

Assessment of the Airport Operational Dynamics Using a Multistate System Approach

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Abstract—The analysis of the airport operational reliability is fundamentally linked to the knowledge of the system’s behavior and dynamics. This paper proposes a model for assessing airport performance at a tactical level (time scale), focusing on the airspace-airside turnaround operations (space scale) and considering different areas: delay, capacity, environmental impact and operational complexity. Airports are transportation systems that can complete their tasks with partial performance levels: failures of some system elements may lead to partial degradation of the system behavior, which cannot be assessed with the traditional binary reliability view (working – not working). To consider this performance granularity, our model uses a multistate approach. A Markov-chain based methodology allows us to predict the system’s reliability evolution and move from reactionary measures to predictive interventions. It also considers the impact of stochasticity on performance prediction by assessing the system operational dynamics. The methodology is developed through a case study at a major European hub airport: a collection of 160,460 turnaround operations (registered at 2016) is used to statistically determine the system characteristics. Results for the appraised case study show that the airport tends to evolve towards repaired states, and that delays are major drivers for airport performance dynamics. The contribution of the paper is twofold: it presents a new methodological approach to evaluate airport operational dynamics and it also provides insights on how different factors influence performance.

Keywords: reliability analysis; turnaround operations; multistate systems; Markov processes; performance prediction; system dynamics.

I. INTRODUCTION

Airports play a prominent role in supporting connectivity, facilitating air cargo, promoting regional accessibility and also driving the economies of the areas they serve [1], [2]. Moreover, airports are inter-modal transportation infrastructures that act as nodes in the air transport network [3]. Failures or degradation of airport operations may easily propagate through the network and generate system-level effects [4]. Therefore, understanding airport operational dynamics, while measuring its performance, is one key to successful and efficient management of transport systems [5]. As airport dynamics depend on various and heterogeneous factors, a holistic view that considers different key performance areas is then essential [6].

The analysis of the operational dynamics for major transport infrastructures is closely related to reliability appraisal, since it provides an assessment of the progress of the system operation [7]. In traditional system reliability evaluation, both the system and its components are considered binary elements, which work in two working levels: perfect functioning and failure [8]. Airports are complex systems that have different components and performance levels, and where we cannot formulate an “all or nothing” type of failure criterion. Failure of some of the system elements may lead only to performance degradation, and a binary model (100%, 0%) would be a poor representation of the system. In this paper, we propose a reliability analysis with different performance levels for the system and its components, which is known as a multistate system (MSS) approach [9].

An attribute of an air transport network, which is similar to many large complex network systems, is that most of elements in the network (including airports acting as nodes) are subject to stochastic influence (e.g. the operation of an airport is subject to weather conditions, which influence the practical runway capacity and some processes). Additionally, the components of the system can transit from perfect functioning to degraded performance levels during working periods. The combined stochastic and time-varying nature of the airport operational system creates a set of “dynamics” that affect the way an airport is managed and can complete its tasks [3]. Moreover, since reliability levels of the system components change during functioning periods, this variability should be considered when performing the reliability assessment. The theory of system dynamics provides a framework to capture the dynamic behavior of complex systems [10]; in air transport studies, this methodology has already been applied to airport terminals [11].

Probabilistic approaches are common in the risk assessment of complex engineering systems, nevertheless it is usually difficult to combine empirical data with expert knowledge of the problem [12]. In this paper, a novel approach based on Markov processes has been used, which merges prior expert knowledge with real data to obtain posterior distributions of transition intensities [12], [13]. Adaptive and stochastic methods can be a useful tool for solving airport operational problems [14].

The time scale for the analysis is related to the tactical phase (the day of operations), as the model predicts, monitors and updates the airport operational performance based on the current

situation. This could be used as an input for continuous performance assessment according to real time traffic demand (capacity optimization and delay management). However, the method could also be applied in a pre-tactical phase (to predict future performance levels according to foreseen initial operational situations) and the results could be further reviewed in a post-operational phase (to measure performance targets or evaluate operational processes).

Regarding the space scale of the analysis, we focus on the Airport Transit View (ATV). This concept describes the “visit” of an aircraft to the airport [15] and includes the main Collaborative Decision Making milestones (Figure 1).

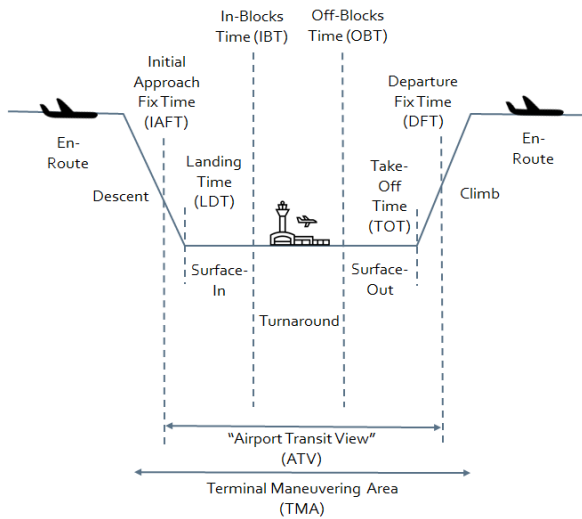


Figure 1. Extension of the ATV concept. Source: adapted from [15].

