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Stochastic Tail Assignment under Recovery

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***Abstract*—The tail assignment problem (TAP) is an pivotal part of the airline planning process with the goal of enabling efficient and safe operations. As the airline industry faces in increasing number of schedule disruptions, taking into account uncertainty during scheduling receives increasing interest by researchers. This paper presents a novel stochastic model for TAP in order to provide robust flight schedules despite operational perturbations, as induced by, e.g., flight delay and airport closure. The model is formulated in a stochastic programming framework. We propose a solution algorithm based on improved column generation and Benders decomposition with the objective to minimize operational cost and expected recovery cost under a user-defined collection of disruption scenarios. The benefits of our stochastic TAP model are demonstrated with a computational study based on real airline data. Our experimental results highlight the efficiency and effectiveness of our new model.**

***Keywords*—Airline planning, Robustness, Stochastic programming**

I. Introduction

The rapid growth of the air transportation puts immense pressure on civil aviation's stakeholders, given manifold requirements on efficiency, safety, and greener transportation. The complexity of overall system together with severe weather conditions and over-capacity usage make delay and disruption inevitable [1]. Even minor disruptions might cause cascading failures/effects on operation efficacy. In the year 2018, inside the US around 18.77% of all arrival flights were delayed by more than 15 minutes [2]. With a predicted growth of air traffic, such irregular perturbations will occur more frequently and have an increasing impact on aviation.

The airline sector has always been a leader and driver in developing advanced optimization techniques in order to cope with complex planning process. The planing process is, so far, mostly carried out under the assumption that operation will be executed as originally planned, with disruptions as exceptional cases. The increase in irregular delays/cancellations, however, led to a novel view in the field: The major short-term challenge is not avoiding delays/disruptions, but rather how to cope with delays and ensure a robust operation in case of disruptions. Therefore, we take the view that an original flight schedule is susceptible to disruption and infeasibility considering various operational constraints(e.g. maintenance requirements); and the goal of the optimization is to increase the robustness of the schedule towards these events.

Existing research on robust scheduling can be classified into three major categories: (1) domain-specific approaches [3]–[5], (2) propensity-diminishing approaches [6]–[8] and (3) feedback approaches [9]–[11]. These categories are widely applied in airline planning for the major sub problems, including schedule design problems (SDP), fleet assignment problems (FAP), aircraft maintenance routing problems (AMRP), crew scheduling problem (CSP), and tail assignment problem (TAP). The flight schedule is designed by airline marketing department based on revenue analysis. Based on this schedule, different aircraft fleets are allocated to flights considering demand and capacity. Subsequently, a route of flights is constructed for each aircraft while maintenance requirements are respeced. CSP assigns crew trips to cover all aircraft routes. And finally, at operational level, TAP is solved to manage individual aircraft by assigning routes to them in contrast to the AMRP at tactical level. An overview of robust airline planning is provided in TABLE I. These approaches are reviewed briefly below.

Domain-specific approaches identify particular features for robustness approximation in airline planning. Typical features include: reasonable allocation of buffer time [3], [4] which is expected to absorb delay propagation; short cycles and less fleet types to provide more opportunities for airline recovery [5], [13]; isolating disruption in a specific airport to protect other flights [12] and evaluation criteria such as flight service level [15]. The performance of these approaches mainly depends on their efficacy gap of robustness and can fluctuate among datasets with mark difference.

Propensity-diminishing approaches, on the other hand, combine robustness within model formulations to decrease the probabilty of disruptions and delays occurrence. One common way is to minimize the expectation of propagated delay by retiming the flight schedule. [6] proposed the delay propagation and rearranged slack time to reduce delay along downstream flights. [7] integrated ARMP and CSP to capture joint reaction for aircraft and crew in minimizing total delay time. Adopting a new compact formulation, [8] minimized the propagated delay in the weekly line of flight (WLOF) network for AMRP and partly extended to TAP. Besides that, [14] solved robust SDP through retiming and refleeting, considering demand uncertainty. [16] presented an extreme-value based framework for robust AMRP where flight delays lied in a correlated uncertainty set. [19] set up new models in extreme-value and chance-constrained paradigms to minimize delay for AMRP which had the potential for more general resource-allocation problems.

TABLE I
SUMMARY OF LITERATURE FOR ROBUST AIRLINE PLANNING.

Literature	Approaches			Covering aspects	Robustness feature
	Domain-Specific	Propensity-dimishing	Feedback		
[5]	✓			AMRP	Aircraft swap opportunities
[3]	✓			CSP	Sufficient/tight ground time
[12]	✓			FAP	Hub isolation & short sycles
[13]	✓			FAP	Station purity
[6]		✓		AMRP	Retiming to minimize propagated delay
[9]			✓	CSP	Crew switching aircraft delay
[14]		✓		FAP	Retiming & refleeting for demand uncertainty
[15]	✓			SDP	Considering block time uncertainty
[7]		✓		AMRP, CSP	Minimizing total propagated aircraft & crew delay
[10]			✓	TAP	Easing recovery process
[11]			✓	TAP	Penalizing maintenance misalignment
[8]		✓		FAP, TAP	WLOF model to minimize propagated delay
[16]		✓		AMRP	Extreme-value paradigm
[17]			✓	SDP, FAP	Integrated model under demand uncertainty
[18]	✓			AMRP	Retiming with Monte-Carlo-based procedure
[19]		✓		AMRP	chance constraint & extreme-value paradigm
[4]	✓			AMRP, CSP	Integrated compact model with sufficient buffers

Feedback approaches utilize the weighted information via recourse (second stage) problem under simulated scenarios. [9] proposed a model for this approach. Specifically, robust CSP was solved by minimizing delay of short connection within a set of scenarios. [10] presented a recoverable TAP model to develop a schedule that is recoverable with limited efforts. Similarly, [11] constructed single day routes for aircraft under the recoverable robustness framework. By penalizing the under supply of routes ending at maintenance stations, they got maintenance plan which is recoverable from disruption. [17] took market demand uncertainty into account and proposed a two-stage mixed integer non-linear model for SDP and FAP jointly. A key feature of feedback approaches lies in their dynamic adjustment of scheduled plan according to the subproblem's feedback on reaction to the plan. Compared with the other two approaches, feedback approaches are able to build robustness in a more direct and realistic way.

