, a trade-off between efficiency and equity might arise through (i) inter-airline variations in intra-day flight schedule patterns (we refer to it simply by ‘differentiated schedules’), (ii) network connections, and (iii) intra-airline variations in flight valuations (we refer to it simply by ‘differentiated valuations’). These conditions are shown in Figure 3 and discussed below.

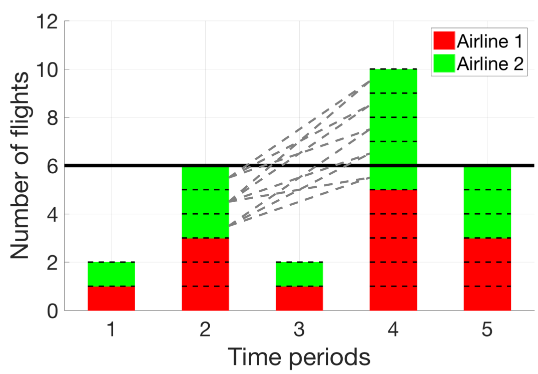
Figure 3a shows that weighted efficiency and equity may not be jointly maximized with differentiated airline schedules, in a 7-period case with 2 airlines and 26 flights per airline, and a simple capacity constraint that ensures that no more than 10 flights may be scheduled per period. We assume that all flights are valued equally and that there are no connections. We also assume that airline 1’s flights (shown in red) are concentrated at later periods, and airline 2’s flights (shown in green) are concentrated at earlier periods. Since capacity is only eceeded during period 5, when all flights scheduled are airline 1’s flights, efficiency would be maximized by displacing 2 flights from period 5 and 2 flights from period 6 (all from airline 1) to later times, by 1 period each. The resulting total displacement is equal to 4 periods, and the airline disutilities are equal to 4/26 for airline 1 and 0 for airline 2. In contrast, equity is maximized by displacing 3 flights of each airline to earlier times, by 1 period each (2 flights from airline 1 from period 5 to period 4; 1 flight from each airline from period 4 to period 3; 2 flights from airline 2 from period 3 to period 2). The resulting total displacement is equal to 6 periods, and each airline’s disutility is equal to 3/26.

Figure 3b shows that weighted efficiency and equity may not be jointly optimized with network connections, with 5 periods, 2 airlines with 13 flights each, and a capacity of 6 flights per period. We represent connections by dashed, gray “links” between flight pairs, and we assume that each connection requires a 2-period interval between the flights in the connection at a minimum. Airline 2’s network involves a number of connections, whereas airline 1’s network has no connections. In this case, efficiency is maximized by displacing 4 of airline 1’s flights (from period 4 to period 3) by 1 period each. The resulting total displacement is equal to 4 periods, and the airline disutilities are equal to 4/13 for airline 1 and 0 for airline 2. In contrast, equity is maximized by displacing 3 flights of each airline, by 1 period each (only 3 flights from airline 1 from period 4 to period 3, as well as one flight from airline 2 from period 4 to period 3 and 2 flights from airline 2 from period 2 to period 1 to maintain 2-period connection times). The resulting total displacement is equal to 6 periods, and each airline’s disutility is equal to 3/13.