The ATV framework connects inbound-outbound airborne segments, providing a tool to optimize airport operations and to enable a more efficient and cost-effective deployment of operator resources. It integrates airside operations (landing, taxiing, turnaround and take-off) and surrounding airspace operations (holding, final approach and initial climb) [16]–[18]. From an air transportation system view, a flight could be seen as a gate-to-gate or an air-to-air process [19]. Whereas the gate-to-gate is focused on the aircraft trajectory flown, the air-to-air process concentrates on airport operations [20]. ATV processes act as major drivers for delays and capacity constraints [19], [21], [22]. Therefore, to comply with air traffic management challenges over the day of operations, a change to an air-to-air perspective is necessary, with a specific attention on integrated airside/airspace operations (coordination of local airport management decisions with the surrounding airspace decisions). In Europe, local turnaround delays accounted for 35% of all departure delays in 2016, where the average departure delay per flight reached 11.2 min [23]. This demands a sustainable improvement of the turnaround performance monitoring and predictability.

The main aim of our study is to produce a mechanism to monitor and forecast the system’s operational state, as a way to proactively assess the reliability of airports (how the system performance evolves, what is the mean time to system degradation or reparation). Therefore, by applying a new

methodological approach we seek to evaluate airport operational dynamics and provide insights on how different factors influence performance.

Expected direct benefits of the model are better performance due to better predictability of airport dynamics and significant resilience benefits through better management of forecasted or unexpected operational shortfalls. It also supports the improvement of airspace/airside operations, situational awareness and collaborative decision-making processes through the integration and monitoring of aircraft flows throughout the ATV cycle.

The remainder of this paper consists of five primary sections: Section 2 provides a literature review of existing approaches and models for airport performance analysis, while Section 3 introduces the data and scenario that will be used in the study. Section 4 presents the methodology for assessing the airport operational dynamics, introducing the concepts of MSS, Markov chains (MC) and reliability indicators. Main results arising from the application of the model to the appraised scenario can be found in Section 5. This case study demonstrates the use of the proposed methodology for modelling and analyzing airport dynamics. Finally, the paper presents the main conclusions, recommendations and potential future work (Section 6)

II. BACKGROUND

Performance measurement at airports is evolving in a dynamic regulatory, ownership, and market environment in the context of rapid demand growth and technical innovation [24], [25]. Airport Operators (AOs) and Air Navigation Service Providers (ANSPs) require effective performance measures and assessment methods to plan and manage within this complex context. Many studies have investigated the productivity or financial performance of airports, and how changes in the industry may have affected them [26]–[29]. Meanwhile, several works have focused its performance analysis on operational metrics: delays [30]–[33], capacity congestion [34]–[37], weather impact, complexity [38], [39] or safety [40]. However, it is usual to segregate the influence of each area on airport operations. In this paper, we propose a holistic view, aiming to construct performance evaluations on the basis of the multiple outputs which airports produce and the multiple inputs which they utilize. Moreover, in our study we revise the linkage between inbound and outbound flights by assessing the aircraft operational flow (turnaround integration in the air traffic network). This approach is in line with past analyses [30], [41]–[43]. Our main contribution in this field is to extend the spatial scope to the Terminal Maneuvering Area (TMA) boundaries. Instead of considering airspace and airside processes in isolation, our approach contemplates the cross impacts among operations. (e.g. the knock-on effects of prior delays cascading from arrival to departure operations [44]). Hence, we extend the spatial scope of previous works that were centered on airside operations [45] to the airport/airspace environment. With regard to the time scale, while many past performance studies were devoted to post-operational analysis [46]–[48], our paper proposes a model for describing and predicting airport performance at a tactical level (day of operations).

Therefore, this study addresses two gaps in the literature: (i) at present, airport stakeholders lack models able to provide an

integrated view of the ATV processes (airside and surrounding airspace) and analyze the tradeoffs between the various measures of airport operational behavior such as capacity, delays, environmental performance and complexity; (ii) current system-wide congestion problems are worsened due to airport operational inefficiencies [4], hence, there is an opportunity to develop new conceptual tools to support airport management functions. In this sense, we propose a novel approach for assessing and predicting the airport's operational behavior, given certain operational circumstances.

III. SCENARIO & DATA

A. Scenario definition

The analysis of the airport operational reliability and its dynamics is applied to a case study at Adolfo Suárez Madrid-Barajas Airport (LEMD). The observation period corresponds to 2016, when 160,460 turnaround operations were registered [49].

B. Preparation of data

The data preparation phase covered all activities required to assemble the final dataset from the initial raw operational and meteorological data provided by the airport, including locating and refining erroneous measurements. 156,386 final valid observations were appraised. Data include operational timestamps, meteorological features, aircraft and airline information, flight and route details and airport configuration. The most representative attributes when assessing airport performance were chosen using feature selection techniques [50]. The objective of the feature selection step is twofold: improving the representativeness and effectiveness of performance attributes and providing a better understanding of the underlying relationships between variables.

C. Exploratory analysis of data (distribution fitting)

Data were used to statistically appraise the system characteristics. Distribution fitting has three main objectives: (a) to characterize each attribute and its statistical behavior; (b) to use probability distributions as a tool for dealing with uncertainty when assessing performance; and (c) to use probability distributions as inputs for setting the states in the MSS model that is developed later. See Figure 2 for an example of distribution fitting.