This study focuses on the robust TAP, with the following contributions as highlights. The first aspect is *computational tractability*. Because feedback approaches often refer to many recourse subproblems for different scenarios, the corresponding solution algorithms contain two or more stages through benders decomposition. Existing literature usually only deals with comparatively small-sized datasets or short time span, It is desirable to have more efficient solution algorithms to tackle this complex model. The second aspect are *operational considerations*. Existing studies on TAP study delay well, but rather ignore airport capacity constraints and maintenance misalignment; the latter is often crucial as maintenance events tend to be vulnerable towards disruptions and provide little recourse within the whole schedule. In addition, the trade-off between maintenance misalignment and its flexibility for possible aircraft swaps is expected to be better handled to save recovery cost.

In view of these aspects, we summarize our contributions as follows. First, we establish a column-and-row generation heuristic to solve the stochastic tail assignment under recovery (STAR) model where improved column generation and benders decomposition algorithms are adopted to obtain satisfying results in short time. In contrast to traditional approaches, our approach can boost the computation performance by at least one order of magnitude (see experimental results). Second, operational insights are derived for airport capacity and maintenance trade-off. Aircraft routes are generated with lower cost and allow recovery actions (i.e., aircraft swap, flight cancellation and maintenance swap). Our recovery recourse problem is an extension of [20] and generates recovery plans efficiently. Because we use a benders decomposition framework, this subproblem model explicitly considers deviation from TAP solution in model formulations to establish the benders cuts. Conceptually the most similar work to ours is [10]. Compared to the latter, our formulation takes maintenance capacity into account and maintenance misalignment can thus be balanced with other recovery action. Therefore, the pricing subproblems and solving algorithms are devised for better performance.

This paper is organized as follows: Section II introduces the mathematical model with emphasis on explaining how benders decomposition and column generation works on the original deterministic model. Section III presents the overall improved solution techniques towards our complex set of models. Section IV reports on the extensive computational experiments and related features obtained. Finally, Section V concludes the paper and discusses further steps for research.

II. THE MATHEMATICAL MODEL

We formulate the stochastic tail assignment problem under disruption recovery as follows. Given a close to operation schedule (4-7 days), the objective is to assign routes to individual aircraft that minimizes the overall operational cost and disruption cost subject to five operational constraints. (1) Flight coverage: each flight should be covered by one aircraft. (2) Equipment continuity: an aircraft can depart from an airport if it arrives at this airport before. (3) Initial conditions: every aircraft must depart from its initial location. (4) Turn around time constraints: a minimum time has to be reserved between 2 consecutive flights. (5) Maintenance check: mandated by FAA, aircraft type A checks should be guaranteed at maintenance stations after operating for a fixed time period (e.g. flying time, elapsed time etc). To obtain robust solution, we solve tail assignment problem and aircraft recovery problem (ARP) together using stochastic programming framework and we establish representative scenarios to simulate realistic airline operation routine or emergencies.

We use the classical flight string model introduced by [21] and a flight connection network, where every flight serves as a vertex in the network and is connected with other vertices if the aforementioned operational constraints (2)-(4) are satisfied. We define the following notations in Table II

TABLE II
NOTATION USED FOR EXPRESSING THE MODEL

Parameters	Description
F	the set of flights, $i \in F$
P	the set of aircraft, $j \in P$
D	the set of days of planning $d \in D$
M	the set of maintenance stations, $m \in M$
S	the set of scenarios, $s \in S$
R^p	the set of routes for aircraft p, $r \in R^p$
R^p_s	the set of routes for aircraft p in scenario s, $r \in R^p_s$
C_{md}	the maintenance capacity of airport m on day d
C^s_{md}	the maintenance capacity in scenario s
a^1_{ir}	1 if flight i is included in route r
a^2_{mdr}	1 if route r visits maintenance station$m$ at day$D$
w_s	weight coefficient for subproblem in scenario s
d^s_i	the cancellation cost for flight i in scenario s
g^1	the cost for flight deviation
g^2	the cost for maintenance misalignment
c_{rp}	the operating cost for route p flied by aircraft p
c^s_{rp}	the delay cost for route r flied by aircraft p in scenario s
Decision Variables	**Description**
x_{rp}	1 if aircraft p executes route r
x^s_{rp}	1 if aircraft p executes route r in scenario s
y^s_i	1 if flight i is cancelled in scenario s
$\alpha^{s+}_{ip}, \alpha^{s-}_{ip}$	indicate deviation of flight i executed by aircraft p in scenario s
$\beta^{s+}_{mdp}, \beta^{s-}_{mdp}$	indicate deviation of the number of times aircraft p visits station m for day d in scenario s
ϕ_s	objective value of subproblem in scenario s

A. Deterministic Model

With these parameters and variables, our STAR model is formulated deterministically as follows:

$$\min \sum_{rp} c_{rp}x_{rp} + \sum_{s\in S} w_s\phi_s \tag{1}$$

$$\phi_s = \min \sum_{r\in R}\sum_{p\in P} c^s_{rp}x^s_{rp} + \sum_{i\in F} d^s_i y^s_i + \sum_{i\in F}\sum_{p\in P} g^1(\alpha^{+}_{ip} + \alpha^{-}_{ip}) + \sum_{m\in M}\sum_{d\in D}\sum_{p\in P} g^2(\beta^{+}_{mdp} + \beta^{-}_{mdp}) \tag{2}$$

$$s.t. \sum_{p\in P}\sum_{r\in R^p} a^1_{ir}x_{rp} = 1, \qquad \forall i \in F \tag{3}$$

$$\sum_{r\in R^p} x_{rp} \le 1, \qquad \forall p \in P \tag{4}$$

$$\sum_{p\in P}\sum_{r\in R^p} a^2_{mdr}x_{rp} \le C_{md}, \qquad \forall m \in M, d \in D \tag{5}$$