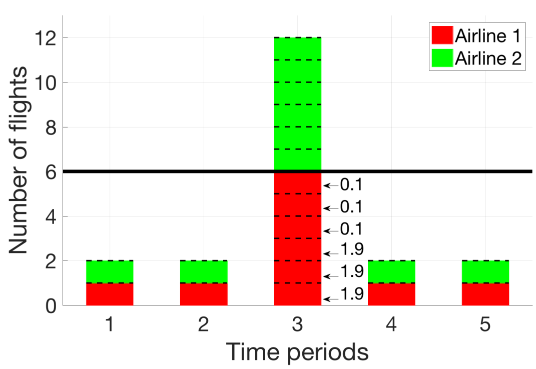
Figure 3c shows that weighted efficiency and equity may not be jointly optimized with differentiated flight valuations, with 5 periods, 2 airlines with 10 flights each, and a capacity of 6 flights per period. Every flight has a value $v_i = 1$, except the 6 flights from airline 1 in period 3, three of which have a value $v_i = 0.1$ each, and the other three have a value $v_i = 1.9$ each. Efficiency is maximized by displacing the three flights of value $v_i = 0.1$ and three flights of value $v_i = 1$ from period 3. The optimal value of the weighted displacement is equal to 3.3 and the airline disutilities are equal to 0.3/10 for airline 1 and to 3/10 for airline 2. In contrast, equity is maximized by displacing four flights of airline 1 (the 3 flights with value 0.1 and one flight with value 1.9) and two flights of airline 2 (with value 1 each). The weighted displacement is equal to 4.2 and the airline disutilities are equal to 2.2/10 for airline 1 and to 2/10 for airline 2.



(a) Differentiated schedules



(b) Network connections



(c) Differentiated valuations

Fig. 3: Example of conditions with an efficiency/equity trade-off

IV. Computational Results

We implement the models developed in Section II for a case study at JFK Airport. We show that, in realistic instances, inter-airline equity can be significantly improved at no (or minimal) efficiency losses if flights are equally valued. We then show that significant equity gains can be obtained even under differentiated flight valuations, at small losses in efficiency. The price of equity is consistently significantly smaller than the price of efficiency even under differentiated flight valuations.

A. Experimental Setup

We consider data from September 18, 2007 at the John F. Kennedy Airport (JFK), one of the busiest US airports with a peaked schedule of flights that offers opportunities for delay reductions through scheduling interventions. Since no scheduling interventions were in place at JFK in 2007, it is representative of airlines' scheduling preferences. Flight schedules were obtained from the Aviation System Performance Metrics (ASPM) database [26]. We consider four groups of airlines: (i) Delta Airlines (DAL) and its regional partners (which operated a total of 320 flights on 09/18/2007 at JFK), (ii) American Airlines (AAL) and its regional partners (260 flights), (iii) JetBlue Airways (JBU) (174 flights), and (iv) all other airlines, each of which represents a smaller share of traffic at JFK (408 flights combined). These scheduling data were used to construct sets $\mathcal{F}$, $\mathcal{F}^{\text{arr}}$, $\mathcal{F}^{\text{dep}}$, $\mathcal{F}_a$, $\mathcal{S}^{\text{arr}}$, and $\mathcal{S}^{\text{dep}}$. We reconstructed aircraft and passenger connections to determine $\mathcal{C}$, $t^{\min}$, and $t^{\max}$ using the ASPM database [26], the minimum aircraft turnaround times estimated in [27], and the passenger connections database developed in [28].

To determine the schedule limits λ_t^{arr} and λ_t^{dep}, we used the results from [16]. This approach starts with airport capacity estimates, i.e., estimates of the average number of arrivals and departures that can be operated per unit of time, obtained from [29]. With the actual schedule of flights on 09/18/2007, the peak expected arrival and departure queue lengths are equal to 14.6 aircraft and 28.1 aircraft, respectively—obtained using the model of airport congestion from [24]. By varying on-time performance objectives (expressed as maximum allowable targets for expected arrival and departure queue lengths), we obtain a variety of values of λ_t^{arr} and λ_t^{dep}. In this section, we name "Test 1", "Test 2", "Test 3" and "Test 4" four computational tests with increasingly stringent on-time performance targets and corresponding progressively lower values of the schedule limits. Details can be found in [25].

B. Results under Uniform Flight Valuations

We first consider the case where all flights are equally valued, i.e., $v_i = 1, \forall i \in \mathcal{F}$. This corresponds to current practice, where the airlines do not provide any inputs on relative timetabling flexibility of their flights, and scheduling interventions are thus performed under the "a flight is a flight" paradigm. We compare the results obtained under an efficiency-maximization objective (Problems **P1** and **P2**) to those obtained with the various levels of inter-airline equity objectives (Problems **P3**(ρ)). This comparison thus shows the extent to which inter-airline equity can be achieved in scheduling interventions under realistic conditions.

