Estimating stochastic air transport process times using the Fuzzy Critical Path Method

Determination of the Estimated aircraft Total Turnaround Time (ETTT)

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Abstract-Predicting both optimal and reliable aircraft turnaround time is one of the most critical tasks in ACDM flight scheduling plans. Considering the effects of randomness and fuzziness on the turnaround process duration, we transform the probability distribution of well-fitted sub-processes into a cumulated density function, equivalent to the fuzzy membership function (FMF), then identify the fuzzy membership grade of each turnaround sub-process by goodness-of-fit. Based on the Critical Path Method (CPM) technique and fuzzy set theory, the turnaround time, considering stochastic effects, is evaluated using the fuzzy critical path method (FCPM). To validate this calculation, we constructed the FMF from historical data and compared it with FCPM-based turnaround times. In addition, we utilize these historical data to generate fuzzy sets of different arrival delays using Frankfurt airport data of summer 2017 and conclude that delays are positively correlated with the FCPMbased turnaround process through a linear regression model.

Keywords—Decision Support System; Airline Ground Operations; Uncertainty; FCPM; Fuzzy Decision Making

I. INTRODUCTION

In a world with increasing air traffic density and rapid growth of the global civil aviation industry, despite the pandemic situation in 2020-2021 [1], considering the need for extremely cost efficient operations due to critical financial conditions for the entire ATM industry and, on the longer run, the return to scarce airport capacities, ground operations are a challenging task with limited flexibility for airlines. Air traffic control (ATC) take main responsibility for on-block times (the time between arriving at the gate and leaving it for departure). Those have been standardized through the implementation of sophisticated decision support mechanisms such as Airport Collaborative Decision Making (A-CDM), supported at major airports by Arrival Manager (AMAN), Departure Manager (DMAN) and advanced approach procedures such as optimized trombones as P-RNAV procedures or Point Merge Systems (PMS). Hence, the turnaround remains the only time frame within which an airline has major but still no exclusive control over the operations of its aircraft. Due to

the very complex nature of the turnaround, which involves the interaction of various stakeholders for multiple simultaneous processes [2]-[4], a large part of ground operations has yet to be transferred into the digital era to unlock further potential for efficiency gains. A practical approach to tackle this issue is to assess aircraft turnaround time accurately for given conditions. However, dozens of sub-processes constituting the aircraft turnaround are uncertain as multiples participants are involved, including airlines, ground handling agents, airport operators, air traffic control, etc. Consequently, the airspace user will have to consider aircraft as stochastic to perform robust net planning. This is not only because proper scheduling of the turnaround time allows for an efficient integration of resources in the turnaround process thus effectively minimizing monetary losses due to flight delays. For the net perspective, the short and precisely scheduled turnaround times may increase the airline's economic efficiency due to improved aircraft utilization.

The way how these ground handling activities are controlled mostly depends on the individual expert knowledge of the involved experts from airline operations centers (AOC) and ramp agents, coordinating the turnaround processes based on deterministic planning tools and via headset/ telecommunication. The introduced concept of information sharing in the context of Airport - Collaborative Decision Making (A-CDM) allows to manage timestamps from touch-down until departure, thus including on- and off-block times per movement (AONT/AOBT) [5], [6]. Further details to these processes are gathered - if ever - on airline level, not yet being shared through A-CDM. Technological support in this workflow is still limited, depending on the local system infrastructure. Consequently, in case of disruptions, the scarce data availability and management capabilities hamper coordination and delay mitigation measures. Also, lead times at detection of such negative events are often too short to allow for effective operational delay compensation. Post-operational analyses of delay codes do not help in identifying workflow weaknesses, since these codes are often assigned very biased [7]. Proactive considerations based on A-CDM time stamps

so far are solely applied to the inbound side, preparing for potential arrival delays by implementing time buffers in the block times planning allowing to absorb such deviations for the price of reduced plan efficiency on average [2], [8]–[10]. The accurate integration of process uncertainties and variable process executions into the aircraft turnaround problem is the fundamental aim of this paper, using the Fuzzy critical path method (FCPM). FCPM is different from probabilistic approaches for critical path analysis. We aim to highlight the advantages of FCPM, to investigate the incorporation of a fuzzy duration of activities within the critical path analysis, and to present a straightforward computational approach for FCPM.