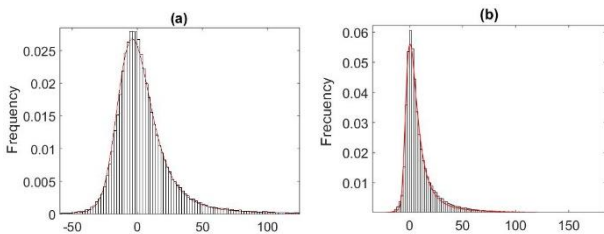


Figure 2. Histogram and distribution fitting for (a) In-Block Delay (seconds) and (b) Off-Block Delay (seconds).

IV. METHODOLOGY & MODEL DEVELOPMENT

A. Continuity study

The reliability model is trained for data corresponding to airport operations under conditions of continuous demand and aircraft queuing. This is when the airport's operational reliability

can be evaluated. Otherwise, recovery indicators may be affected by the transit time between operations. Moreover, when applying the MSS and MC methods, it is necessary to ensure that operations are equally spaced in time. Thus, we performed a continuity study of airport operations to establish which time intervals are the most appropriate for the analysis. During the main operational hours of the airport (i.e., from 5 to 24, local time), we can assume that operations are continuous (3 min of mean time between operations, with an interquartile range of 1 min), and therefore the theory of MSS and MC is applicable.

B. Multistate systems reliability analysis and Markov chains

A system is designed to accomplish a defined task in a determined environment, under different changing conditions. Traditionally, systems have been modelled in a binary way, thus the system has only two possible states: perfect functioning or complete failure [21], [36], [37]. Nevertheless, most real systems can develop their tasks in more than two performance levels. Additionally, real systems are usually composed of elements that can also be found in different states [21]. When the performance rate of the system's elements can vary because of their deterioration (fatigue, partial failure) or because of variable ambient conditions, the entire system may be considered a multistate system (MSS) [37]. An MSS reliability approach allows us to consider a finite number of states for both the system and its components.

Before studying a complete MSS behavior, it is necessary to characterize the elements that constitute it. Any element j of the system can have different k_j states corresponding to the performance levels of the element, represented by the set:

$$g_j = \{g_{j1}, g_{j2}, \dots, g_{jk_j}\} \quad (1)$$

where g_{ji} is the performance level (or performance rate) of the element j at state i ($i \in \{1, 2, \dots, k_j\}$).

The performance level $G_j(t)$ of the element j for any instant $t \geq 0$ is a random variable, which takes values from g_j : $G_j(t) \in g_j$. Therefore, for the time interval $[0, T]$ (in which T is the operation period of the MSS), the performance level of the element j can be defined as a stochastic process. The probabilities associated with each state (or performance rate) of the system element j for any instant t can be represented with the following set of equations:

$$P_j(t) = \{p_{j1}(t), p_{j2}(t), \dots, p_{jk_j}(t)\} \quad (2)$$

where

$$p_{ji}(t) = \Pr\{G_j(t) = g_{ji}\} \quad (3)$$

Since the states of the elements are a complete group of mutually exclusive events (which means that element j can be in one of the states k_j and only one), the following condition must be fulfilled:

$$\sum_{i=1}^{k_j} p_{ji}(t) = 1, \quad \text{for any } t: 0 \leq t \leq T \quad (4)$$

Equation (3) defines the probability function of a discrete random variable $G_j(t)$ at any instant t . The pairs $g_{ji}, p_{ji}(t)$ (with $i=1,2,\dots,k_j$) completely determine the probability distribution of performance (PD) of the element j at any instant t .

When an MSS is composed of n elements, its performance rate is determined in an unambiguous way by the performance levels of the elements that compose it. At each moment, the elements of the system have a performance level that corresponds to their current state. The state of the entire system is determined by the states of its elements. Therefore, the definition of a MSS reliability model must include the performance stochastic process for each element j of the system: G_j ($j=1,2,\dots,n$) and the system structure function that generates the stochastic process corresponding to the output performance of the entire MSS: $G(t) = \varphi(G_1(t), \dots, G_n(t))$ (Figure 3).

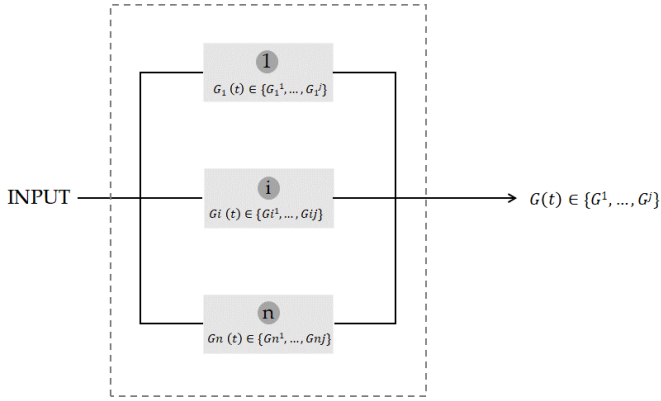


Figure 3. Example of an MSS structure (n components).