Note that the solution of Problem **P2** is arbitrarily "chosen" by the optimization solver from the set of (possibly) multiple optimal solutions. In order to characterize the equity range among efficiency-maximizing solutions, we also determine the solution which *minimizes* inter-airline equity, i.e., which lexicographically *maximizes* airline disutilities, while ensuring the optimal value of efficiency. This characterizes the efficiency-maximizing solution that performs the worst in terms of inter-airline equity. We denote this problem by $\overline{\mathbf{P2}}$.

Table I shows, for each of the four tests, the total schedule displacement faced by each airline (that is, the number of its flights displaced by 15 minutes each, as the maximum displacement δ^* is equal to 1 15-minute period in all our case studies), and each airline's disutility (i.e., its weighted average per-flight displacement) for Problems **P2**, $\overline{\mathbf{P2}}$ and **P3**(ρ^*). It also reports the ratio of the largest to smallest airline disutility. As the schedule limits become smaller, the resulting schedule displacement increases, as noted by [16], but these results show that, for any test considered, the modeling approach developed in this paper provides strong equity gains at no loss in efficiency. Note, first, that Problem $\overline{\mathbf{P2}}$ results in max-min ratios $\frac{\max_a \sigma_a}{\min_a \sigma_a}$ ranging between 10 and 50. For the cases considered, AAL and JBU tend to be much more significantly penalized than DAL, which is reflected through more of their flights being rescheduled and through much higher disutility values. The set of efficiency-maximizing solutions thus contains highly inequitable outcomes. Problem **P2** does not result in the most inequitable outcome in that set, but provides solutions that still impact some airlines (here, AAL, JBU and the "other" airlines) more negatively than others (here, DAL). Inter-airline equity is achieved only by solving Problem **P3**(ρ^*). In that case, airline disutilities are much closer to each other than those obtained by solving Problems $\overline{\mathbf{P2}}$ and **P2**. Note that the differences in airlines' schedules of flights and network connectivities result in all four airlines *not* having the exact same disutility, but differences are very small (i.e., the max-min ratio $\frac{\max_a \sigma_a}{\min_a \sigma_a}$ is very close to 1) under the equitable solution. Most importantly, the equity-maximizing solution (Problem **P3**(ρ^*)) results in the same total displacement as the efficiency-maximizing solution (Problem **P2**) in all cases considered. Only the distribution of schedule displacement across the airlines is modified. In other words, efficiency and equity can be jointly maximized, and the price of equity (ρ^*) and the price of efficiency are both zero.

Therefore, joint optimization of efficiency and equity is achievable under current schedules of flights and uniform flight valuations (which is the assumption widely used in current practice). In light of the results from Section III, this suggests that inter-airline variations in flight schedules and network connectivities are relatively weak and do not create, by themselves, a trade-off between efficiency and equity. This is due to the fact that peak-hour schedules typically include flights from several airlines and the schedules of all airlines exhibit network connections to some extent (so the situations depicted in Figures 3a and 3b are not typical of actual scheduling patterns at busy airports). Under these conditions, incorporating inter-airline equity objectives in scheduling interventions can thus yield significant benefits by balancing scheduling adjustments

TABLE I: Number of flights displaced and airline disutilities per airline under uniform flight valuations