The paper starts with introducing FCPM in Section II and the results of its application to an example aircraft turnaround are described in Section III. Section IV describes the application of the new method to turnaround control with considering historical data and different delay sets, and the results are analyzed and discussed in this section. Section V draws conclusions and provides an outlook onto different areas of research.

II. METHODOLOGY

In this section, the problem characteristics are presented, and it is explained why FCPM is a promising method superseding current strategies for the incorporation of uncertainties in the prediction of stochastic processes such as turnarounds with their target times. Finally, the underlying mathematical procedure is introduced concerning a distinct turnaround process network.

A. Fuzzy numbers and fuzzy sets

The nature of logistic processes can be divided into two broad categories: Deterministic behavior or uncertainty. The latter is often further split into randomness and vagueness. We consequently apply three types of mathematics to study respective data sets. The first type belongs to the group of deterministic mathematics, which explores the object in certainty. There must be some necessary connections between the objects, which could be modelled through differential methods. The second type belongs to stochastic mathematics based on probability theory. Monte Carlo Simulation, Bayesian Network, or Markov Chains are typical methods to search for casual relationships between objects. The last type is found in fuzzy mathematics, studying vague phenomena [11], [12].This paper focuses on the third type of mathematics: the fuzzy number.

Fuzzy sets represent an alternative mathematical framework that provides a solution to the uncertainty arising from the lack of clear criteria for classifying members [13].

As the range of the inputs are not the same, it is necessary to make the fuzzy membership function $\mu_{\tilde{A}}$ normalizes the values to [0,1],

$$\mu_{\tilde{A}} = \begin{cases} U \to [0,1] \\ u \mapsto \mu_{\tilde{A}}(u) \end{cases}$$
(1)

where $u \in U$ and the value of function $\mu_{\tilde{A}}$ is the membership level of fuzzy set \tilde{A} , and the function $\mu_{\tilde{A}}$ represents the membership function of fuzzy set \tilde{A} [14],

$$\iota_{\tilde{A}} = \{ (u, \mu_{\tilde{A}}(u)) | u \in U \}$$

$$\tag{2}$$

Since the operations in this paper involve only fuzzy addition and fuzzy subtraction, these two arithmetic rules are introduced. Assuming that $\forall a_1, a_2, b_1, b_2 \in U$, and $\tilde{A} = (a_1, a_2), \tilde{B} = (b_1, b_2)$, the fuzzy addition and fuzzy subtraction can be formulated by (3) and (4):

$$(a_1, a_2)(+)(b_1, b_2) = (a_1 + b_1, a_2 + b_2)$$
 (3)

$$(a_1, a_2)(-)(b_1, b_2) = (a_1 - b_2, a_2 - b_1)$$
(4)

B. Typical Fuzzy Membership Functions (FMF)

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Much data with ambiguity and uncertainty in practical problems can be represented by fuzzy membership functions (FMF), while there are various types of FMF, such as e.g., trapezoidal FMF, triangular FMF, Bell-shaped FMF, Sigmoid FMF, Gaussian FMF. [15]. In this part, we introduce two typical fuzzy membership functions, triangular and trapezoidal FMF and evaluate which one is more suitable to our model. These two memberships have been selected due to their straightforward arithmetic operations.

a) Triangular fuzzy membership functions (FMF): Supposing the membership function of a triangular fuzzy number $\tilde{A}(a, b, c)$ with the lower limit a, upper limit c and the a value of b, where a < b < c.

In order to perform the arithmetic operations of two triangular FMFs, (5) and (6) define the addition and subtraction formulas.

$$\tilde{A} + \tilde{B} = (a_1, b_1, c_1)(+)(a_2, b_2, c_2)$$

= $(a_1 + a_2, b_1 + b_2, c_1 + c_2)$ (5)

$$\tilde{A} - \tilde{B} = (a_1, b_1, c_1)(-)(a_2, b_2, c_2) = (a_1 - c_2, b_1 - b_2, c_1 - a_2)$$
(6)

b) Trapezoidal fuzzy membership functions (FMF): The membership function of a trapezoidal fuzzy number $\tilde{A}(a, b, c, d)$ has the lower limit a, upper limit d, a lower support limit b, and an upper support limit c, where a < b < c < d.