In our study, the MSS analysis is developed through a Markov-chain methodological approach. Markov chains (MC) are discrete stochastic processes in which the probability of an event only depends of the previous state of the system. Then, this type of systems satisfy the Markov property [51]:

$$P(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, \dots, X_2 = x_2, X_1 = x_1) = P(X_{n+1} = x_{n+1} | X_n = x_n) \quad (5)$$

An MC $\{X(n), n=0,1,2,\dots\}$ is described by a sequence of random variables $X(0) = X_0, X(1) = X_1, \dots, X(n) = X_n$, where X_0 is the initial state. The probability of transition between state $X_{n-1} = i$ and $X_n = j$ is given by $\gamma_{i,j}$. Therefore, the matrix \mathbf{P} represents the one-step transition probabilities [52]:

$$\mathbf{P} = \begin{pmatrix} \gamma_{1,1} & \gamma_{1,2} & \dots & \gamma_{1,k} \\ \gamma_{2,1} & \gamma_{2,2} & \dots & \gamma_{2,k} \\ \vdots & \vdots & \dots & \vdots \\ \gamma_{k,1} & \gamma_{k,2} & \dots & \gamma_{k,k} \end{pmatrix} \quad (6)$$

The vector π_n^T defines the probability of finding the system in a particular state on the n -th transition:

$$\pi_n^T = [\pi_{1,n} \quad \pi_{2,n} \quad \dots \quad \pi_{k,n}] \quad (7)$$

where $\pi_{k,n}$ is the probability that the system is in state k on the n -th transition. The probabilities for each transition are determined iteratively as follows:

$$\pi_n^T = \pi_{n-1}^T \mathbf{P} \quad (8)$$

The stationary distribution π of a system represented by a MC (steady state or long-term, $n \rightarrow \infty$), does not change over time, and thus represents the long-term behavior of the system:

$$\pi = \pi T \quad (9)$$

$$T^\infty = \lim_{n \rightarrow \infty} T^n \quad (10)$$

An equivalent description of the MC can be given by a directed graph called the state-transition diagram of the MC. Figure 4 gives a basic example of a two-state discrete time MC diagram. The one-step transition probability matrix for the example is as follows, where $p_{00} + p_{01} = 1$ and $p_{10} + p_{11} = 1$.

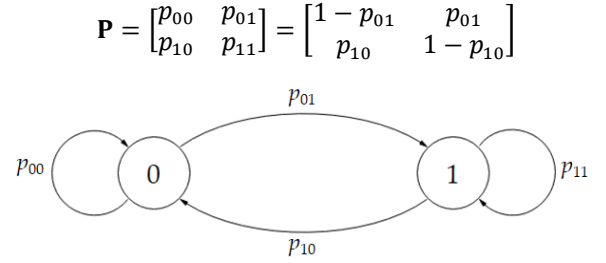


Figure 4. Example of two-state discrete-time Markov chain.

The MC is constructed by all the candidate samples or states. The current state depends only on its adjacent states. The steps are often thought of as moments in time, but they can refer to any other discrete measurement. Formally, the steps are the integers or natural numbers, and the random process is a mapping of these to states. Note that in our case, each step corresponds to an arrival operation.

C. Model development: definition of blocks and state vector

The airport reliability model is divided into four blocks (system components): delay, capacity, environmental impact and complexity, which provide a holistic view of airport operational performance. A number of variables or elements defines each block. This sub-division allows us to perform a specific analysis of each component, so we can get a better understanding of the global model. The three levels (airport system, blocks and partial elements) have different performance rates (all follow a multistate approach). Therefore, the airport is defined as an MSS, which consist in four blocks (system components). Each of these blocks is also divided in different variables/elements with changing performance rates (Figure 5).

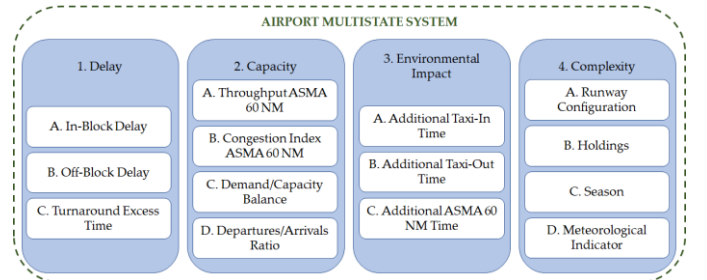


Figure 5. MSS structure for the airport operational behavior.

The performance rate of any component of the system can range from complete failure up to perfect functioning. The failures that lead to a decrease in the component performance rate are called partial failures. After partial failure, components continue operating at reduced performance rates, while after complete failure the components are unable to perform their tasks [51].