$$\sum_{p\in P}\sum_{r\in R^p_s} a^1_{ir}x^s_{rp} + y^s_i = 1, \qquad \forall i \in F, s \in S \tag{6}$$

$$\sum_{r\in R^p_s} x^s_{rp} \le 1, \qquad \forall p \in P, s \in S \tag{7}$$

$$\sum_{p\in P}\sum_{r\in R^p_s} a^2_{mdr}x^s_{rp} \le C^s_{md}, \qquad \forall m \in M, d \in D, s \in S \tag{8}$$

$$\alpha^{s+}_{ip} - \alpha^{s-}_{ip} + \sum_{r\in R^p_s} a^1_{ir}x^s_{rp} = \sum_{r\in R^p_s} a^1_{ir}x_{rp}, \quad \forall i \in F, p \in P, s \in S \tag{9}$$

$$\beta^{s+}_{mdp} - \beta^{s-}_{mdp} + \sum_{r\in R^p_s} a^2_{mdr}x^s_{rp} = \sum_{r\in R^p_s} a^2_{mdr}x_{rp}, \quad \forall m \in M, d \in D, p \in P, s \in S \tag{10}$$

$$x_{rp} \in \{0,1\}, x^s_{rp} \in \{0,1\}, y^s_i \in \{0,1\}, \phi_s \ge 0 \tag{11}$$

$$\alpha^{s+}_{ip} \ge 0, \alpha^{s-}_{ip} \ge 0, \beta^{s+}_{mdp} \ge 0, \beta^{s+}_{mdp} \ge 0 \tag{12}$$

The objective function of Equation(1) is the sum of flight assignment cost (i.e. fuel cost and maintenance cost) along with a weighted sum of aircraft recovery cost among S scenarios. To be specific, the recovery cost is comprised of flight delay d^s_i, deviation from the original schedule. By using variables α^{s+}, α^{s-}, differences in one aircraft's assigned flights can be observed and penalized. In the same way, misalignment of aircraft p's maintenance on station m, day d is expressed with variables β^{s+}, β^{s-}.

Constraint(3) is the flight coverage constraints which means each flight is covered by one aircraft route only. The constraints in Constraint(4) ensures that each aircraft can choose at most one route to fly. Resource constraint(5) limits the number of maintained aircraft in a maintenance station according to the capacity.

For the aircraft recovery recourse stage, the superscript s denotes the recovery scenario s that the model belongs to. Constraint (6) indicates that each flight is either canceled or covered by one route in scenario s. The restriction on the number of available aircraft and airport capacity is described in constraints(7-8) corresponding to constraints(4-5). Constraints set (9-10) capture the deviation of flight and maintenance from original plan x_{rp}. As we add the deviation decision variables α, β with positive objective value, the optimal solution requires these variables to be strictly constrained by their lower bound. Moreover, the upper bound of deviation variables are restricted to 1 in the optimal solution.

From an intuitive view, the deterministic model is a large-scale mixed integer program that solves the TAP and multiple aircraft recovery problems simultaneously, given a set of scenarios. We apply decomposition techniques to solve this complex model. As the model is amenable to the L-shaped method in stochastic programming [22] and the number of feasible flight routes can increase exponentially with the problem size, enumeration can not be used. In this paper, benders decomposition and column generation are used. The following parts present the basic formulation of benders decomposition and column generation.

B. Benders decomposition

Benders decomposition has been successfully applied to combinatorial optimization problems. Its main idea is to decompose a complex model into a master problem (MP) and sub-problem (SP) [23]. For our model, the decomposition problem is that constraints related to variables x_{rp} form the master problem while other decision variables comprise subproblems. The mathematical formulations for primal sub-problems (PSP) and the master problem are given as follows.

(PSP)

$$\phi_s = \min \sum_{r\in R}\sum_{p\in P} c^s_{rp}x^s_{rp} + \sum_{i\in F} d^s_i y^s_i + \sum_{i\in F}\sum_{p\in P} g^1(\alpha^+_{ip} + \alpha^-_{ip}) + \sum_{m\in M}\sum_{d\in D}\sum_{p\in P} g^2(\beta^+_{mdp} + \beta^-_{mdp}) \quad (13)$$

$$s.t. \sum_{p\in P}\sum_{r\in R^p_s} a^1_{ir}x^s_{rp} + y^s_i = 1, \qquad \forall i \in F \quad (14)$$

$$\sum_{r\in R^p_s} x^s_{rp} \le 1, \qquad \forall p \in P \quad (15)$$

$$\sum_{p\in P}\sum_{r\in R^p_s} a^2_{mdr}x^s_{rp} \le C_{md}, \qquad \forall m \in M, d \in D \quad (16)$$

$$\alpha^{s+}_{ip} - \alpha^{s-}_{ip} + \sum_{r\in R^p_s} a^1_{ir}x^s_{rp} = \sum_{r\in R^p_s} a^1_{ir}x^*_{rp} \qquad \forall i \in F, p \in P \quad (17)$$

$$\beta^{s+}_{mdp} - \beta^{s-}_{mdp} + \sum_{r\in R^p_s} a^2_{mdr}x^s_{rp} = \sum_{r\in R^p_s} a^2_{mdr}x^*_{rp} \qquad \forall m \in M, d \in D, p \in P \quad (18)$$

$$x^s_{rp} \in \{0,1\}, y^s_i \in \{0,1\}, \phi_s \le 0 \quad (19)$$

$$\alpha^{s+}_{ip} \ge 0, \alpha^{s-}_{ip} \ge 0, \beta^{s+}_{mdp} \ge 0, \beta^{s+}_{mdp} \ge 0 \quad (20)$$

The optimal solution of the master problem is given as fixed input x^* to primal subproblems for further optimization. As the subproblem's constraints can be always satisfied by setting $y^s_i = 1, \forall i \in F$, the solution status of PSP is always feasible. So, after finding the optimal solution of PSP, optimal benders cuts can be generated for the master problem. Consider the linear relaxation of PSP. Let $(\theta^s_i, \gamma^s_p, \mu^s_{md}, \delta^{1s}_{ip}, \delta^{2s}_{mdp})$ be the dual variable of constraints (14)-(18) respectively. Denote the set of benders cuts as Ω. Given subproblem *s* and $w \in \Omega$, the derived optimal benders cuts can be denoted as constraint (22). These constraints can be understood as an estimation of PSP's objective function baesd on weak duality theory. Through iterating between master problem and subproblem, an optimal solution can be reached by checking the gap of lower bound and upper bound.