In order to perform the arithmetic operations of two trapezoidal FMFs, (7) and (8) again define the addition and subtraction formulas.

$$\tilde{A} + \tilde{B} = (a_1, b_1, c_1, d_1)(+)(a_2, b_2, c_2, d_2)$$

= $(a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$ (7)

$$\tilde{A} - \tilde{B} = (a_1, b_1, c_1, d_1)(-)(a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$
(8)

C. Probability-Possibility transformations

Statistics are the extension of mathematics from a specific to an accidental phenomenon, while fuzzy numbers are the extension of mathematics from the deterministic to the vague realm. Reference [12] developed the possibility theory in which a FMF could be illustrated as a possibility distribution and stated that the relationship between fuzzy numbers and possibility distributions could be analogous to the relationship between random numbers and probability distributions.

Probability theory is the basis for statistics, which focuses on the study of quantitative laws in chance, i.e., the probability that a given element occurs in a universal set [14], while possibility theory is related to the state of being believed and plausible [16], and reflects the imprecise information with a fuzzy membership degree normalized to [0,1].

Many pieces of research have tried to accomplish probability-possibility distribution transformation in different ways. Reference [17] put forward several methods to convert probability distribution into possibility distribution. The most promising approach was the "interval scale method", which combined the Shannon entropy H(p) the Dempster-Shafer theory V(p). The former attempted to quantify the uncertainties in decision situations using a probabilistic approach, while the latter assumed a fuzzy set \tilde{A} of "U-uncertainties"). Reference [18] established a method for measuring probability under non-deterministic conditions by discussing the connection between probability-possibility transitions, confidence intervals, and probability inequalities. Reference [19] provided a feasible method for transforming statistical data into a fuzzy rule base in accordance with the "hierarchical fuzzy rules" and used it for the diagnosis of medical aphasia. Reference [20] proposed an approach to transform probability- into possibility distributions via FMF from a likelihood perspective. The method constructs a standard fuzzy set, which can be viewed as a triangular or trapezoidal FMF. For that set, the best parameter combinations were fitted by the input probability distribution. Reference [16] put forward another "Hypothesis Test" approach to obtain fuzzy sets directly from probability distribution by simulating various decision rules.

D. Probabilistic data of A320 turnaround

The accurate calculation of the target off-block time (TOBT) for a given flight represents a crucial process to effectively apply A-CDM. The difficulty arises from the complexity of the prevailing aircraft turnaround and the number of involved resources and staff members, all impacting the TOBT and other A-CDM timestamps. The (IATA) ground operations manual ((I)GOM) for an A320 aircraft lists twelve related processes (see Fig. 1) which split – according to the individual interest of the operator – into about 150 individual sub-tasks and involves up to 30 different actors. Although not all processes need to be executed in every turnaround (i.e., low-cost carriers tend to cater or clean their aircraft less often, depending on the flight characteristics), there are a number of mandatory processes, which often characterize the critical path of the turnaround (red boxes in Fig. 1).

According to the GOM [4] the critical path of a typical turnaround comprises Fuelling out of the set of candidate processes running in parallel (Fig. 1). Consequently, other processes such as cleaning or catering may constitute the critical path. The probability set is then expressed by stochastic process models [21]. Minimizing the turnaround time in case of schedule deterioration may consequently imply interference with several processes to guarantee an effective

schedule recovery. Once these recovery measures are applied, estimating their impact on the TOBT is found to be complex.



Figure 1: MPM Network Graph of a typical Turnaround with critical path and main activities

As mentioned before, it is crucial for an airline to constantly monitor every time stamp and in particular the TOBT considering all uncertainties. As the stochastic interdependencies are hardly manageable manually, this paper suggests automated decision support transferring the knowledge about time distributions and resource dependencies of each individual core process into a mathematical model, which aggregates all distributions into a single one. To the best of our knowledge, this approach is deemed to produce more accurate results than existing - purely stochastic methods which usually depend on Monte-Carlo simulations or involve state-transition models (see [3], [8], [22]). Many pieces of research on modeling TA with uncertainty are mainly based on probabilistic theories, such as Bayesian networks or Monte Carlo simulations [23], [24]. Furthermore, only a few attempts have been made to deal with TA as a project scheduling problem in an uncertain environment. Furthermore, our model type can directly compute the total impact time of recovery measures (translated into parameter adaptations) at a preset confidence interval without repeating the entire simulation.

