Fourteenth USA/Europe Air Traffic Management Research and Development Seminar (ATM2021)

Equity within Air Transportation Management – an Analysis of Inequity Index for Multi-Stakeholders Optimisation

Application to an airlines group operational use-case

Marie Carré[1,2], Eric Nantier[1]
1: Operations Optimisation & Development
Swiss International Air Lines
Zurich, Switzerland
marie.carre@swiss.com
eric.nantier@swiss.com

Séverine Durieux[2], Laurent Piétrac[2]
2: Université Clermont Auvergne, Clermont Auvergne INP, CNRS, Institut Pascal, F-63000 Clermont-Ferrand, France.
severine.durieux@sigma-clermont.fr
laurent.pietrac@sigma-clermont.fr

***Abstract*—Current equity definition hampers innovation within the Air Traffic Management (ATM). Even if different definitions are currently well established, summer 2018 and 2019 highlighted the urgent need for flexibility and efficiency that current equity definitions cannot cover anymore. Moreover, in any group decision, targeting the best performances for the group can be at the expenses of some stakeholders. Therefore, new equity perspectives must be proposed. In our paper, we propose a new paradigm, and explore the first steps. Equity has a multi-criteria-based definition, and should be guaranteed over time instead of ensuring it in every situation. We first introduce the context of our research and point out, thanks to a literature review, three inequity indexes successfully used in other fields. We analyze their sensitiveness and pertinence according to our problem. We discuss then the results and the following research paths.**

Keywords-component; equity; inequity index; airlines operations; multi-stakeholders optimisation.

I. Introduction

Equity triggered numerous discussions, philosophical and political statements and judgments, as well as scientific researches to face the lack of objectivity. Nowadays, this notion is still source of debates and research. In the context of group decisions, in which global performances can conflict with local performance, equity is a main criterion to get all actors of the group accepting the decision. By equity, we refer to a "fair" and "just" distribution of benefits among the actors. Many definitions and concepts, though, can meet this definition [1]. It is then crucial to clarify the understanding of equity in our context and then prove – thanks to equity assessment – to all actors that the decision reaching global performances is equitable.

In a problematic of operations disruption, affecting airlines operating each from different hubs with different business models although belonging to the same strategical group, operational cooperation can be sometimes conflictive, even to enable the best use of resources. Nowadays, each airline tries to find alone its best solution out of the disruption. The assumption is that all airlines of a same group searching for an optimal situation together would generate less impact of the disruption on the global operations. However, being a group compromise, some airlines could face more consequences or sub-problems than the others could, as no perfect solution exists. The same problematic can be extended to the Air Traffic Management (ATM), as all stakeholders collaborates to enable smooth and efficient operations despite the high traffic density (pre-covid situation). Section II elaborates the context of our research and its applicability.

Equity is a subject tackled by mathematical indexes since more than one century. Different approaches and indexes have been developed, and some present interesting modelization for our problem. Section III presents a short literature review and the introduction of the three main indexes developed by economists.

We propose in this paper to adapt and assess these equity indexes. We will confront the three main ones to several solutions from a basic group optimisation in order to analyze their sensitivity to different inequitable situations. Section IV develops the analysis and following discussion validating the potential use and adaptation of the indexes in our operational field.

This paper aims at proposing the first step of a new paradigm for equity calculations: base equity on multi-criteria and guarantee it on the long run (instead of the current approach, which must guarantee equity on a situation level). This change of paradigm would enable far more flexibility and thus efficiency in the group operations optimisation as well as in the European and American ATM.

II. CONTEXT

A. *Necessity for Equity in European and US Air Traffic Management*

Since the beginning of the collaborative European ATM and the creation in 1988 of the Network Management Operations Control position (NMOC, managing the European ATM), ATM stakeholders commonly accepted First Come First Served (FCFS) principle being the base for equity between flights. Safety is the first priority of ATM stakeholders and then comes efficiency closely linked with equity [2]. Nowadays, FCFS remains the main equity principle for air traffic controllers (ATC) and amount of delay minutes per flights is the equity driver for NMOC.

However, these well-acknowledged procedures of equity were undermined during the last few years. FCFS principle hampers efficiency and was overridden many times, as airspaces and airports capacities reached their limits, as it has been shown in SESAR (Single European Sky ATM Research) PJ25 project. By enabling the airlines re-sequencing their flights arriving in ZRH, thanks to proactive and collaborative process with all stakeholders involved, the efficiency raised tremendously. Similarly, strict comparison in terms of delay minutes is missing operational reality to guarantee equity. Two flights with the average amount of delay could either not affect the operations or have huge operational impact on one airline's operations, therefore affecting the entire European network. The ATM Portal from Maastricht proposed, within PJ24 SESAR funded project, new features to enable operational priorities communication. Moreover, after the highly constrained summer 2018, eNM/S19 measures (Enhanced Network Manager measures for Summer 2019) proposed numerous adaptations to make the best use of the European capacity [3, p. 15]. By rerouting flights, restricting flight levels filing possibilities for flights departing from specific European regions and many more short-term solutions, FCFS and delay minutes were not respected anymore. Therefore, there is a need for the current ATM stakeholders to discuss and agree on a new equity definition, which cannot be summarized by one criterion.

From the US ATM perspective, several approaches were proposed to assess the fairness in ATM. For the seasonal slot allocation process, [4] proposed to calculate the deviation of slots received in comparison with the airline's requests, compared with the airline's number of requests. During capacity constraints at an airport, [5] defined the fairness of slot reallocation as the deviation from an ideal slot allocation not taking into account some congestion specificities (such as exempted flights or popup flights). Further articles based their developments on this approach. However, this is taking into consideration only one dimension, which is the delay allocation. [6] proposed recently two dimensions to fairness, one related to changes in the initial sequence of arrival for unmanned vehicle operator, and the other calculating the time-deviation by considering the worst FCFS delay applied. A very valuable contribution of this article is that the overall fairness is impacted by the different operators' definitions of fairness as well as the weight that they attribute to efficiency towards fairness. This highlight greatly the sensitivity of equity definition.

