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Airway Network Flow Management using Braess's Paradox

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***Abstract*—The ever increasing demand for air travel is likely to induce air traffic congestion which will elicit great economic losses. In the presence of limited airspace capacity as well as the saturated airway network, it is no longer feasible to mitigate air traffic congestion by adding new airways/links. In this paper, we provide a "counter-intuitive" perspective towards air traffic congestion mitigation by removing airways/links from a given airway network. We draw inspiration from Braess's Paradox which suggests that adding extra links to a congested traffic network could make the traffic more congested. The paper explores whether Braess's Paradox occurs in airway networks, or more specifically, whether it is possible to better distribute the flow in an airway network by merely removing some of its airways/links. In this paper, We develop a generic method for Braess's Paradox detection for a given airway network. To validate the efficacy of the method, a case study is conducted, for South-East Asian airspace covering Singapore airway network, by using 6 months ADS-B data. The results shows that Braess's Paradox does occur in airway networks and the proposed method can successfully identify the airway network links that may cause it. The results also demonstrates that, upon removing such links, the total travel time for a given day traffic at a given flight level, was reduced from 8661.15 minutes to 8328.64 minutes, a reduction of 332.5 minutes. This amounts to a saving of 3.8% in travel time.**

I. Introduction

Present day air traffic network is reaching its operational capacity [1] and accommodating future air traffic growth will be challenging for air navigation service providers (ANSP) [2], [3]. The International Air Transport Association (IATA) estimates that by 2036 the number of air passengers will double to 7.8 billion [4]. Such growth will put pressure on the infrastructure and airspace capacity. As a result, air traffic congestion is likely to occur [5] which may cause huge economic losses to ANSP, airlines and customers. For instance, according to FAA/Nextor, the average cost of aircraft congestion for U.S. airlines was $68.48 per minute in 2017, an increase of 7.4% over 2016 [6].

Note that air traffic has long been artificially concentrated on airways with intermediate waypoints [7], which could result in congestion [8]. The airway network has evolved, over a period of time, without any scientific basis. As new waypoints and airways are added to airway networks to accommodate the increasing traffic in an as-needed manner, structures of airway networks become more and more complicated [9], [10], [11]. Such a network may also lead to increasing air traffic controller workload [12], [13]. Therefore it is no longer feasible to add new airways/links to the airway network to improve the traffic flow and manage airspace congestion. Applying changes to airway network to force a better distribution of traffic flow may reduce the congestion and can provide more flexibility to flight schedules and routes.

Thus, it becomes necessary to investigate novel methods for restructuring airway networks so as to better manage traffic flow. In line with this idea, researchers have made significant efforts towards designing airspace with optimal airway structure [1], [14], [15]. These efforts often end up with complete new airway structure for a given airspace. The challenge is that, due to operational constraints, complete redesign of airway networks, so as to obtain the global optimal airway structures (for a given airspace/FIR), is not feasible. Also some new concepts are proposed for the operation of airways, such as defining new airways dedicated to the most important traffic flows [16], [17] which is similar to the highways for ground traffic that accommodate flows of car traffic between big cities, and 4D airways design [18], [19] that can reduce congestion by planning the space and time of flights.

In this paper,we focus on how to make minimal changes to an existing airway network such that it can improve the traffic flow. Can we achieve this instead by removing some of the airways, i.e. network links, in air traffic network? The answer may be yes for some networks. This counter intuitive phenomenon has been observed in 1969, when a new road was constructed in Stuttgart. Traffic flow was worsened and only improved after the road was closed. In 1990, when 42^{nd} street was closed in New York City, the traffic flow was actually improved instead of the anticipated traffic congestion. These paradoxical phenomena are named as the Braess's Paradox (BP). The BP was first recognized by an economist Arthur [20] and it was named after Braess, a mathematician who articulated the phenomena [21] [22]. Braess noted that when a new link is added in a user-optimized network, change in the equilibrium flows might result in a higher cost.

Since its discovery in 1968 [21], BP has generated significant change to research in ground traffic network. BP has been studied concisely in its original network form [23], [24], [25] and in a more general context. Characterizations of the occurrence of the BP are obtained using separable affine travel cost functions [26] and using separable monotone travel cost functions [27]. It is also proved that BP is likely to occur in a natural random network model [28]. More precisely, for a given appropriate total flow, it is shown that in almost all networks, there is a set of links whose removal improves the travel time at equilibrium [28]. Further, under the future air traffic management paradigm where aircraft operators can make the user preferred route (UPR) choices, BP could be a common phenomenon in the network. However, to the best of our knowledge this phenomenon has not been explored or investigated in air transportation networks. One of the possible reason might be the centralized air traffic management and the lack of methods for large air traffic data processing.

This paper aims to answer the question whether BP occurs in airway networks. More specifically, this research investigates if it is possible to improve the performance of an airway network by removing certain airways/links. If BP does occur in a given airway network, this paper further aims to identify the airways/links that lead to BP. To achieve this goal, this paper first quantifies the performance of an airway network as the total duration spent by all the aircraft on the given airway network. Then we propose a generic method for BP detection for a given airway network. The proposed method consists of three key steps. The first step mainly deals with air traffic data pre-processing and building the network. The second step is about BP detection and identification of the potential links that may cause BP. The third step is BP verification which aims to validate if the links suggested by step 2 are indeed the links that cause BP.

In order to verify the efficacy of the proposed method, this paper carries out a case study on South-East Asia airspace covering Singapore Airway Network (referred to as SAN hereafter), using ADS-B data recorded over 180 days during the calendar year 2017. In this paper, we choose the flight level 330 (33,000 ft) and apply the proposed method to the underlying airway network. In summary, we propose that traffic flow, on the airway networks, can be improved by removing airways/links from airway networks.

The remainder of the paper is organized as follows. Section II introduces the fundamental idea of BP. Section III describes our studied problem. Section 4 delineates in detail the proposed generic method for BP detection for a given airway network. Section V demonstrates the case study on SAN and discusses the results. Section VI concludes the paper.

II. Braess's Paradox

A. Introduction to BP

BP provides an explanation for the situation where an alteration to a traffic network to improve traffic condition actually has the reverse effect and impedes traffic through it. General characterizations of change in travel costs at equilibrium were obtained by adding paths or by varying demands. In a traffic network consists of links associated with non-decreasing function of flow known as cost functions, 'users' unilaterally choose least cost (or shortest) path on the network. The resulting flow is known as User Equilibrium (UE), also known as Wardrop Principle 1 [29]. However, there is a better flow pattern if the users cooperate with each other to come out with a flow that is best for all the users in the system. 'Best for all users' means that the total costs incurred to all the users is minimum. Such a flow is termed as System Equilibrium (SE), also known as Wardrop Principle 2 [29]. When a new link is added, it is aimed to reduce the total cost in the system (SE), but the users pursue a new UE. If we calculate total costs incurred to all the users of this new UE, it might become larger than the total costs on the previous UE. BP is rooted in behavior of the users: they choose their own least cost routes without any regard for how their choices may affect others. If the users decided to collaborate to constitute SE, there would be no place for BP. In Air Traffic this is analogous to the way airlines do their flight planning exercises. For a given flight, the Airlines Operations Center (AOC) chooses the best possible route accounting for aircraft performance and flight profile, subject to safety and operational constraints. AOC carries out this exercise using its own business model, without any consideration to other airlines. In Air Traffic Management, ATCs largely adhere to the flight plan, however do makes tactical changes to ensure safety of the flight in conflict situation or bad weather conditions.