Test	Model	Number of flights displaced: DAL	AAL	JBU	Others	All	Disutility: $\sigma_a = \frac{1}{\lvert\mathcal{F}_a\rvert}\sum_{i\in\mathcal{F}_a}\lvert u_i\rvert$: DAL	AAL	JBU	Others	$\frac{\max_a \sigma_a}{\min_a \sigma_a}$
Test 1	$\overline{\mathbf{P2}}$	1	13	1	5	20	0.3%	5.0%	0.6%	1.2%	16.00
	P2	1	9	2	8	20	0.3%	3.5%	1.1%	2.0%	11.08
	P3(ρ^*)	4	5	3	8	20	1.3%	1.9%	1.7%	2.0%	1.57
Test 2	$\overline{\mathbf{P2}}$	1	29	9	7	46	0.3%	11.2%	5.2%	1.7%	35.69
	P2	7	18	8	13	46	2.2%	6.9%	4.6%	3.2%	3.16
	P3(ρ^*)	13	10	7	16	46	4.1%	3.8%	4.0%	3.9%	1.06
Test 3	$\overline{\mathbf{P2}}$	1	28	27	9	65	0.3%	10.8%	15.5%	2.2%	49.66
	P2	10	27	10	18	65	3.1%	10.4%	5.7%	4.4%	3.32
	P3(ρ^*)	18	14	10	23	65	5.6%	5.4%	5.7%	5.6%	1.07
Test 4	$\overline{\mathbf{P2}}$	37	113	39	17	206	11.6%	43.5%	22.4%	4.2%	10.43
	P2	50	57	32	67	206	15.6%	21.9%	18.4%	16.4%	1.40
	P3(ρ^*)	57	46	31	72	206	17.8%	17.7%	17.8%	17.6%	1.01

more fairly among the airlines at no efficiency losses.

C. Results under Differentiated Flight Valuations

We now consider the case where all flights are not equally valued, and compare the outcomes of scheduling interventions when only the efficiency objectives are considered to the outcomes when equity objectives are also considered. This captures potential extensions of existing and other previously proposed mechanisms for airport scheduling interventions that would allow the airlines to provide the relative timetabling flexibility of their flights (e.g., auction, credit-based mechanism). Since the flight valuations rely on information that is often private to the airlines and since they are challenging to estimate using available public data, we use a sampling approach to simulate them. Specifically, we consider the case where the average flight valuation is identical for all airlines, to identify the impact of the *distribution* of flights valuations.

We keep the average flight valuation of all airlines equal to 1 (without loss of generality), and vary the distribution of flight valuations for one given airline a. We set $v_i = 1, \forall i \notin \mathcal{F}_a$. We partition the set of flights $\mathcal{F}_a$ of airline a into two subsets $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$ such that $\mathcal{F}_a^{(1)} \cap \mathcal{F}_a^{(2)} = \emptyset$ and $\mathcal{F}_a^{(1)} \cup \mathcal{F}_a^{(2)} = \mathcal{F}_a$. We can think of $\mathcal{F}_a^{(1)}$ (resp. $\mathcal{F}_a^{(2)}$) as the set of the more flexible flights (resp. the less flexible flights) of airline a. We choose to represent the valuations of the flights in $\mathcal{F}_a^{(1)}$ (resp. $\mathcal{F}_a^{(2)}$) by a Gamma distribution $\Gamma_1(\mu_1, k)$ (resp. $\Gamma_2(\mu_2, k)$) with mean μ_1 (resp. μ_2) and shape parameter k, with $\mu_1 < \mu_2$. We adjust the shape parameter of these distributions such that the 95$^{\text{th}}$ percentile of the former distribution coincides with the 5$^{\text{th}}$ percentile of the latter. These choices of distributions and parameters are made in order to provide a transparent and flexible bimodal characterization of flight valuations such that the valuations of flights in $\mathcal{F}_a^{(1)}$ are, in most cases, lower than the valuations of flights in $\mathcal{F}_a^{(2)}$. Finally, we set the values of flights in $\mathcal{F}_a^{(1)}$ (resp. $\mathcal{F}_a^{(2)}$) equal to $\Theta_1^{-1}\left(1/\left(\left|\mathcal{F}_a^{(1)}\right|+1\right)\right)$, $\Theta_1^{-1}\left(2/\left(\left|\mathcal{F}_a^{(1)}\right|+1\right)\right)$, ..., $\Theta_1^{-1}\left(\left|\mathcal{F}_a^{(1)}\right|/\left(\left|\mathcal{F}_a^{(1)}\right|+1\right)\right)$ $\Big($resp. $\Theta_2^{-1}\left(1/\left(\left|\mathcal{F}_a^{(2)}\right|+1\right)\right)$,$\Theta_2^{-1}\left(2/\left(\left|\mathcal{F}_a^{(2)}\right|+1\right)\right)$, ..., $\Theta_2^{-1}\left(\left|\mathcal{F}_a^{(2)}\right|/\left(\left|\mathcal{F}_a^{(2)}\right|+1\right)\right)\Big)$, where Θ_1 (resp. Θ_2) denotes the cumulative distribution function of $\Gamma_1(\mu_1, k)$ (resp. $\Gamma_2(\mu_2, k)$). This sampling strategy ensures that the resulting set of flight valuations is distributed "smoothly" across the distributions considered without sampling these values multiple times. For each airline, we vary two parameters: (i) the fraction of flights in $\mathcal{F}_a^{(1)}$, denoted by $\eta = \frac{\left|\mathcal{F}_a^{(1)}\right|}{\left|\mathcal{F}_a\right|}$ (so that $1-\eta = \frac{\left|\mathcal{F}_a^{(2)}\right|}{\left|\mathcal{F}_a\right|}$), and (ii) the mean valuations of flights in $\mathcal{F}_a^{(1)}$, i.e., μ_1 (such that $\eta\mu_1 + (1-\eta)\mu_2 = 1$). Within each set, $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$, we sort flights from the least valuable to the most valuable using 10 random permutations. In other words, the 10 tests have the same sets of flight valuations, but differ in terms of which actual flights are more flexible and which are less flexible.