Equity is a very important concept, which is philosophically, politically and even psychologically sensitive. [7] showed that humans are more sensitive to losses than gains with comparable amounts. Therefore, if one stakeholder is not fairly managed towards the others, and that no balance of equity is foreseen, injustice feelings can interfere into the group optimisation, resulting possibly to some actor refusing the cooperation. Before any change in the current procedure is implemented, equity must be validated. A good example in the European ATM is the User-Driven Prioritisation Process (UDPP) project. The equity is one of the main constraints assessed and guaranteed during each validation exercise, in order to get the stakeholders acceptation and trust in the new process [8]. Further discussions still take place in Exploratory Research 4 (ER4) projects like BEACON and SlotMachine about the trade-off between equity and efficiency. Developments of safe, more flexible, more efficient, and environmentally friendly processes are often hampered by strict equity considerations.

Equity is necessary, but a strict equity on each situation should not reduce the potential improvements on efficiency. We aim in this paper at offering the reader a new paradigm about equity and launch reflection on a new kind of equity within multi- stakeholders' decision and optimisation, especially in ATM.

B. *Multi-Airlines Optimisation – an Operational Use-Case*

Airlines started to cooperate and create alliances to handle the growing concurrence. Several types of cooperation exist, strategically broadening airlines' networks and connection possibilities: airlines alliances and airlines group [9]. Alliances allow codeshare flights opportunities, which offers more connections, more frequencies and thus more flexibility to the passengers. This increases attractiveness of the airlines. Airlines' group implies stronger alignment and integration. Generally, one airline acquires other airlines, and departments such as high-level strategy, pricing, revenue, even techniques could be merged. In some cases (like United Airlines), Operations Control Centers (OCC) are joined and one OCC monitors all operations. In other cases, each airline remains independent, and coordination is organized. This is the case of the Lufthansa Group airlines operations. During disruptions, the OCC managers coordinate the cancellations publication time together to avoid automatic rebooking to the other airlines' group flights, which are planned to be cancelled. However, each airline optimises by itself its own disrupted operations and rarely takes into account the possibilities of deeper coordination with the other OCCs.

We aim at improving this collaboration during disruptions by sharing operational resources and thus may offer better solutions to the disrupted passengers of all involved airlines. The acceptance of the compromise bringing the best performances for the group is a crucial aspect, which needs clarification so that the Network Operations Controllers can trust the new system. Indeed, best performances for the group could be reached at the expenses of some airlines. These disfavored airlines would then be more affected than what would have happened in their local optimisation. Therefore, equity is a primordial topic, to guarantee for each stakeholder involved. As explained above, strict equity at each operation reduces the flexibility and possibility to reach greater benefits for the group. Thus, we aim not only to find an index with the right sensitivity to inequity but also an index, which allows an overview of inequity status of each airline.

Therefore, we would like to propose equity guarantee on the long run instead of in each unique situation. Before developing a trustable long-term equity system, equity in the context of multi-stakeholder optimisation must first be defined clearly, and means to calculate it must be introduced. This is the goal of this paper.

C. *Use-Case Introduction*

We consider the five major hub-airlines of the Lufthansa Group, operating to the virtual hub airport XYZ. We define a virtual hub as an airport, being no involved airlines' hub, to which more than 25 rotations are operated daily. This enables sufficient traffic from the same group to allow enough group operational flexibility and offers to the passengers multiple connections via the different group's hubs. In our use-case, all airlines are operating together 52 flights from/to XYZ (26 rotations) with flights spread over the day and high amount of connecting passengers. The share of flights initially planned to XYZ operated by each airline is presented in table I. A disruption occurring in XYZ lead to a mandatory traffic reduction of 20% requested by the airport or ATC. Delays are not an accepted mitigation measure in this study case. Each airline operating to the airport must therefore reduce its amount of operated flights by 20%. As the Lufthansa Group is operating initially 52 flights, 10 flights must be cancelled, as we have to cancel complete rotations. Table I gives the theoretical repartition among the airlines.

TABLE I. REPARTITION OF FLIGHTS PLANNED BY AIRLINES

Airlines IATA Code	*Number of initially scheduled flights*	*20% flights cancelled*
SN	4	0.8
OS	10	2
LX	10	2
LH FRA	12	2.4
LH MUC	14	2.8
Total	50	10

We consider that each airline operating from its hub and with its own OCC is independent in his own operational choices. Thus, we consider that LH MUC and LH FRA are two different stakeholders.

D. *Multi-Criteria Equity*

In order to develop a trustable system, a team of operational experts is involved. They answered questionnaires about the criteria to consider during the equity assessment process. All agreed on the necessity of several criteria defining equity, as no unique criteria could represent alone the full reality. Let us illustrate it with a simple operational example: if uniquely the number of cancellations was considered in equity measurements (as nowadays by ATC during disruption), all flights would be considered equal. However, depending on many parameters like the number of passengers booked and the connections fed by these flights, each flight has a different value. Similarly, considering only the minutes of delay on each flight is just one aspect of the reality. Depending of the flights, crews flying as passengers to operate the next rotations, as well as operating crews and aircraft rotations constraints bring new dimensions to the reality. Each flight is different from an operational point of view and has a different priority [10]. Therefore, no unique criteria can be considered, and multi-criteria approach is necessary to understand the multi-dimensionality of the reality.

Consequently, we steer our research towards multi-equity-elements based on the experts' feedback.

III. STATE OF THE ART OF INEQUITY CALCULATIONS

Before deepening into the multi-criteria equity analysis, a literature review is necessary to highlight the different equity approaches developed since several centuries, as well as the mathematical indexes proposed.

A. *Concept of Justice and the Different Approaches Proposed in Transportation Research*

The problem of equity, especially in justice or politics, exists, is identified and analyzed over more than a century. [11] published a first article outlining the inequity in poverty and [12] tackled the first social effects of transportation, especially the spatial distribution on railways. Later on, economics and public transportation network design problems based their equity definition on the justice approach and developed it further. The definitions mainly related to poverty, inequity of salary and inequity of access to the public transports. Some authors even proved extensively the link between inequity in the public transport accessibility (spatial equity), and the social equity (mainly jobs opportunities, social services access like healthcare and educational access) [13]–[18]. As an article presenting an extensive literature review on equity approaches applied to transportation field has been recently published, the reader could find more detailed information on this topic [1]. We will focus on the theory and scientific articles necessary to the reader to understand the current science status on which we based our analysis.