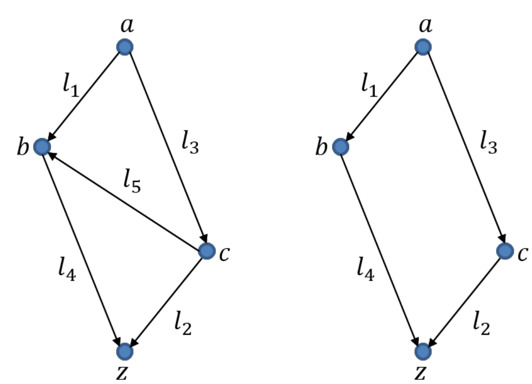


Figure 1. The original network model introduced by Braess. Figure in the left panel shows the network with extra link l_5. Figure in the right panel shows the network without extra link l_5.

Previous investigations on BP were commonly based on the classical, symmetric four-link network introduced by Braess [21]. There are few studies on BP in practical large scale transportation networks with realistic transport demand. Researchers prove that BP detection is highly intractable especially in real networks and no efficient method has been introduced. A heuristic methodology based on Genetic Algorithm has been studied to detect BP [30], [31], in which links that might cause BP is identified by simply testing their closure one by one and then an algorithm is adopted to run over these links to find a combination whose closure improves traffic cost. This method is effective for detecting BP in small scale networks

Table I
DEFINITIONS OF RELATED NOTATIONS AND VARIABLES.

BP	Braess's Paradox
UE	User Equilibrium
Link	A link of a network is one of the connections between the nodes of the network
Path	A path in a network is a sequence of links which connect a sequence of nodes
Φ	Total flow from origin to destination
T_{Eq}^{+}	Total time spent for all the users to go from origin to destination with the extra link under user equilibrium
T_{Eq}	Total time spent for all the users to go from origin to destination without the extra link under user equilibrium
ϕ_i	Flow on link i
f_i	Flow on path i
T	Time spent for each user to go from origin to destination under user equilibrium
T_i	Time spent for users to go from origin to destination by path i

and therefore is not applicable to large, complex real-world networks. In a different context, a path-based formulation has been developed [32] to detect BP in a stable dynamic network. The major limitation of this path-based (not link-based) formulation is that its computational time cost increases exponentially if the network grows in size.

Note that most studies on BP detection mainly identify the links/paths that cause BP by enumerating all possible removals to see if the removals indeed improve travel cost on the focal transportation network, with their critical drawback being the time consuming implementations. In order to assist better identification of the links/paths that cause BP to a traffic network, the authors in [33], [34] presented a variant to BP model which aims to explore a solution that will make some users better-off but no user is worse-off compared to the solution to UE. To keep consistency with UE and SE, here we name it as Braess Equilibrium (BE). As BP detection in realistic networks is NP-hard [35], the BE model is proved to be promising for detecting BP from medium scale networks. Inspired by [33], [34], in this paper we adopt the BE model to assist in identifying the potential airways/links that cause BP.

B. An Example of BP

We use the simple network shown in Fig. 1 to explain BP. As illustrated in Fig. 1, many network users travel from the origin 'a' to the destination 'z'. Assume that each user chooses an a-z path independently and selfishly, so as to minimize their own time cost.

In the network shown in the left panel of Fig. 1, there are three paths from origin a to destination z: $1 : l_1 \to l_4, 2 : l_3 \to l_2, 3 : l_3 \to l_5 \to l_4$.

The flow on each link is:

$$\phi_1 = f_1, \phi_2 = f_2, \phi_3 = f_2 + f_3, \phi_4 = f_1 + f_3, \phi_5 = f_3 \quad (1)$$

Each link of the network is characterized by its cost function, which describes the time spent by the users travelling through the link. Time cost on each link is:

$$t_1 = \phi_1 + 50, t_2 = \phi_2 + 50, t_3 = 10\phi_3, t_4 = 10\phi_4, t_5 = \phi_5 + 10 \quad (2)$$

Time cost on each path is:

$$\begin{array}{l} T_1 = 11f_1 + 10f_3 + 50 \\ T_2 = 11f_2 + 10f_3 + 50 \\ T_3 = 10f_1 + 10f_2 + 21f_3 + 10 \end{array}, \quad (3)$$

We assume that traffic in the network reaches an equilibrium state, the natural outcome of "selfish routing" in which all users find their own optimal paths. In accordance with UE, travel times are equal on all used paths and are smaller than that on any unused paths, namely:

$$T_1 = T_2 = T_3 \quad (4)$$

The total travel time of all users on the network is:

$$T_{Eq}^{+} = \sum_{i=1}^{3} T_i = \frac{31\Phi^2 + 1010\Phi}{13} \quad (5)$$

In the network shown in the right panel of Fig. 1, there are two paths from origin a to destination z: $1 : l_1 \to l_4, 2 : l_3 \to l_2$.

The flow on each link is:

$$\phi_1 = f_1, \phi_2 = f_2, \phi_3 = f_2, \phi_4 = f_1 \quad (6)$$

Time cost on each link is:

$$t_1 = \phi_1 + 50, t_2 = \phi_2 + 50, t_3 = 10\phi_3, t_4 = 10\phi_4 \quad (7)$$

Time cost on each path is:

$$T_1 = 11f_1 + 50, T_2 = 11f_2 + 50 \quad (8)$$

According to UE, we have:

$$T_1 = T_2 \quad (9)$$

The total travel time for all users on the network is:

$$T_{Eq} = \sum_{i=1}^{2} T_i = \frac{11\Phi^2 + 100\Phi}{2} \quad (10)$$

We assume that the traffic demand - the total amount of traffic flow - in the network is 6, which is represented as:

$$f_1 + f_2 + f_3 = 6 \quad (11)$$

Then in the equilibrium state of the network shown in the left panel of Fig. 1, each of the three paths, i.e., a $\to b \to z$, $a \to c \to z$ and $a \to b \to c \to z$, has a flow of 2 with the corresponding cost being 92. However, if we remove the link $b \to c$ to obtain the second network shown in the right panel of Fig. 1, then in the ensuing equilibrium state, half of the flow are attached to path $a \to b \to z$ and the other half to path $a \to c \to z$, resulting a cost of 83 for each user which is better than that of keeping link $b \to c$.Thus removing links can improve the performance of the equilibrium flow of a selfish routing network.