To consider the stochastic processing times of each activity, deterministic GOM values are substituted by fitted Gammaor Weibull-distributions as shown in Table I and Fig. 2. Both distribution types are well-suited as they comprise no negative time values and have demonstrated a good fit when matched with data from operational analyses [4], [21]. The parameters for deboarding, fuelling, catering, cleaning, and boarding were, thereby, adopted from previous studies of our chair [4], [21]. All the remaining processes were also matched to Gamma-distributions, containing the respective GOM values as 80% quantiles. It should be noted that all parameters are variables in the model and can be adjusted to airline-/ airport-specific operational characteristics at any time.

In this approach, all processes are assumed to be independent of each other, as typically executed by different resource entities. According to Table I, standard operating procedure (SOP) schedules, for instance, three handling agents for the cargo processes UNL and LOA, while CLE, CAT, and FUE are executed by a single servicing vehicle or crew, respectively. Possible interactions between parallel cabin and ramp processes are neglected since they are supposed to be centrally coordinated by the ramp agent. Strategic buffer times are also omitted, as, in case of delays, they would naturally be consumed with priority and have no direct effect on the

Process/Activity			Mean	St.Dev.
Abbrev.	Abbrev. Name Distribution		in min	in min
IB	In-Block ^a	-	-	-
ACC	Acceptance	Gamma (2.0,1.0)	2.00	1.41
DEB	Deboarding	Gamma (6.81,1.47)	10.01	3.83
CLE	Cleaning	Weibull (2.16,11.29)	9.99	4.88
CAT	Catering	Weibull (2.18,17.37)	15.38	7.44
FUE	Fuelling	Gamma (9.12,1.64)	14.96	4.95
UNL	Unloading	Gamma (11.29,1.24)	11.10	3.71
LOA	Loading	Gamma (15.34,1.24)	15.48	4.38
BOA	Boarding	Gamma (14.36,1.47)	21.10	5.57
FIN	Finalization	Gamma (4.0,1.0)	4.00	2.00
OB	Off-Block ^a	-	-	-

TABLE I. Stochastic turnaround process distribution [21]

^aIB and OB as start and end events have no duration.



Figure 2: Stochastic turnaround standard process distributions

execution of the individual processes [4]. This leaves the MPM network graph as depicted in Fig. 1, for which the stochastic processing time will be calculated using FCPM. Equally to the critical path algorithm, sequential processes are added with their duration, while parallel processes wait for their longest (critical) counterpart to be finished before the next activity can be started. As the resulting function is not trivial and does not follow a known distribution, it is necessary to describe it via its constituents.

The most classic project scheduling methods include the Critical Path Method (CPM) and Plan Evaluation Method and Review Technology (PERT), both are project management techniques based on network analysis to illustrate the sequence of the task [25].

Critical Path Method (CPM) is a proven solution to a project scheduling problem, based on network analysis to illustrate the task sequence when the task duration is determined. Therefore, we combine the CPM with fuzzy set theory. Fuzziness and randomness in network planning are given sufficient consideration, i.e., using FCPM to achieve a more realistic membership degree of the fuzzy network's total duration. Reference [26] proposed the combination of the concept of fuzzy sets with the CPM technique, where fuzzy sets could be used to characterize task durations under uncertainty, and CPM was used to illustrate the sequence of the tasks. Reference [27] tried to express the task duration in terms of fuzzy number and to find the earliest start time of the task. In these studies, time parameters of the task in the fuzzy network were obtained from the forward- and backward calculation using the CPM technique. However, in the arithmetic operation of fuzzy numbers, fuzzy number \hat{A} plus fuzzy number \hat{B} equals fuzzy number \hat{C} . But, it cannot be derived that fuzzy number \tilde{A} equals fuzzy number \tilde{C} fuzzy minus fuzzy number \tilde{B} . So, direct application of the CPM technique to a fuzzy set in the backward calculation would not get to correct results for the latest start and the slack time. Therefore, [28] considered an "interactive fuzzy subtraction" to construct the fuzzy critical path and viewed that only the positive part (including zero) of the fuzzy set was practically meaningful. Reference [29] proposed a general form for computing the latest start and slack time of a task to extend backward calculations for all fuzzy networks. [30] used the FCPM technique to obtain a fuzzy critical path for aircraft maintenance projects in an uncertain environment by considering the attitude of decision-makers towards risk.

