Thirteenth USA/Europe Air Traffic Management Research and Development Seminar (ATM2019)

Alternative Resource Allocation Mechanisms for the Collaborative Trajectory Options Program (CTOP)

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Abstract—**In this paper, we identify two weaknesses in the design of the collaborative trajectory options program (CTOP) traffic management initiative. First, CTOP may issue excessive quantities of delay even when the parameters of the program are chosen correctly. Second, CTOP's current design can discourage airlines from accurately disclosing trajectory options. We propose new mechanisms that address these design flaws. We also provide computational results that demonstrate that our proposed mechanisms would reduce delay costs and encourage greater participation in CTOP.**

Keywords-collaborative trajectory options program; traffic management initiatives; air traffic flow management.

I. INTRODUCTION

A new traffic management initiative (TMI), the collaborative trajectory options program (CTOP) was developed and deployed in the U.S. over the past eight years. Initial development and deployment occurred between 2010 and 2013 and deployment under the TFMS system occurred in 2014. CTOP is strongly based on Collaborative Decision Making (CDM) principles [1], [2], [3]. A CTOP program starts with the identification of one or more flow constrained areas (FCAs) for which capacity-demand imbalances are anticipated. Flight operators may provide alternative trajectories through and around the FCAs together with information that allows the airspace manager to choose among them taking into account flight operator priorities and current conditions. The final CTOP tools and concepts evolved from earlier version developed under the name, SEVEN, System Enhancements for Versatile Electronic Negotiation [4], [5].

It is safe to say that CTOP has not lived up to its potential. In particular, it has received limited use since its introduction [6]. Potential reasons for this include its flexibility and complexity, which together lead to a significant overhead in its application in a particular environment. Here the overhead involves a substantial burden both on the FAA and airlines. Another potential reason is the lack of a clear benefit case for specific application settings. Multiple research projects and systems modifications are underway to address these challenges. The research presented here grew out of one of those projects.

CDM in the U.S. originated with the planning and control of ground delay programs (GDPs). GDPs involve a single traffic management action, namely delaying the departure of flights in order to avoid congestion in the air. Additionally, airlines may initiate the further action of canceling flights. CTOP seeks to manage the combined use of ground delay and reroutes. A previously implemented U.S. TMI, the airspace flow program (AFP), also addresses these two control actions but with a more limited set of airline control features.

There is a rich literature that simultaneously considers the assignment of ground delay and route choice. Included are optimization models capable of solving very large instances, e.g. [7], [8] and models that evaluate the basic tradeoffs for a single flight in the presence of uncertainty [9], [10]. Also of relevance are models that identity viable route choices through convective weather conditions [11], [12]. This prior research treats problems strictly from an air navigation service provider (ANSP) or flight operator perspective. CTOP models the ANSP-airline shared decision-making responsibilities. There is limited research that considers the collaborative nature of the problem. For example, [13] takes the structure provided by an AFP as a starting point and develops an optimization model that employs user-supplied information to make slot allocation and rerouting decisions.

There has been research that studies specific elements of the CTOP mechanism. Some existing work studies the strategies which airlines should use to select their alternative trajectories, taking into account the inter-dependence of such strategies among airlines [14], [15], [16]. Similarly, there is work that studies the problem of selecting relative trajectory costs (RTCs) in a CTOP program [17]. Alternatively, statistical models have been used to predict the RTCs that airlines would select [18]. The problem of deciding the rate at which flights should be admitted to each FCA in a CTOP program has also been studied [19], [20]. While this aforementioned research studies how to plan or respond to a CTOP program, we are aware of only two works that propose improvements to the allocation mechanism itself [17], [21].

In this paper we analyze a basic version of the CTOP resource allocation mechanism and show that it is possible that CTOP's performance can fall far below the best possible, even when the rate at which flights are admitted into a resource perfectly matches the capacity of that resource. We also show that it can be beneficial for airlines to strategically manipulate the information they provide, specifically by offering no alternative trajectory options, even though some are possible and desirable under reasonable conditions, i.e. "gaming" the process can be profitable. This is reflected in earlier works that study optimal flight operator strategies [14], [15], [16]. Rather than attempting to identify how the airlines will respond to these incentives, we attempt to improve the mechanism itself in a way that encourages airlines to provide accurate trajectory information. We follow up our analyses, which are based on "stylized" models and examples, by developing specific, practical modifications to the CTOP resource allocation mechanism. We conduct a simulation study that shows that these modifications can produce substantial improvements over the current version of CTOP.

The research conducted in [21] is similar in spirit to that presented here. The existing work also identifies a potential area of improvement in the CTOP mechanism. In particular, the authors demonstrate that serving flights in random order can lead to lower costs than serving flights in the order that they are scheduled. The opportunities that we identify are conceptually distinct. In particular, we demonstrate that the current CTOP mechanism does not accurately reflect the benefits of rerouting a flight, and our proposed mechanism reduces costs by remedying this problem.

While the work presented in [17] focuses on the problem of setting RTCs within the current CTOP framework, the authors of that work also propose a modification to the CTOP scheme that would allow airlines to specify delay costs as piecewise linear functions. This would allow airlines to specify their preferences more precisely, since the current CTOP allocation scheme assumes that delay costs are a linear function of delay. For the purposes of this paper, we will make the assumption that delay costs are linear. The improvements that we discuss could potentially be combined with modifications similar to those proposed in [17]. We leave this as an area of future research.