BP happens when the total travel time on the network without link 5 is less than that on the network with link 5, that is:

$$T_{Eq} \leq T_{Eq}^{+} \tag{12}$$

By solving inequation 12 we get the interval of demand for BP to happen: $0 < \Phi < \frac{80}{9}$, that is, with a set of cost functions for each link of a traffic network, when the traffic demand is within a certain range, BP will appear. In this simple network, the mechanism of BP is shown by simple calculations. When it comes to large and complex networks, BP also widely exists, however, in a much more intractable manner.

III. Problem Description

An airspace, comprising of several airways and intermediate way-points, can be seen as a network of links between way-points. The horizontal projection of such an airway network can be seen as a planar network with the nodes being the waypoints and links the airways. An airway of an airway network is defined as a sequence of segments starting and ending at fixed waypoints [1]. Fig. 2 exhibits an example of an airway network.

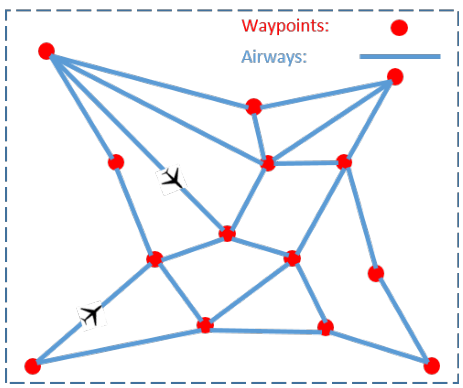


Figure 2. A graphical illustration of an airway network.

In Fig. 2, the nodes in red denotes the waypoints and the links in blue signifies the airways. To keep en-route vertical separation, so as to assure en-route flight safety, an airspace network often has a layered structure, as shown in Fig. 3.

Fig. 3 illustrates the vertical distribution of aircraft trajectories on layered airway network. We can see from Fig. 3 that an airway network is practically in a multi-layer structure. All the aircrafts in a certain layer have to follow the airway network at the corresponding layer.

As an airspace has its capacity limit, when air traffic demand reaches the capacity threshold of an airspace, then air traffic congestion is likely to occur [36]. In order to mitigate congestion, one possible way is to add new airways to the airway network so as to dispatch the congested traffic on alternate routes. Note that the design of an airway network has to take into account many factors like procedural, technical

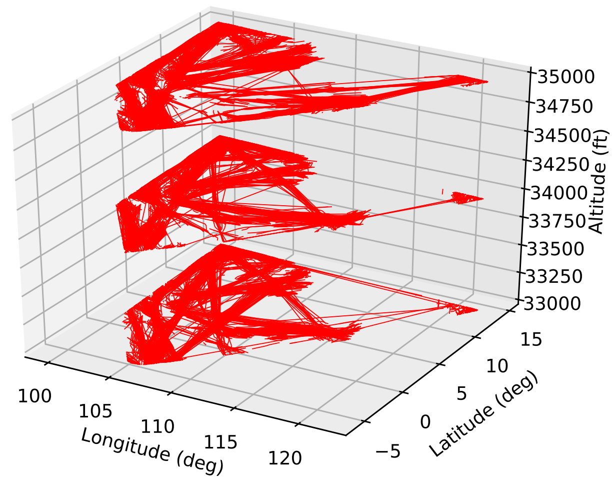


Figure 3. The layered en-route structure airspace network from ADS-B data used in the study.

and geographical constraints. An airway network can possess a very complicated structure as a consequence, and it is no longer feasible to mitigate congestion by adding extra airways/links to an airway network.

In this paper, we suggest a “counter-intuitive” method to mitigate air traffic congestion with the inspiration drawn from BP. Specifically, we aim to detect BP in a given airway network at a given layer (flight level). In doing so, for a given airway network we can identify the airways/links that may cause BP. By removing those airways/links, the performance of the given airway network can be improved. In this paper, we quantify the performance of an airway network as the time duration spent by all the aircrafts on an airway network in a given period of time. Therefore, the problem to be investigated in this paper is to identify airways/links from a given airway network, at a given layer, such that the removal of such airways can reduce the travel duration on the focal airway network. In this paper, we focus on 33000 ft, a heavily congested flight level in South-East Asia

IV. Methodology

A. Methodology Overview

The proposed methodology encompasses three key components: Trajectory Registration, Cost Function Formulation, and BP Detection. To better comprehend how the three components interact with each other, we present a visual representation of the proposed method in Fig. 4.

In Fig. 4, we take SAN as an example to illustrate the proposed method. The Trajectory Registration step mainly aims to map the aircraft trajectory data points, in ADS-B data, to the SAN. Besides, the Trajectory Registration step also abstracts air traffic demand (number of aircraft) information from the ADS-B data for later calculation.

The outcome of the Trajectory Registration step is the relationship between the flow (number of trajectories) and flight duration (total flight time) for each link of the SAN.

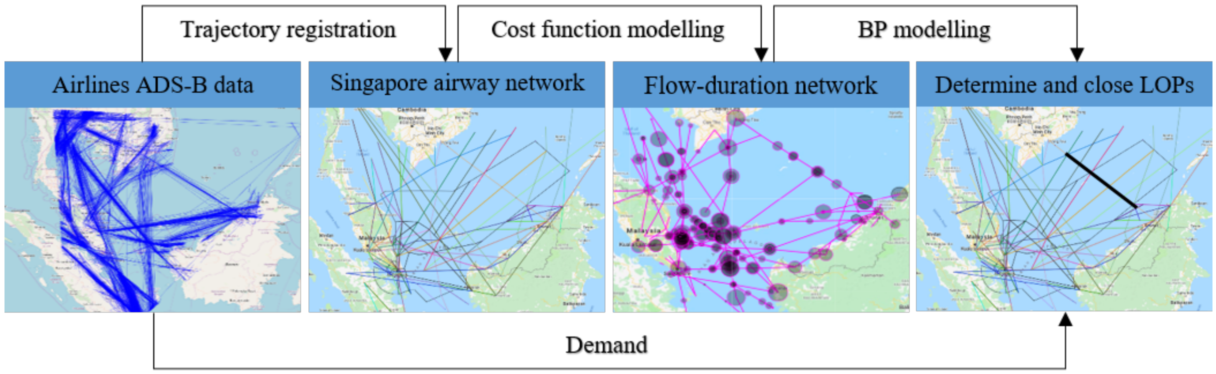


Figure 4. The diagram of the proposed method for BP detection for a given airway network.

In the flow-duration network shown in Fig. 4, the size of 2 circles on both end of a link represent the significance of flight duration on that link. The larger the circles are, the longer the flight duration is. The Cost Function Formulation step develops the cost function for each link of the SAN.

From the first two steps of the proposed method, we obtain the cost functions and total traffic demand information. Then the final step, i.e., BP Detection, is performed through non-linear programming to identify the links of paradox (LOP) in the airway network that cause BP. In what follows, we delineate in detail how each step works.