Philosophically, one could separate in several categories the different approaches about equity. [19] identified three trends, based on the philosophical approaches mainly developed in the XVIII century, to which we added a new one developed by [20]:

- The utilitarian approach, aiming at maximizing the total benefits considering all actors as one, and preferring a total benefit higher even with a very poor fairness, than a slightly slower benefit but with a very high fairness. Translated into our use-case, this would mean that solutions with very good group performances but poor fairness would be considered (which could be also the case, if we can grant a long-term equity between the airlines after a given time).
- The egalitarian approach aims to equalize benefits for all actors, disregarding the intrinsic differences, and capabilities of each actor. This approach would reject (or propose with very bad scores) solutions with high

performances for the group but with a low level of fairness. This is highlighting the difference between equity and equality, equality being the strict same amount of benefits while equity relates to a baseline which can be either the initial repartition or a repartition based on the stakeholder's needs. This is not an interesting approach for us as our goal is to support that a group solution would benefit more than local solution found by each stakeholder.

- The contractarian approach aims at improving the benefits of each actors, still respecting the initial differences, and according to specific contracts or agreements between the actors (maximizing the benefits of the actors having the worse results). This is an interesting approach for our operational use-case.
- The so-called "sufficientarian" approach aims at insuring a sufficient level of benefits for each actor according to its needs. It uses a threshold notion, which negatively influences the equity index if an actor does not reach this threshold. It have been used by [20] to enhance the already proposed vision of equity. This notion of threshold could be very interesting to our case, to differentiate the solutions acceptable with equity imbalance, from the too imbalanced and inequitable solutions.

Research community proposed several indexes through the years, mainly focusing on economical calculations such as income inequity index. We will present three of the main ones, recognized by the scientific community and applied in multiple fields and analyses.

B. *Developed Index for Inequity Measurements*

Inequity indexes are based on years of research, especially in the economics science. They must follow a given number of rules, reminded by [21]. The main ones are: the normalization (for perfect equitable distribution, the index is zero, otherwise it is positive); the symmetry (who gets the highest or smallest income does not matter, this is an anonymity principle); the Dalton-population principle (invariance of the index if the population is replicated), the Pigou-Dalton transfer rule (if a transfer from one "rich" individual to a "poor" individual happens, the inequity index must decrease); the continuity of the index (if a small variation in the income share happens, a small variation in the index should be observed); the relative invariance (if all incomes are multiplied by a constant, the index should not change the index).

1) *Gini Index*

Focusing the literature review on the public transportation equity, numerous approaches and equity indicators have been developed. Historically, one of the first indicators tackling inequity is the Gini index [22]. The Gini index expresses the distribution of different incomes between the different actors. With a comparison for each pair of individual *i* and *i'*, with *N* the number of individuals, and x_i the income of the individual *i*. The Gini index is calculated as following in equation (1):

$$GI = \frac{\sum_i \sum_{i'} |x_i - x_{i'}|}{2 \times N^2 \times \underset{i}{average}(x_i)} \quad (1)$$

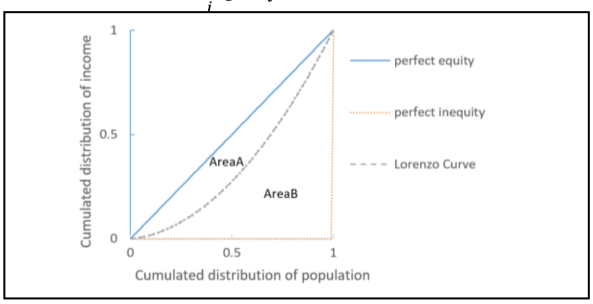

Figure 1: Illustration of the Lorenzo curve

The Gini index calculates the quotient of the area between Lorenzo curve and the strict equality curve and the area under the strict equality curve (Figure 1). The Lorenzo curve is the cumulated distribution of income corresponding to the cumulated distribution of individuals groups. The strict equality curve is a straight line from point (0,0) to point (1,1) (also called "perfect equality line" as all incomes are equally distributed). Gini index is then equal to 0. The "perfect inequality line" corresponds to the entire incomes share belonging to one group of individual. As the individuals are ranked by incomes, the Perfect Inequality line corresponds to the red line plotted in Figure 1. Gini index is then equal to 1. Gini index has the advantages to be easily computable, recognized by the scientific community as one of the fundamental equity index and intuitively understandable.

2) *Theil Index*

Theil proposed another index, more sensitive to the groups' sizes of individuals and their income shares. He based his index on the principle of entropy in information theory. The principle is: the smaller the probability is that an event would occur, the higher the interest. The logarithm function of (1/x) is modelling this requested behavior [23, p. 35]. For each *i=1,…,N,* an event that could happen, we write w_i the event probability. The expected information resulting of the situation is "the sum of the information content of each event weighted by the respective probabilities", defining the entropy *Q(w)*:

$$Q(w) = \sum_i w_i . \ln\left(\frac{1}{w_i}\right) \quad (2)$$

Theil introduced two changes in equation (2) to analyze the income concentration. First, the probability w_i is replaced by the income of one individual $s_i = x_i / N.\mu$, with μ being the average on *i* of x_i (historically the mean income). Secondly, he defined the index as the difference of the maximal entropy (*ln (N)*) and the current entropy *Q(s).* Theil index can be written as in equation (3) and with the expression of s_i, can be expressed as in (4).

$$TH = \ln(N) - Q(s) = \sum_i s_i . \ln(N) - \sum_i s_i . \ln(s_i) \quad (3)$$

$$TH = \frac{1}{N} . \sum_i \frac{x_i}{\mu} . \ln\left(\frac{x_i}{\mu}\right) \quad (4)$$

3) *Atkinson Index*

Atkinson modelled his index differently, aiming at an equity in welfare rather than income. He expressed it as the incomes required to enable total welfare being exactly equal to the welfare generated by the actual income distribution. He formulated as y_e being the "equally distributed equivalent level of income" [24]. With μ being the average income, the best equality is reached for y_e =μ. As for Theil and Gini index, the perfect equality reached should be reflected by the index being equal to zero. Therefore, Atkinson Index (AT) is AT=1- y_e/μ. Atkinson used the generalized mean with exponent *p*, also called Hölder mean to calculate y_e in function of the real distribution of the y_i. We then can traduce with n_i the number of individuals in the income category y_i:

$$\frac{y_e}{\mu} = \left(\sum_i \frac{n_i}{N} . \left(\frac{y_i}{N}\right)^p\right)^{\frac{1}{p}} \quad (5)$$

As the function is strictly concave, the inequity aversion is modeled as the utility elasticity, which is evaluating the relative change of the utility in relation with the individual income changes. For simplification purposes, Atkinson assumed that this inequity aversion is constant, which defined the utility function as:

$$U_e(y) = \begin{cases} \frac{y^{1-\varepsilon}}{1-\varepsilon} \; if \; \varepsilon \neq 1 \\ \ln(y) \; if \; \varepsilon = 1 \end{cases} \quad (6)$$

Using p= 1-ε, with ε representing the aversion to inequality, which characterize the Atkinson index (ε=0: no aversion to inequity; ε =1: each individual has the same weight; ε tends towards infinite: the indicator tends to consider only the worse observation). This gives:

$$AT_\varepsilon(y) = \begin{cases} 1 - \left(\sum_i \frac{n_i}{N} . \left(\frac{y_i}{\mu}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} \; if \; \varepsilon \neq 1 \\ 1 - \left(\prod_i \frac{x_i}{\mu}\right)^{\frac{1}{N}} \; if \; \varepsilon = 1 \end{cases} \quad (6)$$

The greater ε is, the more the transfers of income in the lower end of distribution are influencing the index [25]. This index is interesting to know which end of the distribution is the most unequal and contributed the most to the inequality measure.