II. CTOP Allocation Procedure

A. Inputs Required for CTOP

In order to run the CTOP allocation procedure, the FAA must decide which time slots are available for each FCA. Each airline must declare a trajectory option set (TOS) for each affected flight. The TOS declares several trajectories that the flight could take. We will refer to each flight's original, scheduled trajectory as the "primary" trajectory, while any other trajectories in the TOS will be referred to as alternative trajectories. Airlines must also provide a relative trajectory cost (RTC) for each trajectory. The intention behind the RTC is as follows. Suppose that the cost incurred by ground delay assigned to a flight increases linearly with the amount of ground delay taken and does not depend on the route that the flight takes. Further suppose that for each route in the flight's TOS there is a fixed cost associated with taking that route that does not depend on the ground delay. Under these assumptions, for each route there is a quantity r such that the fixed cost associated with the route is equal to the cost of taking r minutes of ground delay. The CTOP procedure operates on the assumption that the RTC is set equal to this quantity r.

For example, suppose that a flight has two trajectories that it can take. The first trajectory is a direct route that passes through an FCA. The second trajectory is a route that avoids the FCA, but requires an additional 10 minutes of flight time. Suppose that for this particular flight, an extra minute spent in the air is worth roughly two minutes spent on ground. Then the cost of taking the second route is equivalent to the cost of taking 20 minutes of ground delay. The airline can choose to include both of these routes in the TOS of this flight, or can choose to include only one of these two routes. When both routes are included, the intended manner of setting RTCs would be to set the RTC of the direct route to zero while setting the RTC of the less direct route to 20. In practice, airlines are free to select RTCs in any manner, and existing research indicates that it may be beneficial for airlines to select RTCs by a method that does not align with the intended interpretation [17].

B. Running CTOP

When a CTOP program is declared, each flight that would access the affected resources is assigned an initial arrival time (IAT). This would be the time at which the flight would first encounter an FCA according to that flight's declared schedule. Flights are then processed one-at-a-time in order of their IAT, beginning with the flight whose IAT is earliest. For each flight, an adjusted cost is calculated for each route in that flight's TOS. This adjusted cost is calculated as follows. Let s be the earliest usable slot at the first restricted resource encountered along the route. Let d be the minutes of ground delay that the flight would take were it assigned to slot s. The adjusted cost is the sum of d and the RTC of the route. In some cases, an alternative trajectory may avoid all restricted resources. In this case, the adjusted cost of the trajectory is simply the RTC of the route. The flight is assigned the slot associated with the route that has the lowest adjusted cost. If the trajectory with the lowest adjusted cost does not pass through a restricted resource, then the flight is given no slot. Note that if the RTC is defined in the intended manner described in Section II.A, then the adjusted cost is the true cost expressed in terms of an equivalent quantity of ground delay. We will refer to costs represented in this way as Ground Delay Equivalents (GDEs).

C. Airline Response Actions

After a CTOP program has been run, airlines are free to perform intra-airline substitutions, in which a slot assigned to some flight can be exchanged with a slot assigned to a different flight owned by the same operator. Airlines can also choose to reroute flights in order to avoid the FCAs and can choose to cancel flights. In these cases, they surrender the slot associated with the flight. Naturally, intra-airline substitutions can be combined with reroutes or cancellations. For example, suppose that an airline has two flights, Flight A and Flight B. Suppose that Flight A is given an earlier time slot than Flight B, but Flight B is the more crucial flight and can also make use of the earlier slot. The airline may choose to swap the slots of Flight A and

Flight B, and then may choose to reroute Flight A along an alternative trajectory, avoiding the excess ground delays associated with the later slot assignment (although likely incurring costs associated with taking a longer route).

III. Opportunities For Improvement In CTOP

We identify two types of ways in which the CTOP allocation procedure could be improved. First, we believe that it is possible to produce allocations with much lower delay costs than those produced under the current CTOP allocation procedure. Second, we believe that there are often situations in which the current procedure discourages airlines from declaring alternative trajectories.

A. Continuous CTOP Model

In order to demonstrate the potential reductions in delay costs, we present a continuous approximation of a CTOP program. Suppose that a continuous stream of flights is scheduled to approach an FCA at a rate of R flights per minute. Define:

- $NF(t)$ to be the number of flights that are scheduled to reach the FCA boundary within t minutes,
- $T(k)$ to be the time at which the k^{th} flight reaches the FCA boundary.

It is easy to see that

$$NF(t) = Rt$$

and

$$T(k) = \frac{k}{R}.$$

For example, when the scheduled arrival rate R is equal to 0.5 flights per minute, then

$$NF(60) = 0.5 * 60 = 30$$

which is to say that 30 flights would be scheduled to reach the FCA within the 60 minutes, and

$$T(60) = \frac{60}{0.5} = 120$$

i.e., it would take 120 minutes for the 60$^{\text{th}}$ flight to reach the FCA.