B. Trajectory Registration

BP detection requires the flow-duration function on each link of the airway network. With regard to this, we map the aircraft trajectory data points, from ADS-B data for a given period of time, on the SAN. The trajectory registration algorithm is presented in Algorithm 1. The notations are as follows:

T_i : $(T_1^i, T_2^i, T_3^i, ..., T_k^i, ..., T_{n_i}^i)$ is the i-th trajectory with T_k^i being the k-th waypoint on T_i. T_i consists of $n_i - 1$ consecutive links. We randomly choose two way points T_k^i and T_g^i forming the segment ST_{kg}^i. Obviously there are $\binom{n_i}{2}$ different trajectory segments.

A_j : $(A_1^j, A_2^j, A_3^j, ..., A_h^j, ..., A_{m_j}^j)$ is the j-th airway with A_h^j being the h-th waypoint on A_j. A_j consists of $m_j - 1$ consecutive links. We randomly choose two way points A_h^j and A_l^j forming the segment SA_{hl}^j. Obviously there are $\binom{m_j}{2}$ different airway segments.

The distance between segments ST_k^i and SA_{hl}^j is defined as the sum of the distances between the starting point and ending points of each segment: $D = (x_k^i - x_{hl}^j)^2 + (y_k^i - y_{hl}^j)^2 + (x_{k+1}^i - x_{h(l+1)}^j)^2 + (y_k^i - y_{h(l+1)}^j)^2$. The bigger the difference between 2 segments, the larger the value of the distance.

As illustrated in Fig. 5, from a micro perspective, for each pair of matched airway segment and trajectory segment, links composing the trajectory segment will be registered onto links composing the airway segment. Time spent on each trajectory link will be added to the duration of its matched airway link. From a macro perspective, we assign the entire trajectory T_i onto the airway segment that has the minimum D between them, with the constraint $D \leq D_0$. When the minimum D is larger than D_0, we discard this trajectory because it may be corrupted by noise. D_0 is the upper bound of distance. When a distance is larger than D_0, it means there barely exists similarity between the trajectory and any airway segment. Then we match the corresponding segment of the trajectory onto the corresponding segment of the assigned airway. If a part of a trajectory has the minimum distance to an airway, then this part will be registered onto the corresponding airway and the rest part of the trajectory will be regarded as a new trajectory and undergo the registration process again. We repeat this step until every part of the trajectory is either matched onto an airway segment or discarded as a noise. When a trajectory segment is registered onto an airway segment, one-unit flow will be added onto links comprising the airway segment.

Algorithm 1 Trajectory registration algorithm

Input: ADS-B Data

1) Find the best matched airway segments for trajectory T_i: By traversing all airways, find an airway A_j contains a sequence of segments $S = (A_k^j, ..., A_m^j, ..., A_h^j)$ which has the minimum distance D_{min} to T_i.
2) If $D_{min} > D_0$, T_i will be regarded as noise and discarded.
3) If $D_{min} \leq D_0$, find the best matched segments $Ts = (T_l^i, ..., T_p^i, ..., T_q^i)$ on T_i for As, so that Ts has the minimum distance to As.
4) One unit of flow will be added to each segment $As_y = (A_y^j, A_{y+1}^j), y = k, ..., m, ..., h$, in As.
5) Each segment $Ts_x = (T_x^i, T_{x+1}^i), x = l, ..., p, ...q-1$, in Ts will be registered to the airway segment (A_m^j, A_{m+1}^j) in As that has the minimum distance to Ts_x.
6) The rest part of the trajectory T_i, $(T_1^i, ..., T_l)$ and $(T_q^i, ..., T_{n_i}^i)$, will be regarded as new trajectories and being registered from step 1.

Output: Flow-flight duration relation on each link of the airway network.

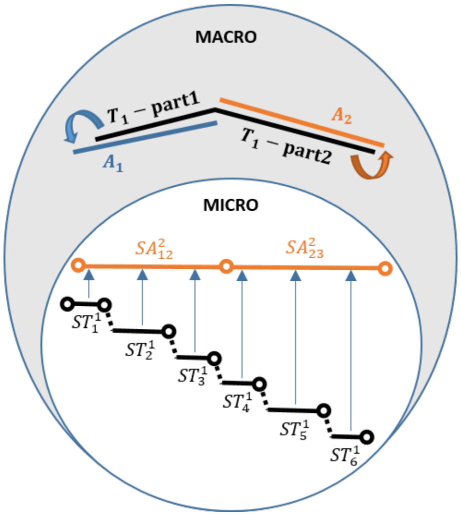


Figure 5. Registration of trajectories onto airways. From a macro view, part 1 of trajectory T_1 will be registered to airway A_1, while part 2 of trajectory T_1 will be registered to airway A_2. One unit flow will be added to A_1 and A_2. From a micro view, the time duration on trajectory links ST^1_1, ST^1_2, ST^1_3 will be added to airway link SA^2_{12}, while the time duration on trajectory links ST^1_4, ST^1_5, ST^1_6 will be added to airway link SA^2_{23}.

C. Cost-Function Formulation

After mapping all the segment of all the trajectories onto their best fitted airway segments, we can get the flow-duration data on each link for a given period of time, e.g, a period of one day. The flow on one link is the number of trajectories matched onto it during the time period, while the flight duration on one link is the time spent by the trajectory segments matched onto it. Fig. 6 gives an example of the flow-duration relation on one link of an airway network.

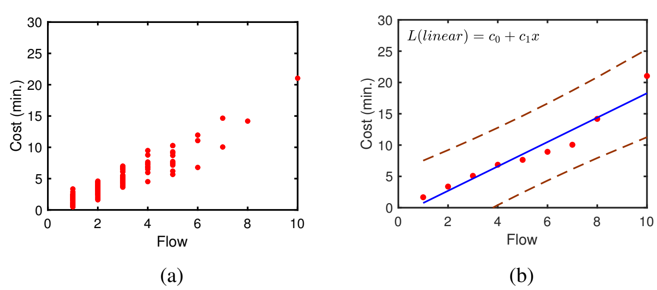


Figure 6. An example of the data-driven cost function calculation for one link of an airway network. (a) shows the original flow-duration relation obtained by the Trajectory Registration step. (b) shows the linear cost function for the focal link.

Upon getting the relation between flow and duration, to determine the overall trend of the relation without loss of accuracy, we fuse data points under the same flow by taking the average of them, whereby the red points shown in Fig. 6(a) are converted into red points in Fig. 6(b). Afterward, we use the least mean square method to determine the coefficients of the linear cost function as displayed in Fig. 6(b).

Note that in the literature when investigating BP on road traffic networks, researchers have utilised linear cost functions [26], [37], [38], polynomial cost functions [39] and general monotonous non-decreasing cost functions [40]. We regress the cost function using linear function, polynomial function and cubic function and the results show that they have similar performance. Therefore in this paper, we adopt the linear function, $L = c_0 + c_1x$, in accordance with the principle of parsimony [41].

D. Origin-Destination (OD) Pair Extraction

From Section II we know that, apart from the cost functions for all the links of an airway network, to detect BP we also need to know the OD pair as well the traffic demand information about the airway network.

From the ADS-B data we know the entry-exit points for each flight in the SAN. After mapping each flight trajectory to SAN, we identify the waypoints that are nearest to the entry and exit points. We then label the two waypoints as an OD pair. We extract the OD pairs for all trajectories on SAN.