Table II shows results (in Test 4, which corresponds to the most stringent schedule limits) under different sets of flight valuations provided by DAL (left) and AAL (right)—similar results are obtained by varying the flight valuations provided by the other airlines. The first row provides a baseline where all flights are equally valued (i.e., $v_i = 1, \forall i \in \mathcal{F}$). In the top half, we assume that $\mathcal{F}_a^{(1)}$ and $\mathcal{F}_a^{(2)}$ both comprise 50% of the flights from DAL or AAL, and we progressively increase the valuation differential $\mu_2 - \mu_1$. In the bottom half, we fix $\mu_1 = 0.75$ and we progressively decrease the proportion of flights in $\mathcal{F}_a^{(1)}$ (and we thus decrease μ_2 to ensure that $\eta\mu_1 + (1-\eta)\mu_2 = 1$). Table II reports, in each scenario, the total schedule displacement $\sum_{i\in\mathcal{F}_a}\lvert u_i\rvert$ of *each* airline a obtained in the equity-maximizing scenario (i.e., Problem **P3**(ρ^*)), as well as the prices of equity and efficiency, averaged across all 10 samples.

The observations from variations in $\mu_2 - \mu_1$ (top) and in η (bottom) are threefold. First, as an airline's flight valuations become more differentiated, the displacement of this airline's schedule increases. In turn, flight valuations create, for each airline, a trade-off between prioritizing *which* flights get rescheduled, on the one hand, and minimizing their total displacement, on the other hand. Second, as the variance in any airline's flight valuations increases, other airlines' displacements do not change significantly (in fact, sometimes they decrease a little). In other words, the model can account for any airline's scheduling preferences without negatively impacting the other airlines. Third, the price of equity is much smaller than the price of efficiency across all the scenarios considered, therefore indicating strong gains in inter-airline equity at small efficiency losses.

V. Conclusion

Any airport demand management scheme involves a trade-off between mitigating airport congestion, on the one hand, and minimizing interference with airlines' competitive scheduling, on the other hand. In this paper, we have developed, optimized and assessed models for airport scheduling interventions that, for the first time, incorporate inter-airline equity considerations. Theoretical and computational results have shown that, under a wide range of realistic and hypothetical scenarios, inter-airline equity can be achieved at no, or small, efficiency losses. In other words, achieving maximum equity requires a small sacrifice (if any) in terms of efficiency losses. On the other hand, for some of our computational scenarios, our results showed that ignoring inter-airline equity (i.e., considering efficiency-based objectives exclusively, or, in some cases, requiring maximum efficiency) may lead to highly inequitable outcomes. This further highlights that it is critical to explicitly incorporate inter-airline equity objectives in the optimization of scheduling interventions. In turn, this offers the potential to extend existing approaches to airport demand management (either the schedule coordination practices in place at busy airports outside the United States, or the scheduling practices at a few of the busiest US airports where flight caps are in place) in a way that balances scheduling interventions fairly among the airlines, thus considerably enhancing their applicability in practice.