Suppose that there is a reduction in the rate at which the FCA can process flights, which will lead to the initiation of a CTOP. Let a (with $0 < a \leq 1$) be the rate reduction factor so that the rate at which the FCA can process flights is now given by aR. Suppose that flights are admitted to the FCA at the correct rate aR, so that none of the delay incurred in this example is due to misspecification of the admittance rate. Let:

- $T'(k, a)$ be the time at which the kth flight reaches FCA boundary assuming that the flights arrive according to the rate aR (that is, assuming that no flights are rerouted).
- $d(k, a)$ be the delay incurred by the kth flight assuming that flights arrive according to the rate aR (again, this assumes that no flights are rerouted).

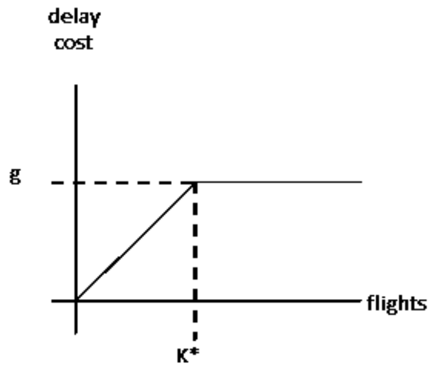

Figure 1. Graph of CTOP delay cost function.

Note that

$$T'(k, a) = \frac{k}{aR}$$

and

$$\begin{aligned} d(k, a) &= T'(k, a) - T(k) \\ &= \frac{k}{aR} - \frac{k}{R} \\ &= \frac{(1-a)k}{aR}. \end{aligned}$$

For example, when the arrival rate R is equal to 0.5 flights per minute and the rate reduction factor a is equal to 2/3 then the delay taken by the 40$^{\text{th}}$ flight is given by

$$\begin{aligned} d\left(40, \frac{2}{3}\right) &= \frac{\left(\frac{1}{3}\right) * 40}{\left(\frac{2}{3}\right) * 0.5} \text{ minutes} \\ &= 40 \text{ minutes.} \end{aligned}$$

Then, the total delay for K flights without CTOP is:

$$\begin{aligned} TD(K, a) &= \int_0^K d(k, a) dk \\ &= \frac{(1-a)K^2}{2aR} \end{aligned}$$

Under CTOP, when a flight's delay gets large enough, it may opt to take an alternative trajectory. We assume that all flights have a single alternative trajectory that does not pass through the controlled region, and all flights declare the same RTC g for their alternative trajectory. We will assume that the RTC for the primary trajectory is always zero. Let:

- $d'(k, a, g)$ be the delay cost for the kth flight under a CTOP when the rate reduction factor is a and all flights have an RTC of g.

It is easy to see that:

$$d'(k, a, g) = \min\left\{\frac{(1-a)k}{aR}, g\right\}.$$

We now compute total delay cost with CTOP. First we define:

- K^* to be the flight index at which a flight first switches to an alternative trajectory.

Such a switch occurs when ground delay is equal to the RTC g, so that:

$$\frac{(1-a)K^*}{aR} = g,$$

which implies that

$$K^* = \frac{gRa}{(1-a)}.$$

Then the total delay cost with CTOP is given by:

$$TD(k,a,g) = \begin{cases} TD(k,a) & \text{for } k \leq K^*, \\ TD(K^*) + g(K-K^*) & \text{for } k \geq K^*. \end{cases}$$

This function is illustrated in Figure 1.

It is informative to understand exactly how flights incur their cost as one moves from left to right in Figure 2. The early flights (prior to K^*) incur increasing ground delay. None of these flights would choose to go to its alternative trajectory. Flight K^*, by definition, uses its alternative trajectory. The GDE for that flight is g. As one moves beyond K^*, some flights use their primary trajectory and proceed through the FCA after incurring g minutes of ground delay, whereas others use their alternative trajectory and incur the same GDE cost.

We can now compute the delay savings from CTOP for processing K flights:

$$DS(K,a,g) = \begin{cases} 0 & \text{for } K \leq K^* \\ TD(K-K^*,a) & \text{for } K \geq K^* \end{cases}$$

This function is illustrated in Figure 2.

Figure 2 could perhaps be deceptive in that for large K, the savings (blue triangle) could be much larger.

We now discuss an intriguing implication of this analysis. Note that after K^* all flights incur the same cost equal to g. Basically, the CTOP allocation creates a queue of flights that are waiting to be assigned to a slot. Once this queue reaches g minutes of ground delay, all flights either receive g minutes of ground delay or use their alternative trajectory, which has a GDE cost of g. Suppose it was recognized at the outset that there would be a long period of time where such process would be at work. For illustrative purposes, suppose one half of the flights are assigned g minutes of ground delay while the remaining half take their alternative trajectories. This would occur if the (reduced) FCA capacity were exactly half the demand placed upon it. A "deal" could be struck where, at the outset, every other flight would be placed on its alternative trajectory and the remaining flights would remain on their primary trajectories. Under such a deal, the flights using their primary trajectories would incur no delay at all since the rate of arrival of these flights would be equal to the (reduced) capacity of the FCA. Ignoring the flights before K^*, this process would cut the total delay cost in half, since the flights using their alternative trajectory would incur the same GDE cost as before but the remaining flight would incur no ground delay and no rerouting cost.