For each OD pair, we know the traffic demand, i.e., the number of aircraft travelled across the OD pair. We then obtain the total traffic demand on the focal airway network by adding together the demand on each OD pair. With the cost functions, OD pairs and traffic demand information at hand, we then in the next step illustrate the BP detection process for a given airway network.

E. BP Detection Process

The BP detection process involves two sub-steps. The first sub-step is to minimize the objective functions with respect to the UE, SE and BE models so as to obtain the corresponding optimal solutions. The second sub-step is to compare the corresponding optimal solutions yield by the first sub-step and identify the links that cause BP. In what follows we present the details for each sub-step.

1) Minimize the Objective Functions: In order to detect BP from a traffic network, we need to work out the optimal traffic flow distribution on the focal network based on a given principle like UE. In the literature, the UE, SE and BE models have been formulated as mathematical optimization problems [33], [34]. Because in this work we investigate the BP problem on airway networks, we introduce the optimization problem by first presenting the following pertaining notations and definitions.

Given an airway network denoted by an undirected network $G = (V, E)$ with V being the set of waypoints (nodes) and E the set of links connecting waypoints. Let the nodes of the network be indexed as $i = 1, ..., n$ and links be index as e_{ij} if the link connects nodes i and j. Let $\mathbf{A}$ be the adjacency matrix of G, with its entry $a_{ij} = 1$ if there is a link e_{ij} between nodes i and j, otherwise, 0. Let $EE = \{EE_1, ..., EE_t, ..., EE_T\}$ be the set of the T OD pairs on G. The t-th OD pair is $EE_t = \{O_t, D_t\}$ with O_t and D_t respectively being the origin and destination waypoints. Let $D = \{d_1, ..., d_t, ..., d_T\}$ be the demand vector for the T OD pairs on G, i.e., d_t is the demand on the t-th OD pair EE_t. Assume that for the t-th OD pair there are maximum R_t accessible paths respectively denoted by $P_t^1, P_t^2, ..., P_t^r, ..., P_t^{R_t}$. The flow on path P_t^r is represented by f_{tr}. The set of all paths in airway network G is denoted by $P = \{P_1, P_2, ..., P_T\}$. We say a link $e \in p$ when the link is traversed by the path p. Define a function $\delta_{(tr,ij)}$, with $\delta_{(tr,ij)} = 1$ if link $e_{ij} \in P_t^r$, otherwise, $\delta_{(tr,ij)} = 0$. Let $\mathbf{L}$ be the latency matrix of G with its entry $l_{ij} = c_{ij}^0 + c_{ij}^1 x_{ij}$ being the cost function for link e_{ij}. x_{ij} is the amount of flow on link e_{ij}.

With all the above notations and definitions, under the context of airspace network, we then formulate the optimization problem with respect to the UE model as follows:

$$\begin{aligned} \text{UE}: \quad \min_{x_{ij}} \quad & F_{UE}(\mathbf{X}) = \sum_{i=1}^{n}\sum_{j=1}^{n}\int_0^{x_{ij}} a_{ij} l_{ij}(t)dt \\ s.t. \quad & f_{tr} \geq 0 \\ & \textstyle\sum_{r=1}^{R_t} f_{tr} = d_t \\ & \textstyle\sum_{t=1}^{T} d_t = D \\ & \textstyle\sum_{t=1}^{T}\sum_{r=1}^{R_t} \delta(tr,ij) f_{tr} = x_{ij} \\ & l_{ij}(x_{ij}) = c_{ij}^0 + c_{ij}^1 x_{ij} \end{aligned} \tag{13}$$

In the UE optimization problem as presented above, f_{tr} denotes the flow on path P_t^r. Constraint $f_{tr} \geq 0$ assures that flow on every path of an airway network is non-negative. Constraint $\sum_{r=1}^{R_t} f_{tr} = d_t$ denotes that the summation of the flows on all the paths between the t-th OD pair should satisfy its demand requirement. Constraint $\sum_{t=1}^{T} d_t = D$ means that the summation of the demands on all the OD pairs should equal the demand on the studied airway network at a given spatial-temporal scale. Constraint $\sum_{t=1}^{T}\sum_{r=1}^{R_t} \delta(tr,ij) f_{tr} = x_{ij}$ converts path flows into link flows since the objective function is involved with links flows. The last constraint presents the linear cost functions for each link of an airway network. By minimizing F_{UE} we obtain the optimal flow distributions which are the outcome of users' selfish routing.

$$\begin{aligned} \text{SE}: \quad \min_{x_{ij}} \quad & F_{SE}(\mathbf{X}) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ij} l_{ij}(x_{ij}) \\ s.t. \quad & f_{tr} \geq 0 \\ & \textstyle\sum_{r=1}^{R_t} f_{tr} = d_t \\ & \textstyle\sum_{t=1}^{T} d_t = D \\ & \textstyle\sum_{t=1}^{T}\sum_{r=1}^{R_t} \delta(tr,ij) f_{tr} = x_{ij} \\ & l_{ij}(x_{ij}) = c_{ij}^0 + c_{ij}^1 x_{ij} \end{aligned} \tag{14}$$

Analogous to the UE optimization problem, we formulate the SE optimization problem in Eq. 14 which shares the same constraints as that in Eq. 13. The SE model is the outcome of users' collaborative routing and as a result the minimization of F_{SE} will generate the optimal solutions for the ATC side. The BE optimization problem can then be formulated as follows.

$$\begin{aligned} \text{BE}: \quad \min_{x_{ij}} \quad & F_{BE}(\mathbf{X}) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ij} l_{ij}(x_{ij}) \\ s.t. \quad & f_{tr} \geq 0 \\ & \textstyle\sum_{r=1}^{R_t} f_{tr} = d_t \\ & \textstyle\sum_{t=1}^{T} d_t = D \\ & \textstyle\sum_{t=1}^{T}\sum_{r=1}^{R_t} \delta(tr,ij) f_{tr} = x_{ij} \\ & l_{ij}(x_{ij}) = c_{ij}^0 + c_{ij}^1 x_{ij} \\ & u_{tr}^* \geq u_{tr} \end{aligned} \tag{15}$$

It can be clearly seen from Eq. 15 that the BE optimization problem is the SE optimization problem plus one more constraint. In the constraint $u_{tr}^* \geq u_{tr}$, the variables u_{tr}^* and u_{tr} denote the time costs on path P_t^r that are obtained by respectively optimizing F_{UE} and F_{BE}. Constraint $u_{tr}^* \geq u_{tr}$ requires that the optimal solution to F_{BE} makes some users better-off but no user worse-off when compared to the UE model.

By minimizing each of the three objective functions, we can obtain the corresponding flow distribution matrix $\mathbf{X}$ with its entry x_{ij} being the optimal flow on link e_{ij} of the studied airway network.