III. APPLICATION TO TURNAROUND CONTROL

Knowing a reliable target time for any turnaround operation is just the first step towards efficient operations management of an airline. Due to tight schedules and the complex itineraries of aircraft, crew, passengers and servicing resources, potential deviations of the turnaround target time from the original schedule can cause significant propagation effects throughout the entire airline network [31], [32].

A. Fuzzify of the A320 turnaround data

Fuzzification of data covers the transformation of a probability distribution to a possibility distribution. There are some approaches to implement the probability-possibility transformation (see [17], [18], [20]), and the Hypothesis Test Method is tested to preserve the most original information in the conversion, whose main idea is through the continuous distribution function of the decision rule α to obtain the corresponding membership level of a fuzzy set [16]. The flowchart to conduct the probability-possibility transformation is as Fig. 3, where x stands for the period covered by each probability with α significance level.

The p-value data distribution of the test statistic in this approach could be calculated by (9),

$$p - value(x) = 2 * (0.5 - |0.5 - CDF_D(x)|)$$
(9)

The p-value refers to the probability that the observed sample would seem more extreme if H_0 is not rejected. However, if p-value > a, it seems appropriate not to reject H_0 and vice versa. The parameter $\delta(H_0)$ in (10) is to ensure that the decision-rule α in the transformation cannot be rejected H_0 .

$$\delta(H_0)(x,a) = \begin{cases} 1, & if[p-value(x) > \alpha] \\ 0, & if[p-value(x) < \alpha] \end{cases}$$
(10)

Decision rule α is repeated from α_{min} to α_{max} and the probability density distribution function PDF_A is summed to obtain a cumulative distribution function CDF_A . Regarding (10) the possibility distribution could be written as (11). The possibility distribution is computed as the integral of the probability distribution from a decision rule α , which



Figure 3: Flowchart of probability-possibility transformation algorithm

is determined by simulating different probability density functions.

$$\Pi_{A}(x) = \int_{a_{min}}^{a_{max}} \delta(H_{0})(x, a) * PDF_{A}(a) da$$

$$= \int_{a_{min}}^{p-value} PDF_{A}(a) da$$

$$= CDF_{A}(p-value) - CDF_{A}(\alpha_{min})$$
(11)

where $\Pi_A(x)$, the possibility distribution, could be treated as fuzzy membership function $\mu_{\tilde{A}}(x)$ [12].

Knowing each process distribution and its parameters (see I), the probability density function (PDF) and the cumulative distribution function (CDF) can be obtained through the distribution calculator. After implementing the transformation algorithms as depicted in Fig. 3, the transformation results for each sub-process duration are presented in Fig. 4, where the x-axis represents the task duration (in minutes), and the y-axis represents both the probability of the Probability Density Function and possibility function beside the red line which depicts the PDF of their relative process.

B. FCPM Algorithms

As mentioned in Section I, FCPM is a combination of CPM techniques and the fuzzy set theory, which is effective in project scheduling problems with uncertain task durations. FCPM has two basic principles:

- **Principle 1:** Only the non-negative part of fuzzy numbers has a physical explanation because the uncertain factor in this paper is time, and as such is always positive. [28].
- **Principle 2:** The fuzzy critical path is represented by the minimal ranking indices of all fuzzy members. In other words, after calculating the possible fuzzy set of



Figure 4: Transformation Result for Process Duration (X-axis shows the duration in Min)

all paths, the one with a minimum flexibility (minimal ranking indices) is the critical one [33].

To find the best fuzzy membership function between trapezoidal and trianglar FMFs described in section II-B, the Kolmogorov-Smirnov test is used in this paper to compute a goodness-of-fit for the possibility distribution to ascertain whether Triangular or Trapezoidal is the best fitted function for each process. The results are illustrated in Fig. 5.

Table II shows the optimal parameter combinations for each fuzzy membership function. The K-S test showed stability between 0.08 and 0.09 for each fuzzy membership function.