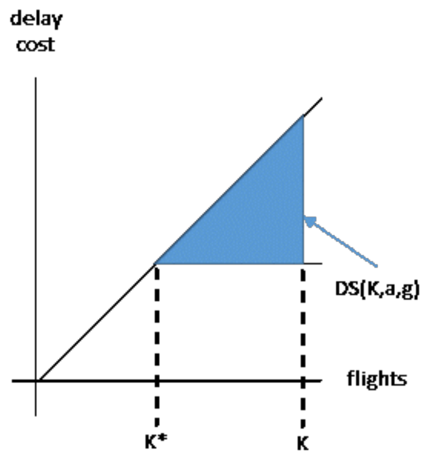

Figure 2. Graph of CTOP delay cost function.

Of course, there are many practical issues in making such a "deal" workable. The TOSs of individual flights vary, each carrier will have varying numbers of flights and these will be distributed over time in different ways. CTOP is quite robust to dynamic changes in FCA capacity over time, whereas this analysis required a long period of substantially reduced capacity. However, let us note an essential feature of this deal: some flights are routed on an alternative trajectory when this individual flight would incur less GDEs if it were routed on its primary trajectory. While this is suboptimal for each of these flights as an individual, the reroute produces benefits for other flights that are waiting in the queue. The downstream benefits of this reroute outweigh its immediate cost. This suggests that we could improve the CTOP procedure by factoring these downstream benefits into our decision of which trajectory to select.

B. Incentives in CTOP

There can be situations in CTOP in which an airline would be better off if it did not announce an alternative trajectory for one of its flights. To give an example, suppose that some airline owns two flights: flight f_1 and flight f_2. Suppose that the flight f_1 has two possible trajectories, where the primary trajectory passes through the region controlled by CTOP and where the alternative trajectory avoids the CTOP. If the airline does not declare this alternative trajectory in the CTOP program, then the flight f_1 will receive some slot s_1; suppose that the flight f_2 can make use of this slot. Further suppose that the resulting adjusted cost of the primary trajectory is higher than that of the alternative trajectory for flight f_1. In this case, if the airline announces the alternative trajectory, the flight f_1 will be rerouted and the airline will lose the slot. On the other hand, if the airline does not announce the alternative trajectory, it will keep the slot. Then, it can perform an intra-airline swap to give slot s_1 to flight f_2 and it can reroute the flight f_1 itself. This produces the same result for flight f_1 as CTOP, while likely providing a better result for the flight f_2 than CTOP would. In this example, the airline would be better off if it did not declare the alternative trajectory for its flight. Furthermore, this is not an atypical example, since

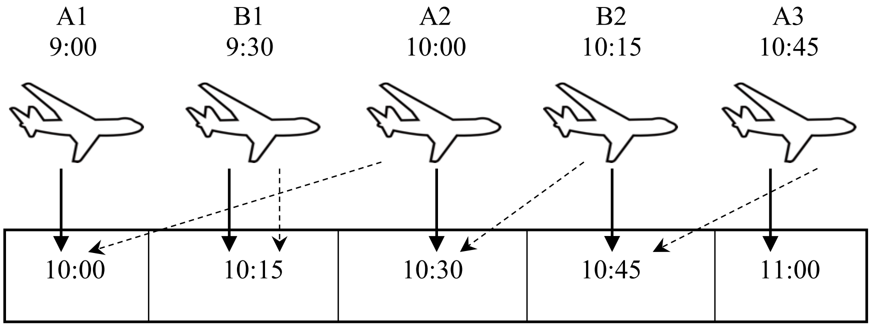

Figure 3. Example of compression algorithm; flight A1 is canacelled.

airlines often have later flights that could make use of the slots given to its earlier flights. This is a very simple example, but is based on the same principles underlying the analysis in [14], [15], [16].

IV. Alternative Allocation Methods

We examine several alternative allocation methods that could be used in a CTOP. For sake of simplicity, we focus on the case in which there is a single impacted region, and we assume that each flight has one primary trajectory that passes through the impacted region and one alternative trajectory that avoids the impacted region. We will assume that airlines have the option not to declare this alternative trajectory within the CTOP mechanism, in which case the flight will always be assigned to its primary trajectory. Unlike in the prior section, we allow an arbitrary flight arrival process and arbitrary RTCs. We believe that the concepts presented here could be generalized for application to any CTOP program, e.g. with multiple FCAs, although it will require further work to produce such a generalization.

A. General Schema for Allocation Methods

All of the methods that we examine can be described by general schema:

1. Produce a tentative schedule of flights to slots in a first-come-first-serve basis in order of IAT. This is the allocation that would occur in current air traffic management practices if all flights declared only their primary trajectory. This schedule will be adjusted by the allocation mechanism as flights are rerouted.
2. Initialize F to be the set of all flights involved in the CTOP program. In each iteration of the procedure we finalize the slot assigned to one flight of F.
3. If no more flights remain in the set F, then stop. Otherwise, let f be the flight in F with the earliest assigned slot in the current allocation, and remove this flight from F.
    a. Estimate whether it would be more beneficial for the airline that operates f to send flight f along its primary trajectory or whether it would be more beneficial to send the flight f along its alternative trajectory.
    b. If it is determined that the flight f should be sent along its alternative trajectory (and if the airline has announced such a trajectory), then f vacates the slot it currently occupies. The tentative allocation for the remaining flights of F is adjusted to make use of this slot.
    c. If it is determined that the flight f should be sent along its primary trajectory (or if the airline did not announce an alternative trajectory), then it is assigned to the slot it currently occupies. This assignment will not be changed in later iterations, and no adjustment to the tentative schedule is necessary.