2) BP links identification: By optimizing the three objective functions we get three flow distribution matrices $\mathbf{X}^{UE}$, $\mathbf{X}^{SE}$ and $\mathbf{X}^{BE}$. We than compare the entries of $\mathbf{X}^{UE}$, $\mathbf{X}^{SE}$ and $\mathbf{X}^{BE}$ and determine the potential links that may cause BP. Specifically, if the following conditions are satisfied

$$\begin{aligned} & \left| x_{ij}^{SE} - x_{ij}^{BE} \right| \to 0 \\ & x_{ij}^{UE} \geq \delta x_{ij}^{BE} \end{aligned} \tag{16}$$

then we regard link e_{ij} as a potential link that causes BP.

The first condition requires that x_{ij}^{SE} and x_{ij}^{BE} are close to each other. $\mathbf{X}^{UE}$ is the outcome of users' selfish routing which is in a way similar to the Nash solution in Game Theory [33], [34]. $\mathbf{X}^{SE}$ is the global optimal solution. $\mathbf{X}^{BE}$ can be regarded as a local optimal solution nearby $\mathbf{X}^{SE}$. In the second condition, the parameter δ determines the difference between the values of x_{ij}^{UE} and x_{ij}^{BE}. According to the BP introduction presented in Section II we see that, if the difference between x_{ij}^{UE} and x_{ij}^{BE} on link $e_{ij}$is large, it suggests that the flow on link e_{ij} could have detrimental influences on the flight cost. Thus, link e_{ij} will be a potential link that causes BP. In this study, we assume that if the difference is of one order of magnitude, then the corresponding link will be regarded as a potential link. As a consequence, we set $\delta = 10$ in our case study.

Based on the above condition we figure out all the potential links that may cause BP. Afterward, we remove those links from the SAN and re-optimise F_{UE} based on the new airway network, yielding a new flow distribution matrix $\mathbf{X}^{UE'}$. If the following condition is satisfied

$$F_{SE}(\mathbf{X}^{UE'}) < F_{SE}(\mathbf{X}^{UE}) \tag{17}$$

which implies that after removing those links, from the airway network, if the total cost on the network is reduced, then the links identified previously are indeed the links that cause BP to the focal airway network.

V. Case Study

The above Section describes the proposed method for BP detection for a given airway network with given trajectory data. To verify the efficacy of the proposed method, in this Section we carry out the case study on SAN using six months ADS-B data.

A. Cost Functions

We first construct the network structure of SAN using ADS-B data. Fig. 7 displays the airway structure of SAN which consists of 198 nodes and 273 links.

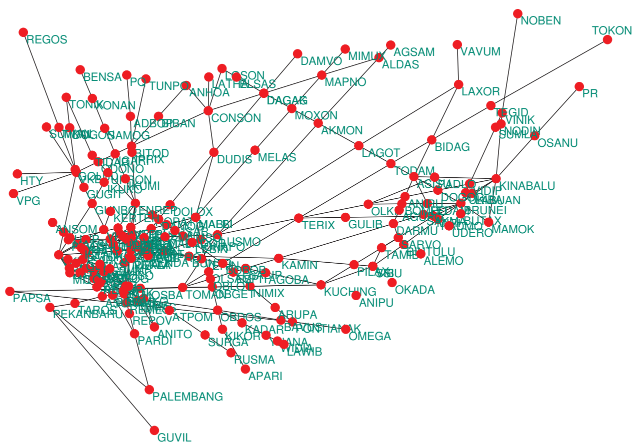
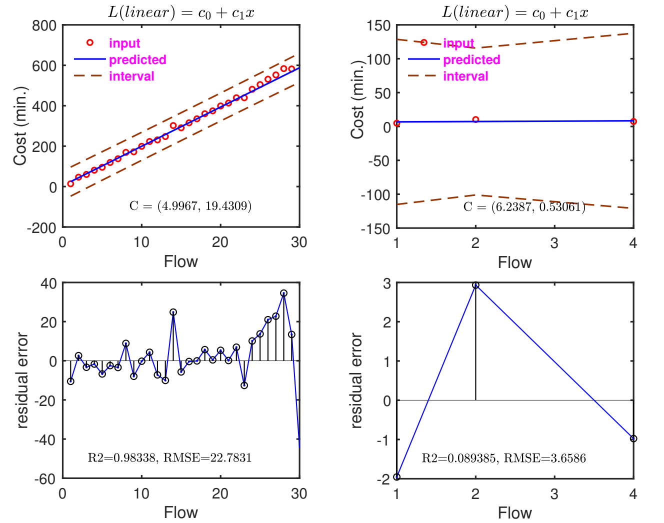


Figure 7. Airway structure of SAN.

We select the trajectory data at flight level 330 and do the trajectory registration thereby obtaining the flow-duration relations for all the links of SAN. Specifically, for each airway of an airway network we do registration for a given time period of one day. The trajectory matching is implemented over 180 days and the averaged results are used to approximate the cost functions of airways. As a consequence, for some days 30 or more aircrafts could fly over a particular airway as show in the top panel of Fig. 8a. Note that the cost function of a link depicts the resistance of a link. The cost function is a function of air traffic flow. As long as the flow, e.g., 2-3 aircraft, on a link is known, we can then obtain the flight duration that the aircraft will spend on an airway through cost functions. In this way we can achieve the flow scheduling thereby detecting BP. The cost function matrix $\mathbf{L}$ is determined using the linear function $y = c_0 + c_1 x$ approximation. The entry l_{ij} of $\mathbf{L}$ contains two elements, i.e., c_{ij}^0 and c_{ij}^1, which are used to calculate the time cost on link e_{ij} of SAN.

Fig. 8 displays the cost functions on two representative links, i.e., link 88 and link 201, of SAN. It can be seen from Fig. 8(a) that link 88 had been frequently travelled by aircraft, while link 201 was relatively less travelled by aircraft. During the calculations of matrix $\mathbf{L}$, we make use of three statistical



(a) (b)

Figure 8. Exhibition of the data-driven based cost functions for two representative links of SAN. (a) cost function for link 88. (b) cost function for link 201.

metrics, i.e., MSE (mean square error), RMSE (root mean square error), and R^2 (R-square), to measure the goodness of the cost functions. The statistical results for the 204 links of SAN are shown in Fig. 9.

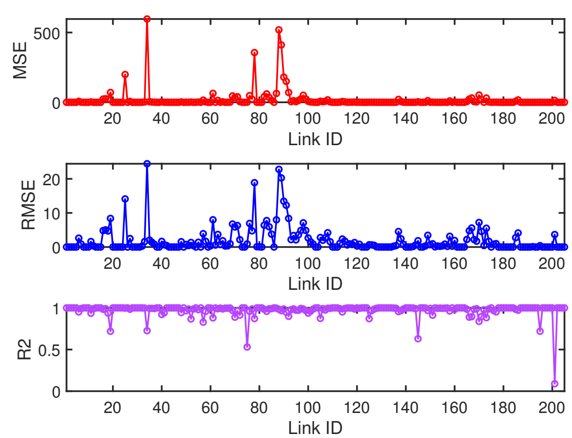


Figure 9. Quality assessment of the data-driven based cost functions using statistical metrics. MSE is the mean square error and RMSE is the root mean square error. R^2 is the R-square statistical metric.