(MP)

$$\min \sum_{rp} c_{rp}x_{rp} + \sum_{s\in S} w_s\phi_s \quad (21)$$

$$s.t.(3)-(5)$$

$$\phi_s \ge \sum_{i\in F}\theta^{sw}_i + \sum_{p\in P}\gamma^{sw}_p + \sum_{m\in M}\sum_{d\in D} C_{md}\mu^{sw}_{md} + \sum_{p\in P}\sum_{r\in R^p}\left(\sum_{i\in F} a^1_{ir}\delta^{1sw}_{ip}x_{rp} + \sum_{m\in M}\sum_{d\in D} a^2_{mdr}\delta^{2sw}_{mdp}x_{rp}\right) \quad \forall s \in S, w \in \Omega \quad (22)$$

C. Column generation

In this paper, we use column generation to solve our master problem(TAP) as well as recourse subproblems(ARP). This methodology avoids enumerating decision variables and is efficient in solving a large-scale optimizing problem. Within each iteration of column generation, the problem's LP relaxation is first solved to optimal. Then, based on the dual information, a pricing subproblem is invoked to check whether variables with negative reduced cost (if it is a minimizing problem) exist. Then add this particular variable to the problem or terminate the process. The pricing problem in this context is a shortest path problem with resource replenishment to find a most negative flight route and meet the 5 operational constraints.

As for the TAP, let π_i be the dual variable of constraint(3), π_p be the dual variable of constraint(4), π_{md} be the dual variable of constraint(5), and π_{sw} be the dual variable for constraint(22). Then the reduced cost $\bar{c}_{rp}$ of route *r* is

$$\bar{c}_{rp} = c_{rp} - \sum_{i\in F}\pi_i a^1_{ir} - \pi_p - \sum_{m\in M}\sum_{d\in D}\pi_{md}a^2_{mdr} - \sum_{s\in S}\sum_{w\in\Omega}\left(\sum_{i\in F} a^1_{ir}\delta^{1sw}_{ip} + \sum_{m\in M}\sum_{d\in D} a^2_{mdr}\delta^{2sw}_{mdr}\right)\pi_{sw} \quad (23)$$

Where assignment cost c_{rp} consists of flights' fuel consumption and aircraft maintenance cost. Similarly. the reduced cost $\bar{c}^s_{rp}$ of route *r* in scenario *s* in ARP is computed as follows.

$$\bar{c}^s_{rp} = c^s_{rp} - \sum_{i\in F}\theta^s_i a^1_{ir} - \gamma^s_p - \sum_{m\in M}\sum_{d\in D}\mu^s_{md}a^2_{mdr} - \sum_{p\in P}\sum_{r\in R^p}\left(\sum_{i\in F}\delta^{1s}_{ip}a^1_{ir} + \sum_{m\in M}\sum_{d\in D}\delta^{2s}_{mdp}a^2_{mdr}\right) \quad (24)$$

Delay cost c^s_{rp} of route *r* comprises of delay cost on every flight in this case. As described at the beginning of section II, we model our pricing problem on a directed connection network *G=(V, A)*. In the network, vertices *V* are flights and maintenance events. The arcs *A* exists if the arrival airport of a previous node is identical to the current node's departure airport (maintenance vertices connect from and to the same airport) while minimum turn around time (maintenance time)

Algorithm 1 Multi-label shortest path algorithm for TAP

Input Connection Network *G(V,A)*, Dual values (π_i, π_a, π_{md})
Initialize the source node's label pool as $[(-\pi_a, 0, 0)]$
for each node *n* in G's topological sort **do**
    for each label *l* in node *n*'s label pool **do**
        for each node n_2 such that $(n, n_2) \in A$ **do**
            $l_{12} = (c_n - \pi_n, t_n, t_n + t_{n,n_2})$
            if the node is a flight **then**
                Generate new label $l_2 = l + l_{12}$
            end if
            if the node is a maintenance **then**
                Generate new label $l_2 = (l[0], 0, 0) + l_{12}$
            end if
            if l_2 valid, not dominated
                Insert l_2 in n_2's label pool
                Remove labels in n_2 dominated by l_2
            end if
        end for
    end for
end for
Output Sink node's label set

needs to be reserved as well. To find such a negative cost path within *G*, a multi-label shortest path algorithm is used and summarized in Algorithm 1.

As the connection network is an acyclic graph, the main loop of this algorithm checks every vertex in topological order. Initializing the source node with a label *l* to track the changes in reduced cost, flying time after last maintenance check and elapsed time (i.e. flying time and ground time) since last maintenance check. A new label is generated and validated by guaranteeing not being dominated by other existing labels or exceeding maintenance rules. Similarly, we apply the multi-label setting algorithm to solve ARP by adding one new label element as delay time and calculating the reduced cost based on delay time.

III. Accelerating Techniques

Although section II has introduced two decomposition skills applied to our STAR model, the basic implementations still suffer from slow convergence. In this section, accelerating techniques are introduced for both benders decomposition and column generation. Finally, an overall solution framework is presented with the two improved algorithms.