There are two ambiguous steps in this schema. In step 3.a., we do not define how the decision is made whether to allocate the flight to its primary or alternative trajectory. Likewise, we have not defined how the schedule is adjusted in step 3.b. The mechanisms that we examine differ in these details, and we will specify them for each mechanism.

Note that the CTOP allocation procedure can be expressed in this schema. In this case, step 3.a. would be conducted by comparing the adjusted costs of the trajectories of the flight. Step 3.b. would be conducted by advancing each flight remaining in F to the next slot.

In all of the alternative mechanisms that we consider, step 3.b. will instead be implemented by using the compression algorithm The compression algorithm is currently used in GDP planning to reallocate slots that are vacated when an airline cancels a flight (see [1], [2], [3]). This was designed to avoid disincentives that would prevent airlines from sharing cancellation information. We believe that it could be beneficial in the context of the CTOP by removing incentives that would discourage airlines from disclosing alternative trajectories for their flights. Its strategyproofness properties were studied in [22], which found that while the compression algorithm is immune to certain types of strategies, it is susceptible to certain others. In the same existing work, an allocation method was proposed that exhibits some beneficial game-theoretical properties that the compression algorithm does not (specifically, the proposed allocation method always produces an allocation in the weak core). Here, we chose to use the compression algorithm rather than the more recently developed mechanism due to the established use in practice. Future work could examine the potential benefits of using more sophisticated alternatives.

For the sake of brevity, we do not provide a detailed definition of the compression algorithm; such a description can be found in existing sources such as [22]. In essence, when the compression algorithm adjusts a schedule to fill a slot, it gives higher priority to the airline whose flight occupied the vacated slot. Figure 3 provides a simple example. In this example, there are two airlines A and B. Airline A has three flights: A1, A2 and A3; airline B has two flights: B1 and B2. The scheduled arrival time of each flight is listed below the name of the flight. The time within each box represents a slot. The solid arrows represent the original allocation of the flights (before the cancellation), while the dashed arrows represent the new slot allocation after flight A1 is cancelled and compression is applied. The steps used to produce this allocation were as follows. When flight A1 is cancelled, the 10:00 slot becomes available. Since airline A owns this flight, the compression algorithm attempts first to fill this slot with a flight from airline A. For this reason, the compression algorithm reassigns flight A2 to the 10:00 slot. This in turn frees the 10:30 slot. Again, the compression algorithm attempts to fill this slot with a flight from airline A. However, there are no remaining flights from airline A that can fill this slot (flight A3 cannot use this slot because its original time of arrival is later than 10:30), so the compression algorithm instead fills it with the next available flight, which is flight B2. Similarly to before, this frees the 10:45 slot, and the compression algorithm attempts to fill this slot with a flight from airline A. Here, flight A3 can be assigned to the 10:45 slot.

While the inclusion of the compression algorithm in step 3.b. is used to address the incentive compatibility problem discussed in III.B, each of our five proposed mechanisms employ alternate strategies for improving overall CTOP performance, i.e. addressing the problem illustrated in IIIA. The alternative mechanisms differ in the implementation of the step 3.a. that determines whether the flight will be sent along its primary trajectory or its alternative trajectory. Three of these mechanisms make use of the same information that is used by the current CTOP allocation procedure. However, the last two require that the flight operators provide additional information, which would increase the information exchange requirement of CTOP. In the first of these mechanisms, which we will refer to as the "Difference in Adjusted Costs" method, step 3.a. is conducted in the same fashion as the CTOP procedure (but this mechanism is different from CTOP due to the use of compression in 3.b). All of the other mechanisms consider the substitution of earlier flights into the slot under consideration when deciding whether to reroute a flight or not. In this way they seek to take advantage of the potential savings illustrated in III.A.

B. Difference in Tentative GDEs Method

Note that when a flight is rerouted, later flights may receive an earlier assignment. Since the current CTOP procedure does not consider these benefits, it may overlook reroutes that would be desirable to an airline once the downstream effects are taken into account. Our second method, which we refer to as "Difference in Tentative GDEs" method attempts to approximate these benefits. We define v_1 to be the total GDEs incurred by the airline operating flight f in the current allocation, and we define v_2 to be the total GDEs incurred by the airline operating flight f were the flight allocated to its alternative trajectory and compression applied. The flight is assigned to its alternative trajectory if and only if v_2 is smaller than v_1.

The "Difference in Tentative GDEs" method provides a more accurate estimate of which trajectory is more beneficial to the airline than using the adjusted cost of the individual flight alone. However, this method may actually overestimate the benefits of sending a flight along an alternative trajectory. This is because when this procedure decides whether a flight should be assigned to its primary or alternative trajectory, all later flights are still tentatively assigned to their primary trajectory. Then, if a slot vacates, all of these flights could potentially receive adjustments in the tentative schedule. The "Difference in Tentative GDEs" method would take all of these adjustments into account. However, some of these later flights may take an alternative trajectory and would receive no benefits if a slot is vacated.