Normally the larger the values of R^2 are, the higher confidence the linear cost functions. We can see from Fig. 9 that the values of MSE and RMSE are large only for a few links, while for the majority of them the values are quite small, which means that using linear functions as cost functions is feasible. The values of R^2 for all the links except link 201 are large. The cost function for link 201 is shown in Fig. 8(b). Although the value of R^2 is small, the cost function is still of high confidence. This is because that the R^2 metric makes some exceptions, i.e., when data samples have a approximately uniform distribution, the value of R^2 will be close to zero.

B. BP Detection

Before BP detection, we work out the demand and OD pair information. Fig. 10 displays the demand of 180 days in SAN at flight level 33000ft.

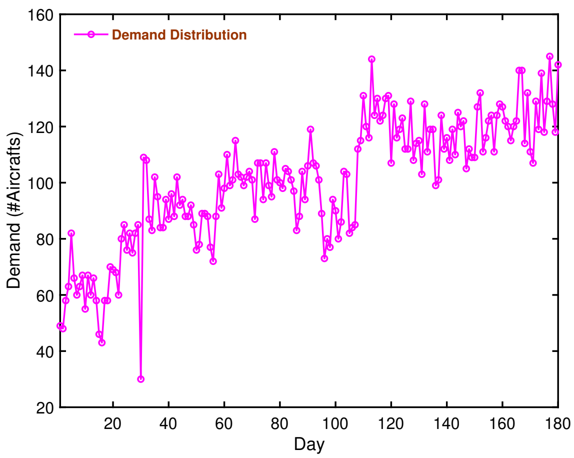


Figure 10. Air traffic demand of 180 days in SAN at flight level 330.

With the cost functions as well as the daily demand and OD pair information about SAN, we minimize $F_{UE}(\mathbf{X})$, $F_{SE}(\mathbf{X})$ and $F_{BE}(\mathbf{X})$, thereby getting three optimal solutions $\mathbf{X}^{UE}$, $\mathbf{X}^{SE}$ and $\mathbf{X}^{BE}$. Each of the optimal solutions represents the optimal flow distribution with respect to the corresponding model and a given demand. By substituting $\mathbf{X}$ into $F_{SE}(\mathbf{X})$, we can obtain the total cost with respect to $\mathbf{X}$.

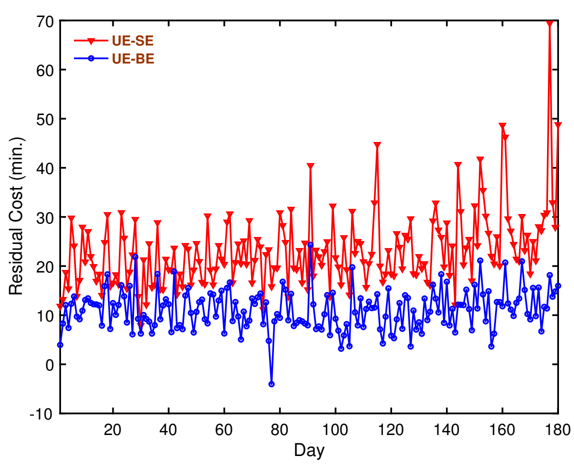


Figure 11. Comparisons of total costs with respect to UE, SE and BE over the studied period of 180 days. The red curve represents the residual costs of the total costs with respect to UE and SE, while the blue curve denotes the residual costs of the total costs with respect to UE and BE.

In Fig. 11 we show the differences between the total costs respectively corresponding to UE and SE and that to UE and BE. Note that BP could occur if the difference between the total costs is significant, as a small difference between total costs indicates that the Nash solution $\mathbf{X}^{UE}$ is close to the global optimal solution $\mathbf{X}^{SE}$ or to the Pareto solution $\mathbf{X}^{BE}$. We can see from Fig. 11 that, for some days the differences are significant while for some days the differences are small (less than 10 minutes).

Fig. 11 only presents a holistic view of the total costs with respect to the three optimization models. In order to identify the links that cause BP, we need to compare between the optimal solutions $\mathbf{X}^{UE}$, $\mathbf{X}^{SE}$ and $\mathbf{X}^{BE}$. As each of the optimal solutions is a matrix, to facilitate comparisons, we then respectively, turn them into three vectors $\mathbf{x}_1 = \{x_1^1, x_2^1, , ..., x_{204}^1\}$, $\mathbf{x}_2 = \{x_1^2, x_2^2, , ..., x_{204}^2\}$ and $\mathbf{x}_3 = \{x_1^3, x_2^3, , ..., x_{204}^3\}$, with x_i being the flow on the i-th link of SAN.

Without loss of generality, we here take day 175 as an example to show the BP detection result. Fig. 12 shows the three optimal flows $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ with respect to UE, SE and BE on day 175. Because for $\mathbf{x}_1$, $\mathbf{x}_2$ and $\mathbf{x}_3$ we have $x_i^1 = x_i^2 = x_i^3$ for some i-s, therefore in Fig. 12 those flows are not shown.

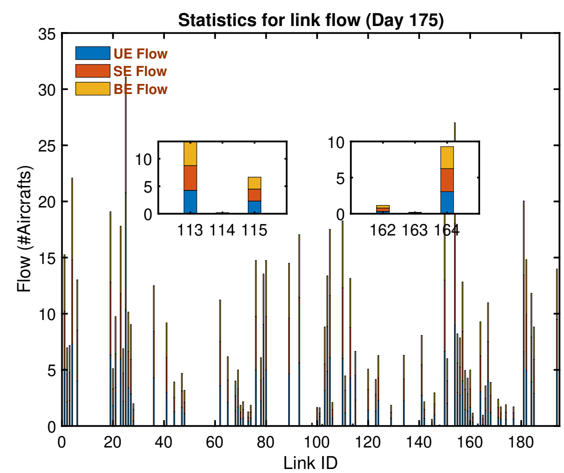


Figure 12. Exhibition of the optimal flows with respect to UE, SE and BE on day 175.

In Fig. 12, the length of a bar with unique color signifies the flow on the corresponding link. It can be seen from Fig. 12 that for most of the links the flows are very close to each other. Only for links 114 and 163 we have $x_{114}^1 = x_{163}^1 = 0.1656$ and $x_{114}^2 = x_{114}^3 = x_{163}^2 = x_{163}^3 = 0$, which satisfies the condition presented in Eq. 16. Therefore, links 114 and 163 are regarded as the potential links that can cause BP.

The waypoints associated to links 114 and 163 are shown in Fig. 13. To verify whether links 114 and 163 indeed cause BP, we remove them from the SAN and re-optimize $F_{UE}(\mathbf{X})$. Then we recalculate the total cost on the network. We show the related results in Fig. 13. We can see from Fig. 13 that before we remove links 114 and 163, the total time duration on the network was 8661.15 minutes.Removing those two links saves the travel time on the network by 3.8%. Thus, links 114 and

Table II
STATISTICAL RESULTS FOR BP DETECTION FOR SAN AT FLIGHT LEVEL 330. TC_1 AND TC_2 ARE RESPECTIVELY THE TOTAL COSTS OBTAINED BY MINIMIZING $F_{UE}(\mathbf{X})$ BEFORE AND AFTER REMOVING THE BP LINKS FROM SAN. $SC = TC_1 - TC_2$ IS THE SAVED COST.