A. Improved bender decomposition

The speed of benders decomposition is closely related to the strength of the benders cuts generated. To cope with the degeneracy of subproblems, where multiple optimal dual solutions exist, [24] selected the most dominated cuts in terms of pareto optimality. For the sake of simplicity, we rewrite the MIP as $\min c'x + d'y : A_1 x + A_2 y \geq b, x \geq 0, y \in \mathbf{Y}$. All vectors and matrices are of suitable size. Then introducing the dual variable α to express the dual problem $\max \alpha'(b - A_2 \overline{y}) : A_1' \alpha \leq c, \alpha \geq 0$. So Magnanti-Wong made up an auxiliary problem to get that pareto-optimal cut. The auxiliary problem get the form as: $\max \alpha'(b - A_2 \hat{y}) : A_1' \alpha \leq c, \alpha'(b - A_2 \overline{y}) = Q(\overline{y})\alpha \geq 0$. where $Q(\overline{y})$ is the optimal objective value of the regular dual subproblem. In order to circumvent the numerical problem of Magnanti-Wong method, [25] provided a more practical definition of Magnanti-Wong point and thus changed the direction of cuts with Pareto-optimal SP: $\max \alpha(b - By^{MW}) : A'\alpha \leq c, \alpha \geq 0$. Here y^{MW} is the Magnanti-Wong point which can be updated during every iteration: $y^{MW} = (1 - \lambda) y^{MW} + \lambda \overline{y}$. If $\overline{y}$ leads to a bounded solution of SP, then 0.5 is the most effective value for λ. Recall that our ARP model will always be feasible for a given TAP model solution, the Magnanti-Wong point in this case can be obtained from a convex combination: $x_{rp}^{MW} = 0.5 * \overline{x}_{rp} + 0.5 * 0$.

B. Improved column generation

The most significant speed-ups of column generation come from accelerating the pricing subproblem [26]. In our case, the multi-label setting algorithm takes up a great amount of computational effort and is thus the bottleneck for our methodology. Because preprocessing is proven to be quite effective in boosting the multi-label setting algorithm [27], we develop our preprocessing method to provide relevant information for the multi-label setting solver to accelerate the column generation. Our method is established based on the work of [28].

Our method starts from the shortest path algorithms from the source node to all other nodes and then run it again backwardly from sink node to other nodes. We refer to this method as forward shortest path on cost (FSPC) and backward shortest path on cost (BSPC). Meanwhile, accumulated resources (flying time etc.) since last maintenance along with validity check (if the path violates resource limitation) have to be recorded. If the path from the source node to sink node exist and meet maintenance rules, we accepted as most negative reduced cost path and no need for multi-label shortest path algorithm. Otherwise, we need to run shortest path algorithm four more times (correspond to two resources) on minimizing flying time or elapsed time both forward and backward (refer to FSPR1, BSPR1, FSPR2, BSPR2 respectively) and update the lower bound (LB) as the reduced cost of FSPC from source node *s* to sink node *t*: η_{st}^{fc}. Check these resource shortest paths' reduced cost $\eta_{st}^{fr1}, \eta_{st}^{fr2}$ and feasibility to update the upper bound (UB). Finally, concatenate these forward paths and backward paths on potential vertex with updated UB after checking feasibility. The algorithm terminates with global LB, UB and recorded shortest reduced cost/resource consumption from every node *i* to the sink node *t* ($\eta_{it}^{bc}, \eta_{it}^{br1}, \eta_{it}^{br2}$), as described in Algorithm 2. Note that we do not concatenate all forward paths with the backward path but heuristically choose to concatenate paths generated, for instance, in FSPC with those in BSPR1 (BSPR2) because they are more likely to become feasible paths in terms of resource consumption.

Algorithm 2 Preprocessing algorithm

Input Connection Network *G(V,A)*, Dual values
Initialize $LB = -\infty, UB = 0, incumbentpath = \emptyset$
Run FSPC and BSPC to get $\eta_{si}^{fc}, \eta_{it}^{bc}, path_{si}^{fc}, path_{si}^{bc}$
$LB = \eta_{st}^{fc}$
if $valid_{st}^{fc}$ **then**
  $UB = \min(UB, \eta_{st}^{fc})$
  $incumbentpath = path_{st}^{fc}$, go to **Output**
end if
for *j* in Resource set **do**
  Run FSPR*j*, BSPR*j* to get $\eta_{si}^{frj}, \eta_{si}^{brj}, path_{si}^{frj}, path_{si}^{brj}$
  if $valid_{st}^{frj} \& UB > \eta_{st}^{fc}$ **then**
    $UB = \eta_{st}^{fc}, incumbentpath = path_{st}^{frj}$
  end if
end for
for node $n \in V$ **do**
  if $valid_{sn}^{f*} \& valid_{nt}^{b*} \& UB > \eta_{st}^{f*} + \eta_{nt}^{b*}$**then**
    $UB = \eta_{st}^{f*} + \eta_{nt}^{b*}$
    $incumbentpath = path_{sn}^{f*} + path_{nt}^{b*}$
  end if
end for
Output LB, UB, η_{it}^{bc}
* denotes *c, rj*

Solution results from Algorithm 2 can help to accelerate multi-label setting algorithm by removing labels whose reduced cost (resource consumption) has or will be larger than current UB (resource limitations) while not removing any label that affects the proven optimality.

C. The overall algorithm

We developed an overall solution framework to solve our model with the decomposition techniques discussed above. As flight string model for TAP usually generates a tight linear relaxation. The overall algorithm will solve the linear relaxation and introduce integrality constraint to the TAP model afterwards. Instead of solving the integral TAP model with precise but time-consuming branch and price method. We adopt a diving heuristic without backtracking to get the final result. This diving heuristic is also utilized in [27]. The whole algorithm is illustrated in Algorithm 3.

The algorithm starts with a linear relaxation of the master problem (MP), and reintroduce the integrity requirements after the relative gap between UB and LB meet a given value. The MP and primal subproblem (PSP) are all solved with Algorithm 1 and Algorithm 2. The multi-label setting algorithm is only called when the preprocessing algorithm can not find negative variables or optimality need to be proved. After that, the MIP MP is solved with dive-and-price algorithm using a branching scheme called follow-on branching [29]. This strategy differs from traditional variable fixing and is widely used in scheduling problem which avoids unbalanced search tree.