The *delay block* is defined by three parameters: *In-Block Delay* to represent punctuality in arrivals, *Off-Block Delay* to represent punctuality in departures and *Turnaround Excess Time* to represent punctuality during the airport ground processes (as well as potential Air Traffic Flow and Capacity Management - ATFCM- regulations). Off-Block Delay is defined as the difference between the Actual Off-Block Time (AOBT) and the Scheduled Off-Block Time (SOBT) [53]. In-Block Delay is defined as the difference between the Actual In-Block Time (AIBT) and the Scheduled In-Block Time (SIBT) [53]. Turnaround Excess Time is defined as the difference between the Actual Turnaround Time (AIBT-AOBT) and the Scheduled Turnaround Time (SIBT-SOBT) [53]. These indicators are the most transparent measures regarding airport delays and the ones with the most direct relevance to operations [5]. Note that delays are defined as schedule delays, therefore it may occur that delays are “negative”, meaning early departure/arrival of a flight or process completed ahead schedule. “Negative” delays occur when the schedule is running close to plans and can cause issues for airport operations; e.g., disrupting the sequencing of flights and the allocation of resources (gates, handling equipment), especially during peak hours at busy airports [3]. “Positive” flight delays often cause significant problems for all the involved stakeholders; e.g., they affect operational and financial performance of airports and airlines, schedule adherence and use of resources, passenger experience and satisfaction, and system reliability [3], [54]. Schedule delays are common occurrences in airline and airport operations, given the multiple agents involved, the stochastic nature of operating times, and the unexpected disruptions in tasks [1], [3]. The three possible states of the parameters of the delay block elements are illustrated in TABLE I. We define the thresholds between states (performance ranges) for each element based on operational targets (e.g. the ± 3 min threshold for punctuality set by SESAR’s performance metrics [5] and the 15 min threshold for defining delay that has historically been common in Air Traffic Management [55]–[57]) or based in expert judgement. We interviewed experts from airlines, airport operators, air navigation service providers, regulators and ground handlers; each for a period of one or two days each. The interviewed explained aspects related to procedures and operational interactions and answered questions in semi-structured interviews [58]. The main inputs from experts relate to performance rates and thresholds.

The *capacity block* is defined by four parameters: *Throughput ASMA 60 NM*, *Congestion Index 60 NM*, *Demand/Capacity Balance* and *Departures/Arrivals Ratio*. The spatial boundary of this study is enlarged to a wider context than the airport itself, to consider both the Arrival Sequencing and Metering Area (ASMA) and potential holding patterns. The ASMA is usually defined as a virtual cylinder with a 40 NM radius around the airport, but it can be extended to 100 NM in some analyses [15]. For our case study is particularly important

to consider an ASMA of 60 NM. This is due to the fact that holdings at Adolfo Suárez Madrid-Barajas Airport (LEMD) are located beyond a radius of 40 NM, as depicted in Figure 6.

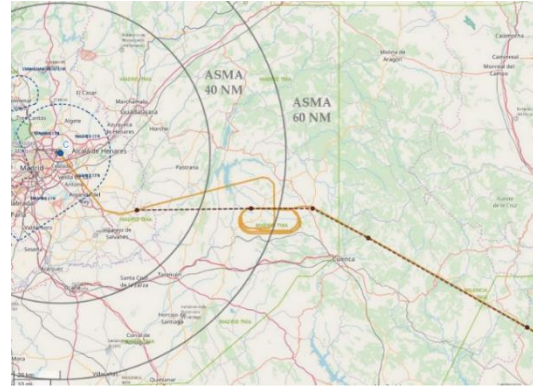


Figure 6. ASMA 40 NM and ASMA 60 NM for LEMD.

Throughput ASMA 60 NM is a theoretical hourly rate based on the truncated 20 min window prior to the arrival of the aircraft to the ASMA. The (theoretical) maximum airport throughput ASMA 60 NM is a determinant of the level of traffic saturation and, thus, the threshold at which effects of congestion can be observed. For each aircraft arriving at the ASMA 60 NM, the number of aircraft that landed in the previous 20 minutes is counted [59]. Congestion Index ASMA 60 NM is the number of arrivals in the queue ahead of the current flight, once it enters the ASMA 60 NM. It is a metric that reflects the level of congestion for the inbound traffic flow. Therefore, these two elements (throughput and congestion index) reflect potential problems related to capacity saturation. The Demand/Capacity Balance (DCB) is defined as the ratio of aircraft landed in the previous hour to the airport’s practical arrival capacity (an operational measure of airport throughput) at this hour. This metric is selected to reflect the importance of scarce arrival capacity in the airport operational performance [60], [61]. The Departures/Arrivals Ratio (DAR) is calculated by confronting the number of departures to the number of arrivals in the previous hour, as a measure to understand the airport’s ability to manage aircraft flows (ability to “absorb” inbound traffic and “produce” outbound traffic). The three possible states of the parameters of the delay block elements are illustrated in TABLE I (throughput and congestion data have been normalized and range from 0 to 1). These thresholds are calculated by analyzing the operational data and using expert judgement.

TABLE I. PERFORMANCE STATES AND THRESHOLDS FOR DELAY, CAPACITY AND ENVIRONMENTAL IMPACT BLOCKS ELEMENTS

States of blocks elements	Elements			
	Time elements (delays)	Throughput and Congestion Index	DCB	DAR
Target state (state 1)	$-3min < d < 3min$	$x \leq 0.2$	$0.8 \leq x \leq 1$	$0.1 \leq x \leq 1.1$
Correct state (state 2)	$-15min < d \leq 3min$ or $3min \leq d < 15min$	$0.2 > x \geq 0.8$	$x < 0.8$	$0.2 \leq x \leq 2.2$
Incorrect state (state 3)	$d \leq -15min$ or $d \geq 15min$	$x > 0.8$	$x > 1$	$x < 0.2$ or $x > 2.2$