Atkinson index has the advantage of taking into account the parameter ε influencing the index results with inequity aversion. This can be very useful to enable more flexibility to the user by defining the aversion level of inequity depending of the disruption extent. Therefore, an adaptation of the Atkinson index to our use-case is relevant.

IV. Critical Analysis of the Inequity Index in the Context of our Use-Case

Based on the different indexes proposed, we analyzed each index behavior on several solutions from our use-case. The solutions and their relevance in the analysis are described in the first part. Then, each index is assessed on several criteria to highlight their behavior. Finally, a discussion positions each index with respect to our use-case.

A. Solutions Proposed for the Analyses

Table II summarizes the different solutions designed to highlight each unique index behavior. Solution A represents a real optimised solution proposed by a basic solver. It suggests a balanced repartition in terms of cancelled flights per airline. To compare with an extreme situation, solution B presents an imbalanced situation, in which one airline cancels all its flights, corresponding to the required number of cancellations for the group. To observe further the indexes' behavior, solution C presents also an imbalanced situation, in which one airline cancels the 10 required flights, which correspond to 70% of its originally planned flights.

A second set of solutions focuses on the indexes' behavior in case of strict equality in terms of cancelled flights amount. However, the flights cancelled does not have the same properties. Solution D cancels flights for each airline with the average number of passengers booked per airline. Solution E cancels also 2 flights per airline with average booked passengers figures, except for two airlines cancelling fully booked flights (SN and LH MUC).

The solution F proposes that only half of the airlines cancel flights for the entire group with average passengers booked figures. Finally the last solution, G, illustrates the behavior of the index when all airlines – except one – are cancelling flights that are fully booked.

These different constellations of solutions enable to analyze the sensitivity of each index in terms of complex reality. Indeed, one can compare balanced solution to several sort of imbalanced solutions, which should lead to a contrast in the indexes interpretation (solutions A, B and C), as well as compare the impact of different indicators (number of cancellation and passengers impacted) on the indexes to highlight their behavior in solutions looking similar (solutions D, E). The last couple of solutions (F and G) are highlighting specific behavior of the indexes in terms of inequity according to different configurations.

In Gini, Theil and Atkinson indexes, the interpretation of the mathematical elements is as follow:

- *i=1,...,N:* an airline with *N* the amount of airlines
- x_i*:* performance of the airline *i* for the analyzed equity elements
- *μ:* average of all airlines performances on the analyzed equity element
- *ε:* aversion to inequity (for Atkinson index only)

It is worth mentioning that x_i must follow the same pattern than the income: x_i greater than *μ* corresponds to an airline *i* with better performances than the average (solution at airline *i's* favor). In our analysis, x_i is expressed as the ratio of non-impacted flights (table III) or non-impacted passengers (table IV) by the disruption, to enable a comparison between the airlines and evolution of the indexes between the proposed solutions.

The data presented in Table II are real data, which has been anonymized for confidentiality reasons.

TABLE I TABLE OF SOLUTION FOR THE INDEX ANALYSIS

Solutions ID	Airlines	Number of planned flights	Number of cancelled flights	Number of operated flights	Number of booked passengers	Number of passengers on the cancelled flights	Number of non-impacted passengers	Ratio of non-impacted passengers	Ratio of operated flights
SOLUTION A	SN	4	0	4	455	0	455	1.00	1.00
	LX	10	2	8	1474	301	1173	0.80	0.80
	OS	10	2	8	1307	251	1056	0.81	0.80
	LH FRA	12	2	10	1555	263	1292	0.83	0.83
	LH MUC	14	4	10	1939	555	1384	0.71	0.71
SOLUTION B	SN	4	0	4	455	0	455	0.00	1.00
	LX	10	10	0	1474	1474	0	1.00	0.00
	OS	10	0	10	1307	0	1307	0.00	1.00
	LH FRA	12	0	12	1555	0	1555	0.00	1.00
	LH MUC	14	0	14	1939	0	1939	0.00	1.00
SOLUTION C	SN	4	0	4	455	0	455	1.00	1.00
	LX	10	0	10	1474	0	1474	1.00	1.00
	OS	10	0	10	1307	0	1307	1.00	1.00
	LH FRA	12	0	12	1555	0	1555	1.00	1.00
	LH MUC	14	10	4	1939	1385	554	0.29	0.29
SOLUTION D	SN	4	2	2	455	218	237	0.52	0.50
	LX	10	2	8	1474	308	1166	0.79	0.80
	OS	10	2	8	1307	267	1040	0.80	0.80
	LH FRA	12	2	10	1555	266	1289	0.83	0.83
	LH MUC	14	2	12	1939	278	1661	0.86	0.86
SOLUTION E	SN	4	2	2	455	280	175	0.38	0.50
	LX	10	2	8	1474	295	1179	0.80	0.80
	OS	10	2	8	1307	261	1046	0.80	0.80
	LH FRA	12	2	10	1555	259	1296	0.83	0.83
	LH MUC	14	2	12	1939	430	1509	0.78	0.86
SOLUTION F	SN	4	2	2	455	228	228	0.50	0.50
	LX	10	0	10	1474	0	1474	1.00	1.00
	OS	10	2	8	1307	261	1046	0.80	0.80
	LH FRA	12	0	12	1555	0	1555	1.00	1.00
	LH MUC	14	6	8	1939	831	1108	0.57	0.57
SOLUTION G	SN	4	2	2	455	280	175	0.38	0.50
	LX	10	2	8	1474	430	1044	0.71	0.80
	OS	10	2	8	1307	430	877	0.67	0.80
	LH FRA	12	4	8	1555	860	695	0.45	0.67
	LH MUC	14	0	14	1939	0	1939	1.00	1.00