Algorithm 3 Overall solution algorithm

Initialize *UB*=$+\infty$,*LB*=0, *GAP*=$+\infty$,y^{MW}.
while *GAP>0* **do**
  LB←**Solve** MP (21)
  for scenario $s \in S$
    $PSP^* \leftarrow$ **Solve** PSP (13)-(20)
    $\lambda^* = 0.5$
    Update M-W point
    Add optimal benders cuts (22) to MP
    Solve Benders Pareto-Optimality PSP
    Add optimal cuts (22) to MP
  end for
  Update *UB*
  GAP=$\frac{(UB-LB)}{LB}$
end while
Reintroduce integrity requirements for MP
Solving MP using dive-and-price

IV. Computational Experiments

A. Data description

The results of computational experiments are reported in this section. The solution framework is implemented in Python and executed on a computer with 2.5GHz Intel i7-6500U CPU and Fedora 27 system. SCIP [30] is called to get the integer solution with CPLEX 12.6.3 as linear programming solver. All experiments are carried out using a single thread for the sake of fair comparison. The experimental dataset is derived from daily operational data of Okay airline, a Chinese domestic low cost carrier. Based on the data, we generated three test instances. Details of these instances are shown in TABLE III.

TABLE III
Characteristics of test cases

Case	Flights	Fleet size	Connections	Airports	Maint.stations
1	53	5	428	21	4
2	77	7	517	23	5
3	104	7	853	26	6

To better simulate daily operations, we generate 56 distinct scenarios. The specifics of these scenarios are presented in TABLE IV. These scenarios cover common operational situations, including flight delay, airport closure, reduction in maintenance capacity, and Aircraft-on-Ground (AOG). Specifically, delay is classified as slight, moderate and severe according to three time minutes range $[15, 30), [30, 60), [60, 180)$ respectively. To generate these delay scenarios, we select delay values randomly from real historical data for that airline. Since the data is extracted from a summer schedule, airports are exposed to the risk of afternoon closure due to thermal thunderstorm. Fog is a comparatively less contributor to the morning closure, thus, we assume the airport closure scenarios happen preferably in the early afternoon. Also, to reveal the impact of maintenance capacity reduction, we reduce the maintenance capacity by half in one operation day. Lastly, aircraft can experience an unexpected mechanical failure which requires the aircraft to be repaired at maintenance stations; referred to Aircraft-on-Ground (AOG) in the airline industry. In our experiments, AOGs take place for two hours and during which the aircraft can not serve any flight. Nevertheless, it should be noted that all assumptions on the daily operations are subject to user-defined changes, and not specific to our model/solution algorithm.

B. Numerical results

We report the computational results of STAR for all instances. Because the mathematical model is large-scale and complex, we proposed our solution algorithm and corresponding enhancements. Two improvements (Pareto optimal cuts and preprocessing algorithm) are implemented and compared with implementations where only one or no enhancement happens. In TABLE V, the different running time are reported as (1) Basic: using the standard benders decomposition and column generation. (2) All enhancements: including Pareto optimal cuts and preprocessing algorithm. The objective values of both linear relaxation solutions and integer solutions are also reported with the corresponding LP relaxation degree. To better represents airline operational situations, we utilize BADA [31] to calculate the fuel consumption cost for specific aircraft type on different cruising flight levels (i.e. simulating short/long range). The average maintenance cost is estimated using statistics from IATA [32]. As delay cost parameters in ARP vary from airlines to airlines, we select two penalty cost values for delay to analyze its impact on the independent delay and propagated delay.

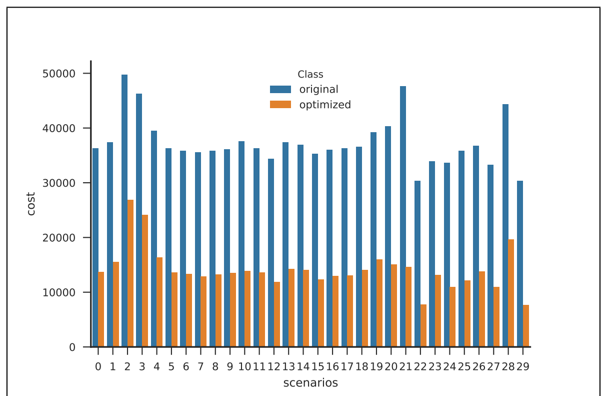
From our experimental results, we observed very long run time for instances without enhancements, as expected. In contrast, pareto optimal cuts along with preprocessing significantly reduces the running time almost one order of magnitude. As the computation time depends heavily on the pricing subproblem and the benders decomposition's iterations, our improved solution usually takes much fewer iterations to converge. Also, the preprocessing algorithm for ARP can often find the most negative path for pricing. Thus, the improved algorithm tends to take advantages over average solution methodology. Moreover, as the last column in TABLE V shows, the linear relaxation of our STAR model is quite tight, with a gap of nearly 0.1% between LP solutions and IP solutions. This motivates us to use a quick diving heuristic for final integer solutions.

C. Effects of stochastic solutions

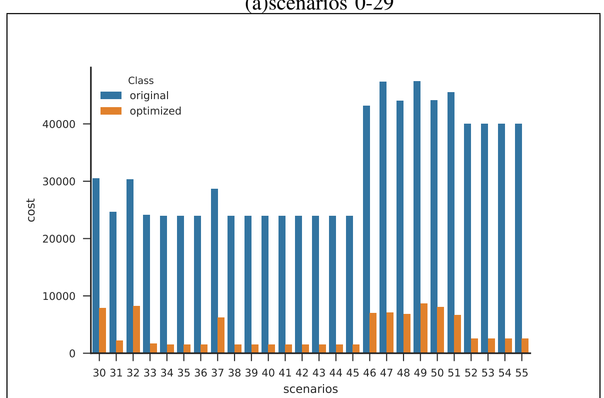
Comparing the result of STAR with that of deterministic TAP alone, we find much improvement margin in the individual ARP objective value across all tested scenarios. Taking the test case 3 as an example (delay cost is 20/min), we make a comparison between the final objective values of ARP models for all 56 scenarios and those from a deterministic TAP solution. The results are shown in Fig 1. STAR has the advantage to produce robust solution's whose ARP objective values will be no worse than that of the solution to the model used in deterministic TAP. Recall that the 56 scenarios were categorized into six classes. The first three classes, 30 cases shown in Fig 1 (a), relating to departure delay constitute a larger cost component because there is much more independent delay at each flight leg. In contrast, airport closure, maintenance capacity reduction, and AOGs in Fig 1 (b) are more prone to cause maintenance misalignment and deviations to planning schedule. When solving ARP, total delay c^s_{rp} including independent delay and propagated delay needs to be minimized. We adopt the formulations in [6] to calculate propagated delay:

$$PD_{ij} = \max(TAD_i - Slack_{ij}, 0)$$

Thus, the propagated delay *PD* from flight *i* to *j* is decided by both total arrival delay *TAD* and slack time *Slack*. In our robust solutions, aircraft have been assigned to flight strings with less probability of propagated delay. This is achieved by leaving more slack time for flights with a long delay time or under the impact of disrupted airports.