C. Difference in Optimal GDEs Method

In order to more accurately estimate the benefits of routing a flight along its alternative trajectory, we propose a third method, which we refer to as the "Difference in Optimal GDEs" method. For a given set of flights F' and a given set of slots S', we let $G(F',S')$ be the minimum amount of GDEs that can be incurred when each flight from F' is assigned a trajectory and a slot s from S' is given to each flight that takes its primary trajectory. This value can be identified by solving an optimization problem. More specifically, we define decision variables:

- x_f – binary variable; takes a value of 1 if flight f takes its primary trajectory, 0 otherwise.
- y_{fs} – binary variable; takes a value of 1 if flight f is assigned to slot s, 0 otherwise.

Let r_f be the adjusted cost of the alternative trajectory of flight f (this would simply be the RTC of that trajectory). Let c_{fs} be the adjusted cost associated with assigning the flight f to the slot s on its primary trajectory. Then, the assignment that minimizes the total GDEs is given by:

$$\min \sum_{f \in F'} r_f x_f + \sum_{f \in F'} \sum_{s \in S'} c_{fs} y_{fs}$$

subject to:

$$x_f + \sum_{s \in S'} y_{fs} = 1 \quad \text{for } f \in F'$$

$$\sum_{f \in F'} y_{fs} \leq 1 \quad \text{for } s \in S'$$

$$x_f, y_{fs} \in \{0,1\}$$

Now, let F' be the set of flights associated with the airline that operates the flight f, and let S' be the set of slots currently associated with this airline (including both tentative slots and slots that have been allocated in previous iterations of the procedure). Let S'' be the set of slots that the airline will have if the flight f is assigned to its alternative trajectory and compression is applied. Then, we allocate the flight f to its alternative trajectory if and only if $G(F',S')$ is greater than $G(F',S'')$. Note that while both of these optimization problems produce a tentative allocation for an airline, neither of these allocations are directly applied. These allocations are used solely to determine on which trajectory to allocate the flight and

TABLE I. RESULTS WHEN AIRLINES ARE TRUTHFUL

Method	Average Per-Flight Costs Before Airline Reallocation						Avg. Per-Flight Costs After Airline Reallocation					
	Ground Delay	***Route Cost***	***GDEs***	***Wtd. Ground Delay***	***Wtd. Route Cost***	***Wtd. Total GDEs***	***Ground Delay***	***Route Cost***	***GDEs***	***Wtd. Ground Delay***	***Wtd. Route Cost***	***Wtd. Total GDEs***
Diff. Optimal GDEs	2.4	18.6	21.0	2.4	18.6	21.0	3.4	16.0	19.5	3.6	14.1	17.7
Diff. Tentative GDEs	1.8	19.2	21.0	1.8	19.2	21.0	2.9	17.4	20.4	3.2	15.5	18.7
Diff. Adjusted Costs	10.5	15.3	25.8	10.4	15.3	25.8	5.7	17.1	22.8	5.7	15.5	21.2
Diff. Wtd. Opt. GDEs	2.6	18.8	21.4	2.7	17.6	20.2	3.5	16.2	19.7	3.6	13.5	17.1
Diff. Wtd. Tentative GDEs	1.8	19.3	21.1	1.8	18.4	20.2	2.8	17.5	20.3	3.0	15.3	18.3
CTOP	14.8	12.6	27.4	14.7	12.7	27.4	8.4	15.1	23.4	8.4	13.4	21.8
RBS	185.8	0.0	185.8	185.8	0.0	185.8	4.9	20.1	25.1	5.0	18.4	23.5
Optimal GDEs	1.3	14.1	15.4	1.3	14.1	15.4	1.6	14.7	16.3	1.6	13.1	14.7
Optimal Wtd. GDEs	2.0	15.0	17.0	1.6	11.4	13.0	2.0	15.0	17.0	1.6	11.4	13.0

otherwise have no effect on the allocation provided by the mechanism.

D. Methods with Flight Weights

The aforementioned methods assume that some quantity of ground delay taken by any flight is worth the same as that taken by any other flight. In practice, some flights may be more or less valuable. If it were possible to solicit information concerning the relative value of each flight, then in step 3.b. it would be possible to more accurately judge the benefits provided by each trajectory option. Suppose that for each flight f, we also had an associated weight $w(f)$ that specifies the relative value of this flight to an airline. We assume that this is proportional to the cost that the airline would incur if the flight took a unit of ground delay. For example, if a per-unit cost of ground delay taken by flight f_1 is twice as expensive as that as flight f_2, then we would expect that $w(f_1) = 2w(f_2)$. We will also assume that these weights have been normalized so that the mean weight of the flights of each airline is equal to one. If a flight has been given a trajectory and a slot (if applicable), we will use the term "weighted GDEs" incurred by a flight to be the product of the weight with the adjusted cost (in GDEs) of the allocation.

When these weights are available, we define two methods. The first method is similar to the "Difference in Tentative GDEs" method. However, instead of comparing the total GDEs in each of the two tentative allocations, the method compares the total weighted GDEs. We refer to this method as the "Difference in Weighted Tentative GDEs". The final alternative method is similar to the "Difference in Optimal Weighted GDEs" method, but compares the optimal weighted GDEs that could be achieved by airline with the given set of flights and slots. More specifically, let $W(F', S')$ be the minimum amount of weighted GDEs that are incurred when each flight from F' is assigned a trajectory and a slot s from S' is given to each flight that takes its primary trajectory. This can be identified by solving a nearly identical optimization problem as that for $G(F', S')$; the only difference is that weighted adjusted costs are used in place of adjusted costs.