TABLE II. INEQUITY INDEXES CALCULATIONS IN REGARDS WITH THE EQUITY CRITERIA CANCELLATION

Solutions ID	Gini	Theil	Atkinson ε=0.5	Atkinson ε=1	Atkinson ε=1.5
SOLUTION A	0.058	0.006	0.003	0.006	0.009
SOLUTION B	0.200	0.223	0.200	1.000	#
SOLUTION C	0.133	0.071	0.040	0.092	0.154
SOLUTION D	0.079	0.016	0.009	0.018	0.028
SOLUTION E	0.079	0.016	0.009	0.018	0.028
SOLUTION F	0.148	0.038	0.019	0.039	0.058
SOLUTION G	0.120	0.025	0.013	0.025	0.039

TABLE III. INEQUITY INDEXES CALCULATIONS IN REGARDS WITH THE EQUITY CRITERIA IMPACTED PASSENGERS

Solutions ID	Gini	Theil	Atkinson ε=0.5	Atkinson ε=1	Atkinson ε=1.5
SOLUTION A	0.059	0.006	0.003	0.006	0.009
SOLUTION B	0.200	0.223	0.200	1.000	#
SOLUTION C	0.133	0.071	0.040	0.092	0.154
SOLUTION D	0.075	0.014	0.007	0.015	0.024
SOLUTION E	0.102	0.032	0.017	0.037	0.059
SOLUTION F	0.147	0.037	0.019	0.039	0.058
SOLUTION G	0.186	0.057	0.028	0.056	0.083

B. Gini Index

To simplify the writing, we will as from now write GI(A) to refer to the Gini index calculated for the solution A (same for TH(A) and AT(A) being the Theil and the Atkinson index).

1) Cancellation

GI(B) and GI(C) present an interesting behavior: in solution B, one airline is totally penalized and do not operate any flights at all, i.e. has a total negative impact on its operations. However, in solution C, the airline cancelling the 10 flights operates initially more flights than this amount. Therefore, even if one airline undertakes all the cancellations for the group, as it still can operates a few flights, GI(C)<GI(B) (See in Table III).

Solution F underlines however the results of the absolute value usage in Gini index. As three airlines are cancelling for the group, the differences in terms of cancellation with the two other airlines are adding up, regardless if this is in favor or not to the considered airline. This leads to GI(F)>GI(C) although C is one of the most inequitable as only one airline undertakes all cancellation for the group. The absolute value captures inequity between two airlines but does not reflect if the inequity is in favor or at the detriment of the airline. The absence of distinction shown previously is a major obstacle to the Gini index utilization, as no one can know if the inequity towards the airline is a negative or positive impact.

2) Booked Passengers

The comparison between solution D and E highlights an essential facet of equity within airlines operations: even if the ratio of cancellation for each airline is identic in both solutions, the number of passengers booked on the cancelled flights greatly affects the perception of inequity. Two flights from a same airline, to the same airport and operated by the same aircraft type are not having the same operational value. Using several equity element to calculate the equity is crucial to tend towards real inequity calculations. SN is able to transport 52% of its passengers in solution D against 38% in solution E. LH MUC gets from 86% to 78% (see solutions description in Table II). This reduction of performances is caught by Gini index sensitivity and GI(D)<GI(E) for the booked passengers equity criteria. Solution B presents a very strong inequity as one airline cancels all its flights initially planned, to hedge the group.

3) Gini Index' Global Assessment

Gini index reflects the inequitable situation with a strict equality approach. If two airlines presents the same performances, both comparison (of *i* with *i'* and of *i'* with *i*) returns the same value. No distinction exists between negatively and positively impacted airlines.

This analysis showed the different behavior of the index depending of the indicator choice. A striking example is the difference of inequity assessment for solution E. The differences of assessment between the Gini index calculated on ratio of operated flights and non-impacted passengers for solution E and G highlight the necessity of well-defined indicator on which the Gini calculations are based, and suggest the need for multi-criteria approach.

Solution G presents a situation in which all airlines except one are actively participating in the disruption resolution by cancelling some flights. What is particularly interesting here is GI(G)=0.186 when GI(C)=0.133, solution in which one airline cancels all the necessary flights for the group (and operates its remaining flights). Gini index considers that it is more inequitable that one actor does not participate and all are "suffering" for the group, in comparison with one "suffering" for the entire group. It worth's it to note this behavior for the discussion comparing the different indexes at the end of this article (IV.E).

Even if the analysis above proved the right behavior of Gini index in regards with inequitable situation, the fact that inequality both in favor of and at the expense of an airline are not differentiated ranks the Gini index as a dysfunctional index for the multi-airlines operations use-case.

C. Theil

1) Cancellation

Theil index for the solution A, TH(A), presents a very closed value (0.006) to zero, the perfect equity represented by TH=0. This small value is attributable to the relative small deviation of all airlines from the mean value. In solution B, one airline is cancelling all its flights for the group. This imbalance is spotted by Theil index, returning TH(B)=0.223. It corresponds to an increase rate of more than 3000%. On the similar solution C, in which one airline cancels for all the required number of flights, but still can operate some, as 14 flights were initially planned, TH(C)=0.071.

Using the ln function needs one adaptation: if $x_i=0$, Theil's convention stipulates that $ln(0)=0$. This convention is interesting: as Theil index is the sum of all inequity contribution, an airline, completely disfavored, is not balancing the total index, and only the positive contribution in favor of the other airlines are added up. Therefore, the global index will be higher, expressing a higher inequity measure, than with a negative contribution from the completely disfavored airline. This is well observable between TH(B) and TH(C). The only drawback of this convention is the decomposition of the index in airline's inequity contribution. Indeed, if the contribution is 0, it must be verified if it is due to a complete unequal situation (x_i=0) or an exact mean performance position (x_i=μ). Then, a new convention should be proposed to express the inequality of the situation towards the completely disfavored airline.

2) Booked Passengers

In solution E, the flights impacted by the cancellation for SN and LH MUC are fully booked. Therefore, the percentage of non-impacted passengers is lower in E than in solution D. Theil index sensitivity to disfavored airlines is clearly observable between TH(E) and TH(D) (see Table IV). It outlines the tendency of Theil index to focus on the negatively impacted cases. The use of ln function explains this tendency:

- If $x_i > \mu$, $x_i/\mu>1$ and $ln(x_i/\mu)$ is increasing slowly towards +∞
- If $x_i = \mu$, $x_i/\mu=1$ and $ln (x_i/\mu)=0$ so no influence to the inequity index as the airline *i* reached the mean value of performance
- If $x_i < \mu$, $0\leq x_i/\mu<1$ and $ln (x_i/\mu)$ is rapidly decreasing towards a vertical asymptote.