(a)scenarios 0-29



(b)scenarios 30-55

Figure 1. Comparison in ARP objective values

Another important part of our analysis is determining the trade-off between explicit aircraft assignment cost and implicit predicted delay costs. To understand how our delay penalty cost parameter affects this trade-off, we analyze the stochastic solution using different delay cost parameters. TABLE VI reports the sum of propagated delay and total assignment cost. For all our experiments, the delay cost

TABLE IV
OPERATION SCENARIOS USED

Type	Affected	Scenarios
Slight departure delay	One scenario for 30% flight legs	0-9
Moderate departure delay	One scenario for 15% flight legs	10-19
Severe departure delay	One scenario for 5% flight legs	20-29
Airport closure	One scenario for each major airport	30-39
Capacity reduction	One scenario for each maintenance station	40-45
Aircraft-on-Ground	One scenario for each aircraft	46-55

TABLE V
COMPUTATION RESULTS OF STAR WITH ENHANCEMENTS

Test case	Delay cost	Running time		LP obj	IP obj	Relaxation
	(/min)	Basic	All enhancements			
1	20	1052.75	66.65	365282.06	365562.06	0.076
	80	1242.50	83.47	376475.02	376752.40	0.073
2	20	1951.39	143.48	490714.95	490731.21	0.004
	80	1686.80	133.80	504852.02	505432.40	0.115
3	20	3387.51	342.66	688098.03	688444.14	0.050
	80	2318.18	297.67	729977.51	729977.51	0.0

parameter does not have obvious effects on the final optimal propagated delay and assignment cost. This indicates our model is robust towards different penalty coefficients which ease the difficulty in quantifying the delay cost.

TABLE VI
DELAY COST EFFECTS ON SOLUTIONS

Test case	Delay cost	Propagated delay(min)	Assignment cost
1	20	1352	361600.00
	80	1352	361600.00
2	20	1132	485498.23
	80	1132	485498.23
3	20	5082	678767.68
	80	5082	678767.68

D. Comparing airline's original plan

Given our experimental results, STAR is obviously advantageous over deterministic TAP model. In this section, we further analyze the strategy used by the airline to arrange their schedules. Because degeneracy is common in airline optimization problems, meaning that many feasible solutions obtain the optimal solution at the same time. Therefore, comparing the optimal solution with airline's original plan directly is unreasonable. Instead, the weighted costs of ARP subproblems and other statistics are used to identify the operation pattern in airline and potential benefits of our model. Specifically, airline's original timetable is given as input x_{rp} to solve every subproblem. The weighted sum of these recovery cost is thus calculated and shown in TABLE VII. The coefficient of delay cost is set to 20, both numeric values and increasing rates are included.

As clearly illustrated in the table, the airline's original plan already contains some robustness in contrast to the solution from deterministic TAP model. For test case 3, the original plan is close to the optimal solution found by our STAR model with a gap smaller than 0.2%. But in test case 2, even TAP solution has a 20% superiority over the airline's solution. Currently, the airline's flight schedule is mainly determined by flight dispatcher from Airline Operation Center (AOC) and staffs from Maintenance Department. While historical operation statistics are taken into consideration, they decide the final aircraft paths and related buffer times empirically. Flights departing from or arriving at congested airports will be given more buffer times than other airports. On the other hand, short flight rotations (an aircraft starts and ends at the same airport) are usually adopted to ensure flow balance at airports but reduce flexibility during disruption or delay circumstances. Our STAR model, on the contrary, is capable to devise good enough robust solution if scenarios are devised accordingly. In addition, [33] indicated that some airlines are willing to experience delay in trade of a shorter schedule block time. STAR can nicely trade-off the overall cost and recovery expense to some extend from the computational results.

TABLE VII
COMPUTATION STATISTICS WITH WEIGHTED SUM OF ARP

Test case	Original plan	TAP solution	STAR solution
1	4699.29	6163.56 (+31.16%)	3873.50 (-17.57%)
2	9099.29	7211.67 (-20.74%)	4987.91 (-45.18%)
3	7657.14	47790.0 (+524.12%)	7642.63 (-0.19%)

V. CONCLUSION

In this paper, we propose a stochastic model for tail assignment problem considering recovery reactions from operational perturbations, as induced by, e.g., flight delay and airport closure, through a stochastic programming framework. To solve the model with efficiency, we propose an improved solution algorithm that combines benders decomposition with column generation. Our experiments show that the model generates tight LP relaxation and can be solved quickly using the presented algorithm. The value of considering robust TAP is also demonstrated from the gap between the deterministic solution and stochastic solution.

Future research involves integrating the tail assignment problem with crew scheduling problem to improve robustness. Models considering airports' slot capacity are under consideration as well. Also, correlation in flights/airports disruptions needs to be handled to better depict realistic operation circumstances.

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References

[1] K. Gopalakrishnan and H. Balakrishnan, "A comparative analysis of models for predicting delays in air traffic networks." ATM Seminar, 2017.

[2] B. of Transportation Statistics, "Year-to-date airline on-time performance for each month."

[3] M. Ehrgott and D. M. Ryan, "Constructing robust crew schedules with bicriteria optimization," *Journal of Multi-Criteria Decision Analysis*, vol. 11, no. 3, pp. 139–150, 2002.