V. COMPUTATIONAL EXPERIMENTS

We ran computational experiments to study the performance of the alternative mechanisms. The flight data for our experiment is based on traffic through one of the FAA's standard FCAs (FCAA05) on September 6, 2016 for the time period 1600Z to 2000Z, when an AFP was in effect. This FCA is used by the FAA to capture flights through Indianapolis Center (ZID) and Cleveland Center (ZOB) from the west, destined to airports in Northern Washington Center (ZDC), New York Center (ZNY) and Boston Center (ZBW). Our data includes the departure time of each flight and its IAT at the constrained region, but does not include the RTCs of any alternative trajectories, nor does it include the relative weight values of the flights. Little data is available on these parameters, so we generated these randomly. The weight of each flight was generated with a triangular distribution with minimum value of 0, maximum value of 2, and mode of 0.5. These weights were then normalized so that the average weight of the flights associated with each airline is equal to 1. The RTC of each flight's alternative trajectory was generated with a triangular distribution with minimum value of 0, maximum value of 90 minutes, and mode of 18 minutes. The RTC of the primary trajectory was assumed to be zero. Due to this randomization, we repeated the generation of the RTCs and the weights 100 times and then applied each method to every resulting instance.

The methods that we considered include the standard CTOP allocation method and all of the previously-discussed alternative methods. For comparison, we report the delay involved if all flights remain on their primary trajectory and receive slots according to their scheduled arrive time, which we refer to as ration-by-schedule (RBS). We also report the costs associated with the allocations that would minimize the total GDEs incurred and the weighted total GDEs incurred. These allocations can be found by solving optimization problems similar to those that occur in the "Difference of Optimal GDEs" and "Difference of Weighted Optimal GDEs" methods respectively. However, instead of using the flights and slots for a specific airline, we include all of the flights and all of the slots in this optimization problem. Note that allocations produced in this way are not necessarily implementable in practice, since they require airlines to report trajectory options and weights (when applicable) in a truthful manner, but pay no heed to the incentives of the individual operators.

We performed two sets of experiments. In the first set of experiments, we assumed that airlines would report alternative weights truthfully even if there were incentives that encouraged

TABLE II. RESULTS WHEN AIRLINES ARE UNTRUTHFUL

Method	Avg. Per-Flight Costs Before Airline Reallocation						Avg. Per-Flight Costs After Airline Reallocation					
	Ground Delay	*Route Cost*	*Total GDEs*	*Wtd. GD*	*Wtd. Route Cost*	*Wtd. Total GDEs*	*Ground Delay*	*Route Cost*	*Total GDEs*	*Wtd. Ground Delay*	*Wtd. Route Cost*	*Wtd. Total GDEs*
Diff. Optimal GDEs	2.7	18.7	21.4	2.7	18.2	20.9	3.7	15.9	19.7	3.8	13.8	17.6
Diff. Tentative GDEs	2.6	18.6	21.2	2.7	17.4	20.1	3.5	16.2	19.8	3.6	13.6	17.1
Diff. Adjusted Costs	10.5	15.3	25.8	10.5	15.3	25.8	5.7	17.1	22.8	5.7	15.5	21.2
Diff. Wtd. Opt. GDEs	2.6	18.8	21.4	2.6	17.7	20.3	3.5	16.3	19.8	3.6	13.5	17.1
Diff. Wtd. Tentative GDEs	2.6	18.6	21.2	2.7	17.1	19.8	3.5	16.2	19.8	3.6	13.5	17.1
CTOP	26.4	17.6	44.1	26.5	17.6	44.1	5.9	16.6	22.5	5.9	14.9	20.8
RBS*	185.8	0.0	185.8	185.8	0.0	185.8	4.9	20.1	25.1	5.0	18.4	23.5
Optimal GDEs*	1.3	14.1	15.4	1.3	14.1	15.4	1.6	14.7	16.3	1.6	13.1	14.8
Optimal Wtd. GDEs*	2.0	15.0	17.0	1.6	11.4	13.0	2.0	15.0	17.0	1.6	11.4	13.0

*No gaming strategy was applied under these methods.

them to behave otherwise. In the second set of experiments, we introduced some gaming behavior. This was implemented in the following manner. We allowed flights to omit alternative trajectories for some of their flights. This would prevent those flights from being rerouted by the allocation procedure. When the allocation method attempts to route a flight along an alternative trajectory, we check to see if airline would prefer to keep the flight in its primary trajectory. This is determined using the "Difference in Optimal Weighted GDEs" method. If the airline would prefer to keep the flight in its primary trajectory, then we assume that the airline would prevent this reroute by omitting the alternative trajectory. Note that this assumes that airlines have full knowledge of how the allocation procedure will proceed, which includes decisions made by other airlines. This also assumes that the "Difference in Optimal Weighted GDEs" method is compatible with the manner in which airlines value a set of allocated slots. Neither of these assumptions may be true in practice, but we believe they serve as reasonable approximations. In general, it is difficult to identify a game-theoretical equilibrium for the mechanisms that we are examining, and it is difficult to predict precisely how airlines would behave. Note that for the experiments in which airlines behave untruthfully, we still report the results for the RBS and system optimal allocations, but the modeled gaming behavior does not apply to these methods and the values used in these allocations are assumed to be truthfully obtained.