The **environmental impact block** aims to include the metrics that reflect emissions of particles and gases (CO₂, water vapor, hydrocarbons, carbon monoxide, nitrogen oxides, sulfur oxides, lead and black carbon) nearby and within the airport, due to airport operations. It can be represented by the extra time invested in aircraft processes related to approach and on-ground operations. That is why this block is modelled by the *Additional Taxi-In Time*, the *Additional Taxi-Out Time* and the *Additional ASMA 60 NM Time*. Additional Taxi-In Time is the difference between the actual taxi-in process time and an estimated unimpeded time to perform the taxi-in operation (depending on the aircraft parking stand and the runway in use). It collects the inefficiencies in the airport taxiways due to traffic congestion or potential incidents. Unimpeded taxi times (flows in the absence of any obstacles) can only be as short as the physics of the process allows, but can grow large in the event of a slow taxi operation [62]. Similarly, the Additional Taxi-Out Time is the difference between the actual taxi-out time and an estimated unimpeded time to perform the taxi-out process (aircraft flow from the parking stand to the runway header for take-off). The Additional ASMA 60 NM Time is a proxy for the average arrival runway queuing time on the inbound traffic flow, during congestion periods at airports. It is the difference between the actual ASMA time of a flight and a statistically determined unimpeded ASMA time based on ASMA times in periods of low traffic demand. The environment block states and performance thresholds are defined in the same way as the delay block states and summarized in TABLE I.

The **complexity block** considers unusual operational situations that introduce complexity into the system. It is represented by four parameters: *Runway Configuration*, *Holdings*, *Season* and *Meteorological Conditions*. Runway Configuration is the layout or design of a runway or runways, where operations on the particular runway or runways being used at a given time are mutually dependent [1]. The operational preferential configuration at LEMD is called north configuration (wind coming from the north). A Holding pattern is a predetermined maneuver which keeps aircraft within a specified airspace while awaiting further clearance from air traffic control [63]. Holding patterns are flown as a delaying tactic and may represent airspace saturation or complex operational situations. Season represents the impact of seasonality and peak traffic periods on airport operations. The Meteorological Indicator considers different variables: cloudiness (height and quantity), visibility, wind (intensity and direction) and special meteorological phenomena (e.g. presence of fog, snow, rain). It ranges from 0 to 7 and it is calculated by weighting the impact of weather elements on the operational conditions of the airport [64]. TABLE II illustrates the states for the complexity block.

TABLE II. PERFORMANCE STATES AND THRESHOLDS FOR THE COMPLEXITY BLOCK ELEMENTS

States of the block elements	Elements			
	Runway Configuration	Number of Holdings	Season (as IATA calendar)	Meteorological Indicator
Normal operation (state 1)	North	$h = 0$	Winter	$m < 3$
Complex operation (state 2)	South	$h \geq 1$	Summer	$m \geq 3$

Once the blocks were characterized, and the states of the elements were established, we determined the different performance states for each block. These were defined by the amount of component failures that lead to the block failure and settled according to expert knowledge. For delay, capacity and environmental impact blocks, the following operational states were considered: (S1) *Optimal* (all parameters in correct or optimal states); (S2) *Correct* (only one parameter in an incorrect state); and (S3) *Incorrect* (two or more parameters in incorrect states). For the complexity block we considered two states: (S1) *Correct* (one or less parameters in complex states) and (S2) *Complex* (two or more parameters in complex states).

The combination of the blocks' states results in 54 possible different states for the global model. This amount of states difficulties the appraisal of the system reliability performance. Therefore, to reduce this number, we used clustering techniques to group states for the global model (associating those states which provide similar operational outcomes). Particularly, we used the Fuzzy c-Means clustering algorithm implemented in MATLAB [65]. The Silhouette criterion gives us the optimal number of clusters. The Silhouette value for each point is a measure of how similar that point is to points in its own cluster, when compared to points in other clusters. The Silhouette value for the i -th point, S_i , is defined as [66]:

$$S_i = \frac{(b_i - a_i)}{\max(a_i, b_i)} \quad (11)$$

where a_i is the average distance from the i -th point to other points in the same cluster (i) and b_i is the minimum average distance from the i -th point to points in a different cluster, minimized over clusters. The Silhouette value ranges from -1 to 1. A high value indicates that i is well-matched to its own cluster and poorly-matched to neighboring clusters. According to the Silhouette criterion the best option is to distribute the system states in 7 clusters (see Figure 7).

Once the groups of states (operational clusters) were obtained, the next step was to assign a performance rate to each Cluster, to order them. We applied the following criterion to weight the influence of each state inside the Cluster:

$$R = \frac{n_1 + 0.5 * n_2}{n_3} \quad (12)$$

where n_1 is the number of optimal states (states S1), n_2 is the number of correct states (states S2) and n_3 is the number of incorrect states (states S3) for each Cluster. TABLE III shows the performance rates of the different performance Clusters.

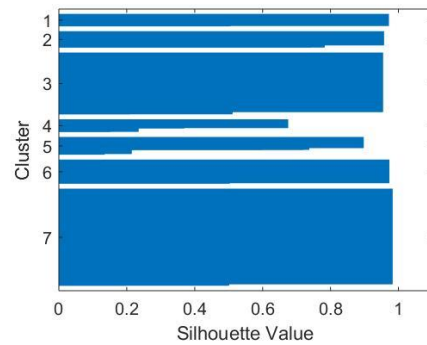


Figure 7. Silhouette value for the chosen number of clusters.