For the same deviation (noted e) above or under the mean value of performances, the disfavored airline has larger impact on Theil index as $|ln((\mu-e)/\mu)| > |ln((\mu+e)/\mu)|$.

3) Theil Index' Global Assessment

Theil index has interesting properties. First, it is differentiating the airlines being favored from the one disfavored, thanks to the positivity or negativity of the ln function. Second, its sensitivity to a disfavored airline, affecting more the total index, reflects the sentiment of injustice experienced by this airline. Human tendency is to pay more attention to his loss than to his gain [7]. Inequity is also about decision-makers feelings, not only mathematical calculations based on just tangible reality.

These abilities also enable a clear understanding of each airline contribution to the inequitable situation. The index can be decomposed without losing any quality in the global index, except for the special case of where the airline's contribution is 0 triggering a doubt between $x_i=0$ or $x_i=\mu$. This is particularly interesting in our use-case, as we could observe adequately the position of each airline in the global inequity of each solution.

D. Atkinson Index

Atkinson unrivaled property in comparison with the other indexes is the aversion to inequity, which can variate depending of the situation. We decided first to test Atkinson index for inequity aversion taking the values 0 (no aversion), 0.5 (small aversion), 0.99 or 1 (great aversion) and 1.5 (huge aversion). From the literature the recommended values belongs to [0.5; 1.5] [25]. We consider that each airline represents one individual, therefore, all $n_i=1$.

1) Cancellation

The first observation is the behavior of Atkinson index when ε=0, which models an absence of aversion to inequity (by the exponents $(1-\varepsilon)$ and $1/(1-\varepsilon)$). This means that whatever the results are, imbalances are not influencing the inequity measurement. Atkinson index is then reduced to the sum of airlines performances divided by the number airline, which is nothing else than the mean value of performance divided by this same mean. Therefore, Atkinson index is always 0 when ε=0.

By comparing the Atkinson index values for the same solution with different ε, we indeed notice that the higher ε is, the higher the index value. However, when ε>1, $x_i^{(1-\varepsilon)}=1/x_i^{(\varepsilon-1)}$. Therefore, as for Theil index, we again tackle the $x_i=0$ issue. As indicated before, this use-case will occur frequently, and therefore cannot be considered as a side effect from the index. For ε=1, an adaptation of the Atkinson index is necessary, as mentioned in the index presentation. The Atkinson indexes between ε=1 and ε=0.99 are very similar over all the described solutions, validating the index continuity. Therefore, for simplification purposes, we propose to set an upper bound to ε at 1 for our use-case. We as from now focus our analysis on $\varepsilon \in [0.5; 1]$.

From a sensitivity point of view, for ε=1, AT(B) returns the highest value possible, which is 1. This matches the very imbalanced solution in which LX cancels all its flights for the group and great aversion to inequity expressed by the ε. Comparing with AT(C), in which LH MUC cancels the 10 necessary flights but still operates its remaining flights, the value is much lower, which is an expected behavior of the index. Comparing for ε=0.5 the same solutions, one can notice the increase for AT(B), but the small aversion to inequity balances the index, therefore not reaching the perfect inequity value.

2) Booked Passengers

Lastly, we can observe that for all solutions (except B which is perfect inequality), Atkinson index for ε=1 fluctuates around twice the values for ε=0.5. The bigger the inequity aversion is, the more sensitive the index is towards the lower class inequalities.

3) Atkinson Index' Global Assessment

This index is very interesting for our use-case. First, its sensitivity to inequity, as highlighted by the analysis on passenger equity criteria, gives a realistic value of the solution inequity. Second, the aversion to inequity is a useful parameter, which is unique in the inequity indexes proposed. This aversion parameter enables the researcher to adapt the index to each situation, depending on the decision-makers wish. In our use-case, we plan to allow each airline expressing its preferences before the optimisation. One airline could express no inequity aversion for cancellations as long as a good share of passengers has a satisfying solution, especially during hub airport capacity reduction due to foggy or snowy weather. In such hub constraints, the hub airline must anyway cancel flights, and could undertake easily the necessary cancellations for the group at the disrupted virtual hub, especially if disrupted passengers are handled thanks to the group collaboration. Moreover, another airline could ask for as few delays as possible if no crew reserve are available and tight crew rotations are planned, to avoid operational snowball effect. This would be expressed as a high aversion to inequity in terms of delayed flights.

The only disadvantage of this index is that each airline contribution cannot be easily observed in the global inequity index. We can not calculate the necessary repartition of resources needed to rebalance the equity.

E. Discussion

As mentioned in our problematic, inequity index must have the right sensitivity, and allow a particular acuity towards advantaged or disadvantaged airlines. These are key aspects to ensure the feasibility of a long-term equity. From the sensitivity perspective, each index has its strengths. Gini enables an equality approach, dealing with the exact same behavior for airlines advantaged or disadvantaged with the same deviation from the mean value. Theil index gives more importance to disadvantaged airline, modeling thus the human sensitivity to loss in contrast with gains. Atkinson index allows even more flexibility by expressing the inequity aversion in the formula itself and influencing thus the index results. All indexes catch the extreme inequitable solutions and returns the greatest value for the most imbalanced solution proposed in our analysis, as well as differentiate fully inequitable solutions from very inequitable solutions (solution B and C). However, as pointed in Gini discussion (B.3), one must be aware of the Gini index behavior, considering that a solution in which all airlines except one are undertaking cancellations is felt less equitable than the solution in which only one airline is cancelling for the entire group (but still operating some flights).

However, a clear finding of this analysis is the index dependency to the elementary performance indicator. When calculated on the ratio of flights operated, the results from the three indexes are not as accurate as when based on the ratio of non-impacted passengers. The analysis was on purpose simple and targeted only two sides of the complex reality to enable clear and understandable outcomes. To represent the full complexity, not only the impacted passengers must be taken into account, but also all the parameters influencing the operations performances for each airlines. The operational experts' feedback outlined this multi-criteria inequity definition necessity (see II.D).