Day	Demand	BP Link	Link Flow	TC_1 (Min.)	TC_2 (Min.)	SC (Min.)	$SC/TC_1(\%)$
1	49	67	2.887e-02	2021.312	2014.060	7.251	0.359
8	63	134	1.804e-01	3503.380	3501.663	1.717	0.049
12	60	121	8.956e-02	2765.423	2764.178	1.245	0.045
17	**58**	**27** **71**	**2.160e-02** **1.445e-01**	**3536.038**	**3516.810**	**19.228**	**0.544**
28	82	144	1.003e-01	5170.283	5161.725	8.558	0.166
30	**30**	**158**	**1.891e-01**	**1131.530**	**1123.548**	**7.982**	**0.705**
38	84	78	1.270e-01	7108.310	7107.388	0.922	0.013
44	92	70	8.820e-02	9642.960	9590.713	52.247	0.542
45	94	156	4.856e-02	8848.947	8846.660	2.286	0.026
48	92	45	8.307e-02	7120.268	7101.940	18.328	0.257
54	88	28	2.964e-02	6265.240	6239.498	25.743	0.411
64	115	45	1.009e-01	13823.800	13814.417	9.384	0.068
66	102	78	2.061e-02	9999.972	9999.807	0.166	0.002
67	99	58	8.068e-02	11940.628	11936.217	4.412	0.037
71	87	134	5.167e-02	7670.403	7668.630	1.773	0.023
72	107	74	5.331e-02	11172.695	11169.220	3.475	0.031
78	111	21	8.447e-02	14093.982	14091.357	2.625	0.019
91	**119**	**114**	**2.279e-02**	**10629.088**	**10397.777**	**231.312**	**2.18**
93	106	186	4.128e-02	10711.080	10709.265	1.815	0.017
96	73	165 177	2.475e-02 1.335e-01	6838.770	6816.277	22.494	0.329
107	**85**	**71**	**5.762e-02**	**8335.593**	**8290.270**	**45.324**	**0.544**
112	116	122	1.747e-01	10854.922	10853.098	1.823	0.017
115	130	101	1.700e-01	10421.997	10400.513	21.484	0.206
126	112	58 169	1.516e-01 1.516e-01	9354.567	9320.438	34.128	0.365
129	114	74	8.605e-02	10751.427	10725.627	25.800	0.240
143	110	177	7.539e-02	8440.668	8434.132	6.538	0.077
146	122	177	1.074e-01	8585.915	8566.662	19.254	0.224
148	112	122	8.963e-02	5675.065	5665.760	9.305	0.164
150	109	186	3.376e-02	9150.042	9149.582	0.459	0.005
165	122	45	3.111e-02	11706.637	11701.168	5.469	0.047
168	114	28	8.776e-02	9694.607	9661.287	33.320	0.344
175	**118**	**114** **163**	**1.655e-01** **1.655e-01**	**8661.152**	**8328.643**	**332.509**	**3.839**

163 are the links that cause BP on day 175. This improvement in the travel time is only for one OD pair, one day, and one layer of the airway network, so when it comes to all layers and all OD pairs in the airway network, the improvement can be more significant.

To verify the validity of our proposed method and the primary result, we then apply our proposed method to the entire data set of 180 days to identify BP links. The statistical results are reported in Table II. The days in bold are the top five days that have reduced flow duration after removal of identified BP links. We can clearly see from Table II that BP does occur in SAN and after removing the BP links, the flow duration on the airway network is indeed reduced.

VI. CONCLUDING REMARKS

In this paper, we have investigated the occurrence of BP phenomenon on airway networks. To achieve this goal, we first developed a generic method to assist in detecting links that can cause BP on airway networks. We then carried out a case study over South-East Asian airspace covering Singapore airway network using six months ADS-B data from Jun. 1, 2017 to Dec. 30, 2017. The proposed method involves two key components, i.e., cost function calculations and BP modelling. Experimental results demonstrate that the BP phenomenon does occur on airway networks. In the case study, we found that after removing BP links, for one day of traffic, the total travel time was reduced from 8661.15 minutes to 8328.64 minutes, a saving of 332.5 minutes. This amounts to a saving in the travel time for the network by 3.8%. just for one flight level. The calculations of the cost functions for the links of an airway network and the traffic demand are totally derived from real-world data instead of any assumptions. We therefore believe that the discoveries of the BP on airway networks are not coincidental. The findings of this work suggest that the "removals" of some airways could reduce the total travelling times for the aircraft that fly through the airway network at a given flight level without cutting down the air traffic demand. In this regard, the proposed method could assist ANSPs with dynamic trajectory planning. Moreover, the idea of this work will contribute to the optimal structure design of airway networks for better air traffic services. Although, we were able to identify possible links that can cause BP, the proposed method cannot identify the conditions under which BP happens on an airway network. In the future, we will

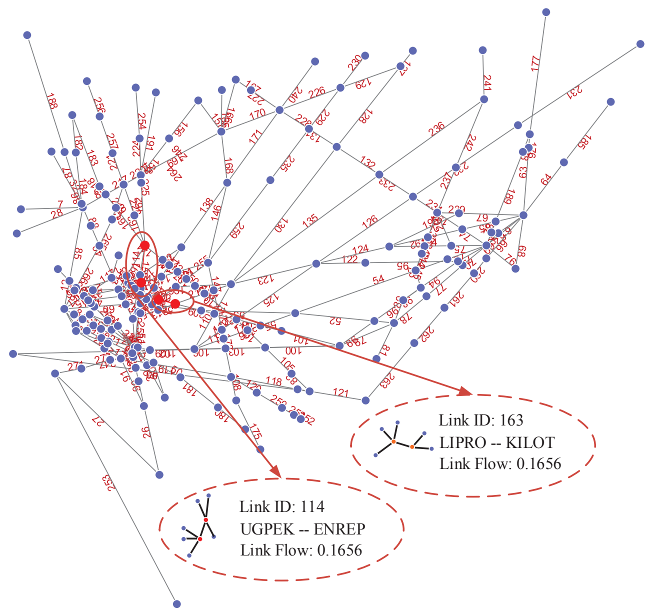


Figure 13. BP detection results on day 175 for SAN. Links 114 and 163 are detected as the links that cause BP. The original time duration on the network was 8661.15 minutes. After removing the two links from the network, the time duration is reduced to 8328.64 minutes.

make efforts to mathematically exploit the condition of the occurrence of BP to airway networks and as well as identifying the critical demand for BP to occur. Although the goal of mitigating air traffic congestion is arduous and still requires interdisciplinary efforts, this study sheds lights on the possible way towards improving air traffic flow.

Acknowledgment

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