TABLE III. PERFORMANCE RATES FOR THE OPERATIONAL CLUSTERS

R	Cluster						
	1	2	3	4	5	6	7
Rate	0.4659	0.5833	0.6500	0.6458	0.7396	0.8333	0.8958

D. Reliability indicators and model functionalities

This section defines the performance indicators used to describe the system's operational performance after a number of steps (arrival operations) [9]: (i) the mean instantaneous performance (E_n), which represents the performance expectation at a given step; (ii) the mean instantaneous performance deficiency (D_n), which represents the system performance deviation from a given demand (w), at a given step; and (iii) the instantaneous availability (A_n), which represents the probability of finding the system in an acceptable state at a given step. These indicators are related to the fact that in systems with weighted components, each component may contribute differently to the performance of the system. Therefore, the system's working/failure principle depends on the total performance of working/failed components [67].

$$E_n = \sum_{k=1}^N g_k p_k(n) \quad (13)$$

where N is the total number of states, g_k is the performance rate (service level) associated to state k and $p_k(n)$ is the probability of the system being in state k at step n .

$$D_n = \sum_{i=1}^N p_i(n) \max(w - g_i; 0) \quad (14)$$

where N is the total number of states, $p_i(n)$ is the probability of the system being in state i at step n , w is the expected performance rate for the system and g_i is the actual performance rate associated with state k .

$$A_n = \sum_{i=1}^K p_i(n) \quad (15)$$

where k is the total number of states with acceptable performance rates and $p_i(n)$ is the probability of the system being in state i at the step n .

If we apply these definitions to the values of the stationary distribution, we can obtain the asymptotic values in the long term ($E_\infty, D_\infty, A_\infty$).

E. System dynamics

The system dynamics methodology models the dynamical behavior of a system over time (or over operational steps in our case), by analyzing the relationships between the different elements of the system. As mentioned above, an MSS has different levels of reliability. Therefore, the system and its components can transit to various performance states during its functioning periods. The MSS transits from higher performance states to lower states with failure rate λ , transits from lower states to higher states with repair rate μ and maintains the same state with stabilization rate γ [10]. The probability of being in each state (j) at step n for component i ($P_{ij}(n)$) is obtained from Chapman-Kolmogorov equations [68]:

$$\left\{ \begin{array}{l} \frac{dP_{i0}(n)}{dn} = \sum_{j=1}^M \lambda_{j0}^i P_{ij}(n) - \sum_{j=1}^M \mu_{0j}^i P_{i0}(n) \\ \quad + \gamma_0^i P_{i0}(n) \\ \quad \vdots \\ \frac{dP_{ik}(n)}{dn} = \sum_{j=k+1}^M \lambda_{jk}^i P_{ij}(n) + \sum_{j=0}^{k-1} \mu_{jk}^i P_{ij}(n) \\ - P_{ik}(n) \left(\sum_{j=0}^{k-1} \lambda_{jk}^i + \sum_{j=k+1}^M \mu_{jk}^i \right) + \gamma_k^i P_{ik}(n) \\ \quad \vdots \\ \frac{dP_{iM}(n)}{dn} = \sum_{j=0}^{M-1} \mu_{jM}^i P_{ij}(n) - \sum_{j=0}^{M-1} \lambda_{Mj}^i P_{iM}(n) \\ \quad + \gamma_M^i P_{iM} \end{array} \right. \quad (16)$$

where λ_{jk}^i and μ_{jk}^i are the failure and repair rates from state j to state k for component i respectively; and γ_k^i is the stabilization rate (transition probabilities as depicted in the example of Figure 4). In addition, Equation (4) must be fulfilled at any operational step, and the initial conditions have to be appraised with regard to the starting operation for the airport.

V. RESULTS

A. Transition matrix for the airport operational system

The transition matrix of the global system is depicted in TABLE IV. This matrix represents the one-step transition probabilities between system states – Clusters (C). The system tends to move towards Cluster 7 for every initial state, except for Clusters 2 and 5, for which the system tends to stay in the same cluster. This means that the airport system shows a tendency towards the state with a highest performance rate (C7); i.e. the airport system is repairable.

TABLE IV. TRANSITION MATRIX P FOR THE GLOBAL SYSTEM

	C 1	C 2	C 3	C 4	C 5	C 6	C 7
C 1	0,117	0,058	0,088	0,258	0,050	0,093	0,335
C 2	0,072	0,414	0,062	0,022	0,381	0,011	0,039
C 3	0,096	0,047	0,103	0,224	0,056	0,085	0,389
C 4	0,074	0,005	0,059	0,318	0,004	0,112	0,427
C 5	0,074	0,384	0,074	0,023	0,400	0,009	0,037
C 6	0,067	0,006	0,060	0,292	0,004	0,120	0,452
C 7	0,063	0,004	0,066	0,271	0,005	0,114	0,478

Given a particular initial state (operational conditions) and the number of steps, the transition matrix allows us to locate the airport system in the most probable operational cluster. This reliability model illustrates the system evolution and provides a framework to detect the need of corrective measures. For the model construction and testing, data was divided into two partial datasets. The first one was used to train and validate the model, with 80% of the total number of observations (building and cross-validation sample); while the remaining 20% (test sample)

was used to test the generalization of the model. Therefore, the process is as follows: (i) randomly split the initial dataset into construction/building and testing sets; (ii) perform cross-validation on the construction set to fit the model (k-fold with k=10) [69]; and (iii) test if results are generalizable, using a test set, which is completely separated from model development. Both the train and the error scores present an average value of 10%, i.e., our model predicts new observations as well as it fits the original dataset. Therefore, we are not overfitting the model and results can be generalizable. By assessing the importance of each block on the system’s operational behavior, we obtain that delayed operations and situations when processes need additional time (delay and environmental blocks) reduce the airport ability to maintain optimal performance rates. Consequently, delays are major drivers for airport dynamics and reduce the system ability to recover itself.