[4] M. B. Ahmed, F. Z. Mansour, and M. Haouari, "Robust integrated maintenance aircraft routing and crew pairing," *Journal of Air Transport Management*, vol. 73, pp. 15–31, 2018.

[5] Y. Ageeva, "Approaches to incorporating robustness into airline scheduling," Ph.D. dissertation, Massachusetts Institute of Technology, 2000.

[6] S. Lan, J.-P. Clarke, and C. Barnhart, "Planning for robust airline operations: Optimizing aircraft routings and flight departure times to minimize passenger disruptions," *Transportation science*, vol. 40, no. 1, pp. 15–28, 2006.

[7] M. Dunbar, G. Froyland, and C.-L. Wu, "Robust airline schedule planning: Minimizing propagated delay in an integrated routing and crewing framework," *Transportation Science*, vol. 46, no. 2, pp. 204–216, 2012.

[8] Z. Liang, Y. Feng, X. Zhang, T. Wu, and W. A. Chaovalitwongse, "Robust weekly aircraft maintenance routing problem and the extension to the tail assignment problem," *Transportation Research Part B: Methodological*, vol. 78, pp. 238–259, 2015.

[9] J. W. Yen and J. R. Birge, "A stochastic programming approach to the airline crew scheduling problem," *Transportation Science*, vol. 40, no. 1, pp. 3–14, 2006.

[10] G. Froyland, S. J. Maher, and C.-L. Wu, "The recoverable robust tail assignment problem," *Transportation Science*, vol. 48, no. 3, pp. 351–372, 2013.

[11] S. J. Maher, G. Desaulniers, and F. Soumis, "Recoverable robust single day aircraft maintenance routing problem," *Computers & Operations Research*, vol. 51, pp. 130–145, 2014.

[12] J. M. Rosenberger, E. L. Johnson, and G. L. Nemhauser, "A robust fleet-assignment model with hub isolation and short cycles," *Transportation science*, vol. 38, no. 3, pp. 357–368, 2004.

[13] B. C. Smith and E. L. Johnson, "Robust airline fleet assignment: Imposing station purity using station decomposition," *Transportation Science*, vol. 40, no. 4, pp. 497–516, 2006.

[14] H. Jiang and C. Barnhart, "Dynamic airline scheduling," *Transportation Science*, vol. 43, no. 3, pp. 336–354, 2009.

[15] M. Sohoni, Y.-C. Lee, and D. Klabjan, "Robust airline scheduling under block-time uncertainty," *Transportation Science*, vol. 45, no. 4, pp. 451–464, 2011.

[16] C. Yan and J. Kung, "Robust aircraft routing," *Transportation Science*, vol. 52, no. 1, pp. 118–133, 2016.

[17] L. Cadarso and R. de Celis, "Integrated airline planning: Robust update of scheduling and fleet balancing under demand uncertainty," *Transportation Research Part C: Emerging Technologies*, vol. 81, pp. 227–245, 2017.

[18] M. B. Ahmed, W. Ghroubi, M. Haouari, and H. D. Sherali, "A hybrid optimization-simulation approach for robust weekly aircraft routing and retiming," *Transportation Research Part C: Emerging Technologies*, vol. 84, pp. 1–20, 2017.

[19] L. Marla, V. Vaze, and C. Barnhart, "Robust optimization: Lessons learned from aircraft routing," *Computers & Operations Research*, vol. 98, pp. 165–184, 2018.

[20] Z. Liang, F. Xiao, X. Qian, L. Zhou, X. Jin, X. Lu, and S. Karichery, "A column generation-based heuristic for aircraft recovery problem with airport capacity constraints and maintenance flexibility," *Transportation Research Part B: Methodological*, vol. 113, pp. 70–90, 2018.

[21] C. Barnhart, N. L. Boland, L. W. Clarke, E. L. Johnson, G. L. Nemhauser, and R. G. Shenoi, "Flight String Models for Aircraft Fleeting and Routing," *Transportation Science*, vol. 32, no. 3, pp. 208–220, 1998.

[22] G. Laporte and F. V. Louveaux, "The integer l-shaped method for stochastic integer programs with complete recourse," *Operations Research Letters*, vol. 13, no. 3, pp. 133–142, 1991.

[23] Y. Xu, W. Dai, X. Sun, and S. Wandelt, "Improved benders decomposition for capacitated hub location problem with incomplete hub networks," in *2017 IEEE Symposium Series on Computational Intelligence (SSCI)*, Nov 2017, pp. 1–8.

[24] T. L. Magnanti and R. T. Wong, "Accelerating benders decomposition: Algorithmic enhancement and model selection criteria," *Operations Research*, vol. 29, no. 3, pp. 464–484, 1981.

[25] N. Papadakos, "Practical enhancements to the magnanti-wong method," *Operations Research Letters*, vol. 36, no. 4, pp. 444–449, 2008.

[26] J. Desrosiers and M. E. Lübbecke, *A Primer in Column Generation*, 2005.

[27] S. Ruther, N. Boland, F. G. Engineer, and I. Evans, "Integrated Aircraft Routing , Crew Pairing , and Tail Assignment : Branch-and-Price with Many Pricing Problems," *Transportation Science*, vol. 51, no. April 2018, 2017.

[28] O. J. Smith, N. Boland, and H. Waterer, "Solving shortest path problems with a weight constraint and replenishment arcs," *Computers & Operations Research*, vol. 39, no. 5, pp. 964–984, 2012.

[29] D. M. Ryan and B. A. Foster, "An integer programming approach to scheduling," pp. 269–280, 1981.

[30] T. Achterberg, "Scip: solving constraint integer programs," *Mathematical Programming Computation*, vol. 1, no. 1, pp. 1–41, 2009.

[31] A. Nuic, D. Poles, and V. Mouillet, "Bada: An advanced aircraft performance model for present and future atm systems," *International journal of adaptive control and signal processing*, vol. 24, no. 10, pp. 850–866, 2010.

[32] IATA, "Airline maintenance cost: Executive commentary."

[33] L. Hao and M. Hansen, "How airlines set scheduled block times," in *10th USA/Europe Air Traffic Management Research and Development Seminar, Chicago IL*, 2013.

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