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TABLE II: Average values of the total displacement per airline ($\sum_{i \in \mathcal{F}_a} |u_i|$), the price of equity and the price of efficiency under differentiated flight valuations

Scenario			For variations in DAL's flight valuations						For variations in AAL's flight valuations					
μ_1	μ_2	η	DAL	AAL	JBU	Others	P_{eq}	P_{eff}	DAL	AAL	JBU	Others	P_{eq}	P_{eff}
1.0	1.0	–	57.0	46.0	31.0	72.0	0%	0%	57.0	46.0	31.0	72.0	0%	0%
0.9	1.1	50%	57.7	44.0	36.2	70.0	4.4%	15.4%	55.5	46.9	36.0	69.5	2.8%	24.8%
0.8	1.2	50%	57.0	44.7	36.4	70.5	4.5%	15.3%	53.4	50.0	34.6	67.9	2.4%	26.5%
0.7	1.3	50%	58.1	44.1	35.9	70.2	5.6%	14.5%	52.2	50.5	33.1	66.6	2.0%	23.3%
0.6	1.4	50%	59.8	43.5	36.2	69.1	6.8%	14.4%	51.5	59.1	32.7	65.1	2.1%	15.8%
0.5	1.5	50%	63.5	43.4	35.3	68.8	7.2%	13.7%	49.9	64.0	32.1	63.4	2.3%	11.6%
0.4	1.6	50%	67.3	43.5	34.9	68.9	8.1%	13.4%	48.8	70.7	31.4	61.4	2.0%	7.8%
0.3	1.7	50%	73.2	42.3	34.0	67.1	9.3%	13.3%	46.5	78.4	31.7	59.0	2.7%	9.7%
0.2	1.8	50%	77.4	42.4	34.8	67.2	11.2%	13.4%	44.4	87.5	29.9	56.5	2.8%	10.7%
0.1	1.9	50%	84.1	42.9	34.0	67.8	13.5%	11.8%	42.0	95.8	29.6	53.4	3.8%	15.7%
0.75	3.25	90%	57.6	44.4	36.6	70.4	4.1%	16.9%	49.3	42.3	32.2	62.4	2.4%	19.9%
0.75	2.00	80%	57.4	44.7	36.6	70.7	4.4%	17.3%	54.9	48.7	35.2	69.0	2.2%	19.6%
0.75	1.58	70%	57.3	44.4	36.3	70.4	4.3%	17.2%	53.5	49.6	35.1	68.4	2.2%	17.9%
0.75	1.37	60%	57.2	44.7	36.2	70.7	4.5%	16.4%	54.3	50.8	35.1	68.2	2.6%	17.7%
0.75	1.25	50%	57.5	44.1	36.4	69.7	5.1%	15.2%	53.0	52.0	33.8	67.6	1.9%	21.2%
0.75	1.17	40%	57.6	44.3	36.0	70.5	5.5%	13.6%	52.7	54.2	34.4	66.9	2.7%	24.2%
0.75	1.11	30%	58.6	44.1	36.5	70.1	6.2%	12.0%	51.9	55.8	33.4	65.9	2.8%	33.0%
0.75	1.06	20%	59.7	44.0	35.6	69.6	6.6%	10.8%	51.0	58.2	33.0	65.0	2.8%	35.8%
0.75	1.03	10%	66.6	43.1	35.4	67.9	6.9%	9.9%	50.0	62.6	31.8	63.3	2.7%	33.8%

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