Some indexes are easier to decompose in order to capture the airline contribution to inequity. Gini and Theil indexes both are straightforward decomposable. The sensitivity to penalized airlines leads in our inequity problematic to favor Theil over Gini index. Atkinson index is more complex to decompose into clear airlines' contribution. However, the scientific literature proposes many approaches of inequity index decomposition into within-group and between-group inequality, including comparison between Gini, Theil and Atkinson indexes [26]. Due to Atkinson and Theil strengths like sensitivity to penalized airlines and especially Atkinson's formalization of the inequity aversion, enabling to catch all the specificities of the situation and reflecting them in its value, we will continue our research with the Atkinson and Theil indexes in our given problematic of operations optimisation of multi-hubs airlines during disruption.

V. CONCLUSION AND PERSPECTIVES

This paper aims to present a new paradigm about equity as well as challenge the equity indexes widely accepted until now. From the FCFS and delay minutes perspectives, used by the ATC and NMOC, we proved that the equity definition cannot be based on only one performance indicator. On a simple operational use-case, based on our research topic about the optimisation of multi-airlines operations during disruption, we proposed different solutions to reach the best group performances, and two elementary performance indicators resulted in different inequity assessment by the indexes.

Three indexes, well known in economics and transportation research, were analyzed with respect to the ratio of operated flights and ratio of non-impacted passengers. A comparison of each index strengths and sensitivity highlighted the potential use for our operational use case. We can conclude that none of the index alone brings out a clear interpretation of the operational complexity. Gini index has an egalitarian approach which does not fit with our operational problematic, while Theil is presenting interesting sensitivity to disfavored airlines as well as easy decomposition into airlines' contribution to inequity calculations, and Atkinson enables interesting sensitivity to the complexity of the solution with the inequity aversion formalization in its formula. Therefore, we propose to continue our research with a combination of Theil and Atkinson indexes for our use-case.

Three main research paths arise from our work. The first one is the integration of several elementary performance indicators in the global inequity assessment of a solution. As shown by our analysis, taking into account only one aspect of equity (ratio of operated flights or ratio of non-impacted flights) overshadows the complex reality. Operational experts' feedbacks confirmed it. Crew, aircraft rotations constraints, connecting passengers but also environmental impact (fuel consumption, noise for direct and indirect users [27]) and many more aspects are needed to enable a trustable inequity calculation. ATM is a very complex environment, for which the equity cannot be summarized to only FCFS or delay minutes. If the analysis confirms clearly the need for multi-criteria analysis, we still need to propose new ways for calculating this multi-criteria inequity. We will focus our future research activities on this aspect.

The second research outlook is the combination of the Atkinson and Theil indexes. Both have specific strengths and one could bind in a clever way their utilization to get the best out of their association. Theil enables an accurate sensitiveness to disfavored airlines, which is needed in our use-case, while Atkinson main strength is the expression of the inequity aversion. We could in future research paths propose a combination of Theil and Atkinson indexes, as [28] proposed between Gini and Atkinson. This could lead to an accurate inequity contribution calculation for each airline and each situation, as well as enable an airline to express its inequity aversion, or link the inequity aversion to the history of inequity, to enable rebalance equity if needed after a given time.

The third and major perspective of this work is to think further about a new equity paradigm: ensuring long-term equity. As shown in the context and literature review, equity is most of the time addressed situation by situation, reducing the flexibility and the efficiency of accepted solution. By allowing temporary inequity between the group's stakeholders, and focusing on an equitable balance after a given time, the flexibility and efficiency would increase by enabling more feasible solutions and thus achieving far better group results. It has to be proved that the equity on a long run is guaranteed, so that all stakeholders accept temporary disadvantages in order to reach better performances for the group and in fine for themselves.

ACKNOWLEDGMENT

The authors would like to thanks Oliver Rueegg (SWISS) for his valuable inputs and helpful discussions, as well as all the operational experts from the Lufthansa Group for their participation in the use-case.

AUTHORS' BIOGRAPHIES

Marie CARRE works in Operations Optimisation departments of SWISS for four years. She developed the First Rotation hours Optimisation Trial locally in collaboration with EUROCONTROL NM, Skyguide and Zurich Airport, contributed to several Very Large Scale Demonstration Projects and participates nowadays in several Exploratory Research (ER4) projects such as BEACON, SlotMachine, ISOBAR and manages SWISS participation in all ER4 projects. Parallel to her job at SWISS, she started a PhD with the University of Clermont-Ferrand on the operations optimisation of multi-hubs airlines during disruption.

Séverine DURIEUX, engineer from the French Institute of Advanced Mechanics, she obtained her master's degree in operational research and production engineering in 2000 and her PhD in 2003 at the Blaise Pascal University of Clermont-Ferrand, France. She is assistant professor at SIGMA Clermont, an engineering school of the INP Clermont Auvergne. Her research topics are decision making, uncertainty management, optimisation problems for product design and industrialization and for the design and management of industrial and logistic systems. Since 2017, she has been the operational director of the Auvergne Rhône Alpes Aeronautical Campus.

Eric NANTIER is the head of "Operations Optimisation & Development" team at SWISS, aiming at ensuring the strategic OPS development and alignment. He is responsible for research, analysis and simulation of Operations Planning and Steering solutions, as well as strategy, deployment and operation. The scope of which includes Crew Management Systems, Ops Control, Dispatch and PAX Flow Management. He works since 21 years at SWISS & SWISSAIR (Ops Research & Air Traffic Management, Dispatch & Control Center and Operations Support). Furthermore, he has directed Steria Zurich, has been Group leader at ABB Transportation and a consultant at Altran.

Laurent PIÉTRAC received the B.Sc. degree in mechanical engineering and the M.Sc. degree in flexible manufacturing systems from the University of Paris VI, France, in 1990 and 1993, respectively, and the Ph.D. degree in automatic control from the Ecole Normale Supérieure de Cachan, France, in 1999. He is currently a Full Professor at SIGMA Clermont, an engineering school of the INP Clermont Auvergne. His research interests include discrete-event systems (DES), supervisory control theory (SCT), operating mode management and manufacturing operations management (MOM). His application are very diversified: manufacturing processes, transportation systems, HVDC networks and embedded systems.

REFERENCES

[1] E. O. Lewis, D. MacKenzie, and J. Kaminsky, "Exploring equity: How equity norms have been applied implicitly and explicitly in transportation research and practice," *Transp. Res. Interdiscip. Perspect.*, vol. 9, p. 100332, Mar. 2021, doi: 10.1016/j.trip.2021.100332.