B. Steady state behaviour

To describe the system’s operational response and evolution, we appraised a particular scenario, as an example of the model applicability. We analyzed the system behavior when the initial state is $(P_0 = [\frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}, \frac{1}{7}])$, and hence it is equally probable to find the airport operating in all the Clusters (random initial operational state). The system reaches the steady state in 30 steps – arrival operations (with a maximum error of the estimation of 0.151%). The stationary state vector is as follows: $X = [0.0733, 0.0525, 0.0679, 0.2551, 0.0513, 0.0998, 0.4002]$. Therefore, the most probable outcome for the steady state behavior of the system is Cluster 7 (40.02%), which shows a reliable nature of the system. The values for the stationary distribution of the performance indicators are: 75.32% for the mean performance (E_∞), 0.25% for the mean performance deficiency (D_∞) and 87.42% for the mean availability (A_∞). We have considered C4, C5, C6 and C7 as the Clusters with acceptable performance rates and $w = 50\%$ as the expected performance rate for the system. The Markov chain for the airport operational dynamics (see TABLE IV) is *irreducible* (it is possible to get to any state from any state) and *aperiodic* (any return to the previous state can occur in just one transition step). A Markov chain is *ergodic* if it is both irreducible and aperiodic [8], [52], [70]. By the Perron-Frobenius Theorem [71], ergodic Markov chains have unique limiting distributions; i.e., they have a unique stationary distribution to which every initial distribution converges. Therefore, the stationary distribution is a long-term behavior indicator of the system.

C. Evolution of performance indicators and state probability

Figure 8 shows the evolution of: (a) the probability of the states, (b) the mean instantaneous performance (E_n); (c) the mean instantaneous performance deficiency (D_n); and (d) the mean instantaneous availability (A_n) towards the steady state (stationary distribution). These graphs characterize the system behavior. The system is repairable, as it increases its performance and reduce its deficiency over time (with steps). Availability is a performance criterion for repairable systems that accounts for both the reliability and maintainability properties of a system. It is defined as the probability that the system is operating properly when it is requested for use. In the case of the airport system, it evolves towards a value of 87.42%.

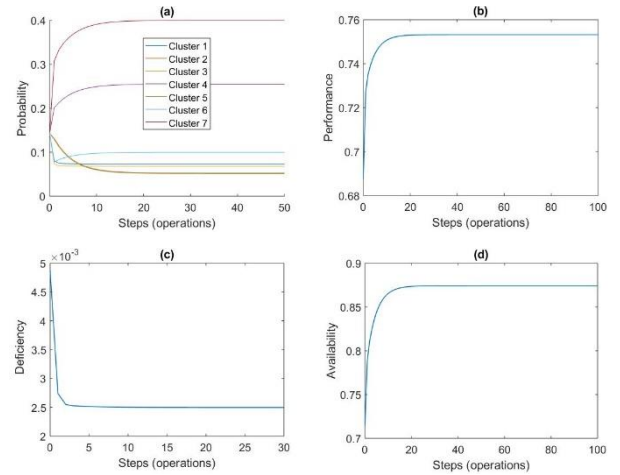


Figure 8. Evolution of (a) states’ probability; (b) E_n ; (c) D_n ; and (d) A_n .

VI. CONCLUSIONS & FUTURE WORK

This paper develops a reliability analysis of the airport transit view process. In this analysis, we use a Multistate system approach, through a Markov-Chain model. Four main elements of the system are used to appraise its behavior: delay, capacity, environment and complexity, with different partial variables. Performance thresholds and states were assigned according to operational targets, data distributions and expert knowledge. Moreover, the global system states were clustered to reflect similar operational characteristics. The application of the model to a case of study of 160,460 turnaround operations (registered at 2016) is used to describe the system operational behavior. The performance indicators allow us to characterize the system’s dynamics and obtain predictions of its most likely evolution. Results show that the airport system tends to a stationary distribution with the highest level of performance achievable, reaching an availability of 87.42%. Therefore, the system is repairable. Delays are major drivers for airport performance dynamics and reduce the ability of the system to recover itself. This reliability model represents a framework to test different ‘what-if’ scenarios and to reduce uncertainty by categorizing different behavior patterns. Therefore, it allows us to predict how probable is for the system to enter a degraded state and highlights the system’s ability to maintain performance (or recover tolerable rates) under varying conditions. This approach marks an evolution from the traditional corrective view regarding reliability analysis (act when the system is degraded) to a predictive perspective (act when the system is providing signs of malfunction, before it reaches degradation). The model allows both a comprehensive view of the airport system and a holistic and performance-based management of the day of operations, enabled by predictions for the future system states.

Future work will focus on improving the accuracy of the model (more complete testing data and methodological enhancements – like weighting the parameters’ influence when defining block states), and on comparing the results when the methodology is applied to other airports (generalization of the case study). We also need to analyze potential response strategies/measures (how specific actions impact the airport system state) and consider other performance areas as safety and financial results.

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