After an allocation is produced by the CTOP mechanism, airlines can perform intra-airline slot exchanges and can choose to reroute flights (as discussed in Section II.C). We assume that the airlines would do so to minimize the total weighted GDEs. This would involve solving an optimization problem, which is the same optimization problem that is used in the "Difference of Weighted Optimal GDEs" method (presented in Section IV.D). In addition to the allocated costs immediately following application of the CTOP mechanism, we also report what the costs would be after the airlines have performed this reallocation.

The results from when the airlines behave truthfully are shown in Table I. All values in the table are expressed in terms of minutes of GDEs. Recall that the CTOP differs from the "Difference in Adjusted Costs" method only in that latter method applies compression when slots are vacated by flights that are sent on their alternative trajectory. Our intent in using the compression algorithm was mainly to better align the incentives of airlines, so we were surprised to see that the "Difference in Adjusted Costs" provided a slight improvement over CTOP in the incurred GDEs (and incurred weighted GDEs) when airlines behave truthfully. The "Difference in Tentative GDEs" and "Difference in Optimal GDEs" performed significantly better than the adjusted cost methods in terms of total GDEs incurred. As compared to CTOP, these methods produce roughly a 25% decrease in incurred GDEs before the airlines reallocate and provide close to a 15% decrease when the reallocation is taken into consideration. The underlying mechanism for this improvement is the phenomenon illustrated in Section IIIA. The performance of these two methods is similar, which indicates that the "Difference in Tentative GDEs" does a good job of estimating the relative benefits of a reroute, and that benefits from a more accurate estimate may be marginal. The weighted variants of the "Difference in Tentative GDEs" and "Difference in Optimal GDEs" have similar performance to the unweighted variants, although these methods incur slightly lower quantities of weighted total GDEs after the airlines reallocate. This indicates that it may be unnecessary to solicit the weight information from airlines, as the benefits of this information do not seem to be large.

The results from the experiments in which the airlines could omit alternative trajectories are shown in Table II. When flights respond to the incentives of the mechanism, participation in CTOP decreases and the incurred costs in the CTOP allocation increase. In this case, the "Difference in Adjusted Costs" method greatly outperforms the standard CTOP allocation mechanism when airline reallocations are not considered. However, when airline reallocations are taken into consideration, the final incurred costs of these two methods end up being similar. While these methods may be similar in terms of total delay incurred in practice, there are advantages to using the "Difference in Adjusted Costs". Under this method, more of the reroutes will be performed within the allocation method itself. This makes the procedure more predictable, as less adjustments will be made to the schedule. When airlines can omit alternative trajectories, there is even less difference in performance between the "Difference in Tentative GDEs", the "Difference in Optimal GDEs" method, and the weighted variants of these methods. This reinforces the suggestion that the "Difference in Tentative GDEs" uses a sufficiently accurate estimate of the relative costs of the trajectory options presented by the airline. As in the case where airlines behave truthfully, these methods outperform the standard CTOP allocation method. This indicates that the performance benefits of our proposed methods hold whether or not airlines behave truthfully. Furthermore, under these methods, the total GDEs incurred before and after airline reallocation are more similar than under CTOP. While this does not directly prove that our proposed methods allow less opportunities for gaming, it does suggest that airlines are more willing to participate and conduct reroutes through the allocation mechanism, rather than implementing the reroutes themselves after the CTOP mechanism has been run.

VI. CONCLUSIONS

We have identified some potential areas for improvement to the CTOP allocation mechanism. First, we noted that the current CTOP mechanism can result in unnecessarily large delays. Our proposed alternatives would address this issue by more accurately recording the relative costs of trajectory assignments. Second, we note that the current CTOP mechanism often discourages airlines from declaring alternative trajectory options. In order to alleviate this second issue, we recommend the use of the compression algorithm to fill slot vacancies. We ran computational experiments that verified that our proposed solutions would result in lower incurred costs for participants in a CTOP program.

Our current methods are limited to the case in which there is a single constrained area and all flights can be routed around this area. Further work would be required to develop a more general solution that would apply to any CTOP program. In our analysis here, we made the assumption that delay costs are a linear function of the amount of delay taken. As discussed in [17], this may not be the case in practice. It may be possible to further improve the CTOP mechanism to allow more generality in the specification of the delay cost function. However, this would likely require soliciting more information from airlines and would lead to a more complex mechanism that is more difficult to understand and implement. Further work would be required to determine whether the benefits provided by the more exact representation of the delay cost function would be great enough to justify the additional complexity.

Another avenue of future work would be to conduct experiments in which airline behavior is simulated more precisely. For example, we generated RTCs randomly, but a more principled method could be used to generate the RTCs. It would also be possible to develop more accurate models of airline gaming behavior. Indeed, more sophisticated game-theoretic models were used in [14], [15] and [16] to identify equilibrium strategies. However, the methods used in these papers do not seem to scale well and could not be used for instances with more than a few flights. Future work in this direction would strive to build models that accurately capture airline gaming behavior while remaining computationally tractable for problems of realistic size.

ACKNOWLEDGMENT

The authors wish to thank Bob Hoffman and Dave Lovell for their feedback on the ideas presented in this paper.

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