[2] International Civil Aviation Organization, "GlobalAir Traffic ManagementOperational Concept - Doc 9854AN/458," 2005. [Online]. Available: https://www.icao.int/Meetings/anconf12/Document%20Archive/9854_cons_en%5B1%5D.pdf

[3] EUROCONTROL, "Network ManagerAnnual Report 2019," EUROCONTROL, Sep. 2020. Accessed: Jun. 04, 2021. [Online]. Available: https://www.eurocontrol.int/publication/network-manager-annual-report-2019

[4] J. Fairbrother, K. G. Zografos, and K. D. Glazebrook, "A Slot-Scheduling Mechanism at Congested Airports That Incorporates Efficiency, Fairness, and Airline Preferences," *Transp. Sci.*, Dec. 2019, doi: 10.1287/trsc.2019.0926.

[5] T. Vossen, M. Ball, R. Hoffman, and M. Wambsganss, "A General Approach to Equity in Traffic Flow Management and Its Application to Mitigating Exemption Bias in Ground Delay Programs," *Air Traffic Control Q.*, vol. 11, no. 4, Oct. 2003, doi: 10.2514/atcq.11.4.277.

[6] C. Chin, K. Gopalakrishnan, M. Egorov, A. Evans, and H. Balakrishnan, "Efficiency and Fairness in Unmanned Air Traffic Flow Management," *IEEE Trans. Intell. Transp. Syst.*, pp. 1–13, 2021, doi: 10.1109/TITS.2020.3048356.

[7] D. Kahneman and A. Tversky, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, vol. 47, no. 2, p. 263, Mar. 1979, doi: 10.2307/1914185.

[8] N. Pilon, A. Cook, S. Ruiz, A. Bujor, and L. Castelli, "Improved Flexibility and Equity for Airspace Users During Demand-capacity Imbalance," p. 9, 2016.

[9] P. E. Fernandez de la Torre, "Airline Alliances:The Airline Perspective," Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, 1999.

[10] M. Carré, A. Rungger, N. Pilon, and L. Guichard, (in press) "User-Driven Prioritisation Process (UDPP) from advanced experimental to pre-operational validation environment," *J. Air Transp. Manag.*, 2021.

[11] L. L. Price and B. S. Rowntree, "Poverty: A Study of Town Life.," *Econ. J.*, vol. 12, no. 45, p. 56, Mar. 1902, doi: 10.2307/2957025.

[12] M. A. Knapp, "Social Effects of Transportation," *Ann. Am. Acad. Pol. Soc. Sci.*, vol. 20, no. 1, pp. 1–15, Jul. 1902, doi: 10.1177/000271620202000101.

[13] G. Currie, "Quantifying spatial gaps in public transport supply based on social needs," *J. Transp. Geogr.*, vol. 18, no. 1, pp. 31–41, Jan. 2010, doi: 10.1016/j.jtrangeo.2008.12.002.

[14] A. Golub and K. Martens, "Using principles of justice to assess the modal equity of regional transportation plans," *J. Transp. Geogr.*, vol. 41, pp. 10–20, Dec. 2014, doi: 10.1016/j.jtrangeo.2014.07.014.

[15] M. Taleai, R. Sliuzas, and J. Flacke, "An integrated framework to evaluate the equity of urban public facilities using spatial multi-criteria analysis," *Cities*, vol. 40, pp. 56–69, Oct. 2014, doi: 10.1016/j.cities.2014.04.006.

[16] F. Cavallaro, F. Bruzzone, and S. Nocera, "Spatial and social equity implications for High-Speed Railway lines in Northern Italy," *Transp. Res. Part Policy Pract.*, vol. 135, pp. 327–340, May 2020, doi: 10.1016/j.tra.2020.03.028.

[17] A. S. van der Veen, J. A. Annema, K. Martens, B. van Arem, and G. H. de A. Correia, "Operationalizing an indicator of sufficient accessibility – a case study for the city of Rotterdam," *Case Stud. Transp. Policy*, vol. 8, no. 4, pp. 1360–1370, Dec. 2020, doi: 10.1016/j.cstp.2020.09.007.

[18] M. Dixit and A. Sivakumar, "Capturing the impact of individual characteristics on transport accessibility and equity analysis," *Transp. Res. Part Transp. Environ.*, vol. 87, p. 102473, Oct. 2020, doi: 10.1016/j.trd.2020.102473.

[19] F. Zagonari, "Responsibility, inequality, efficiency, and equity in four sustainability paradigms: Policies for a shared sea from a multi-country analytical model," *Mar. Policy*, vol. 87, pp. 123–134, Jan. 2018, doi: 10.1016/j.marpol.2017.10.016.

[20] K. Martens, *Transport Justice. Designing fair transportation systems*, New York, NY: Routledge, 2016. 2016.

[21] K. Fourrey, "Décomposition des indices d'inégalité et impact des politiques publiques," Thesis. Economies et finances, Normandie Université, 2019.

[22] C. Gini, *Variabilità e Mutuabilità. Contributo allo Studio delle Distribuzioni e delle Relazioni Statistiche (Variability and Mutability, Contribution to the Study of Statistical Distributions and Relations)*. Studi Economico-Giuridici della R. Cagliari: Università di Cagliari., 1912.

[23] A. Sen and J. E. Foster, *On economic inequality*, Enl. ed. Oxford : New York: Clarendon Press ; Oxford University Press, 1997.

[24] A. B. Atkinson, "On the measurement of inequality," *J. Econ. Theory*, vol. 2, no. 3, pp. 244–263, Sep. 1970, doi: 10.1016/0022-0531(70)90039-6.

[25] F. G. De Maio, "Income inequality measures," *J. Epidemiol. Amp Community Health*, vol. 61, no. 10, pp. 849–852, Oct. 2007, doi: 10.1136/jech.2006.052969.

[26] T. Dast and A. Parikh, "Decomposition of inequality measures and a comparative analysis," vol. 7, no. empirical economics, pp. 23–48, 1982.

[27] E. Nantier, "Airline expectations: Reshape the ATM system around passenger priorities," Feb. 2021. [Online]. Available: https://www.skyguide.ch/wp-content/uploads/2021/02/Airlines-expectations-from-SKY_Blue_Print_2020_Web-2.pdf

[28] V. Jalbert, "Inéquité Horizontale et Reclassement Engendrés par l'Impôt sur le Revenu des Particuliers et les Transferts au Canada," Faculté des sciences sociales, Université Laval, 1998.