In this paper, we choose trapezoidal FMF to describe the



Figure 5: Result of Goodness-of-Fit of K-S test for each Process Duration (X-axis shows the duration in Min)

TABLE II. Parameter combinations of FMF for each process

Process	Triangular FMF	Trapezoidal FMF
ACC	(0.24,0.24,6.00)	(0.36,0.72,0.96,6.00)
DEB	(4.20,6.20,21.00)	(4.20,4.20,6.20,21.00)
CLE	(1.76,22.00,22.00)	(1.76,2.20,3.08,22.00)
CAT	(3.40,4.08,34.68)	(3.40,4.08,5.34,34.68)
FUE	(7.28,8.28,28.56)	(7.28,7.28,8.28,28.56)
UNL	(7.00,9.00,25.00)	(7.00,7.00,9.00,25.00)
LOA	(10.88,11.88,32.00)	(10.88,10.88,11.88,32.00)
BOA	(11.52,28.96,36.00)	(11.52,26.16,28.96,36.00)
FIN	(1.40,9.00,10.20)	(1.40,8.00,9.00,10.20)

whole turnaround duration, since a triangular FMF could be regarded as a special trapezoidal FMF (when b = c), and also trapezoidal FMF has a better fitting than the triangular in some sub-processes, such as Catering with the better K-S test result.

C. Defuzzify of the fuzzy membership functions

There are different defuzzification methods to derive in a crisp number from fuzzy membership function. These methods can turn the parameter combinations of the trapezoidal FMF into a crisp number to verify if the result of the conversion is plausible. Setting the trapezoidal fuzzy number as $\tilde{A} = [a, b, c, d]$, the formula for defuzzification can be expressed as (12)

$$Defuzzy(\tilde{A}) = \frac{(c^2 + d^+cd) - (a^2 + b^2 + ab)}{3[(c+d) - (b+a)]}$$
(12)

The defuzzification result for each sub-process can be obtained by building the trapezoidal FMF based on the optimal parameter combinations illustrated in Table II. As shown in Table III, empirically, the crisp values of each subprocess duration that return from the fuzzy sets are realistic compared to the mean value of each sub-processes in Table I, indicating that the transformation and the goodness-of-fit process can be implemented. Therefore, a fuzzy set of the sub-process duration could be used directly for implementing the computation.

TABLE III. Defuzzification results for each turnaround process

Nr.	Process	Fuzzy Set (a,b,c,d)	Defuzzification
1	IB	(0.00, 0.00, 0.00, 0.00)	0
2	ACC	(0.36, 0.72, 0.96, 6.00)	2.37
3	DEB	(4.20, 4.20, 6.20, 21.00)	9.87
4	CLE	(1.76, 2.20, 3.08, 22.00)	9.69
5	CAT	(3.40, 4.08, 5.34, 34.68)	13.39
6	FUE	(7.28, 7.28, 8.28, 28.55)	14.38
7	UNL	(7.00, 7.00, 9.00, 25.00)	13.07
8	LOA	(10.88, 10.88, 11.88, 32.00)	17.94
9	BOA	(11.52, 26.16, 28.96, 36.00)	25.16
10	FIN	(1.40, 8.00, 9,00,10.20)	6.79
11	OB	(0.00, 0.00, 0.00, 0.00)	0

D. Calculation of A320 Turnaround using FCPM Method

Following is some primary time symbols in the FCPM [34]: A_{ij} The task, which is connected between

		node <i>i</i> and node <i>j</i>
	FET_{ij}	Fuzzy duration of A_{ij}
	FTS_{ij}	Fuzzy slack time of A_{ij}
	FES_j	Fuzzy earliest time of node j
	FLF_j	Fuzzy latest time of node j
	Pred _j	Predecessor tasks
,	Succ _j	Successor tasks
	P_i	Path i from the start node to the end node
	$FCPM(P_i)$	Fuzzy duration of the path i
		in the project
	$FCPM(P_c)$	Fuzzy duration of the fuzzy critical path
		in the project

First, we set the fuzzy earliest time of the first task as $FES_1 = (0, 0, 0, 0)$, then the fuzzy earliest time of the predecessor and the fuzzy duration of this task FET_{ij} are added by fuzzy trapezoidal addition using (7) to obtain fuzzy earliest time FES_j of task j, furthermore, there must be the largest FES_j in the forward calculation of all possible paths, which is considered as the total duration spent on the entire project. The forward calculation is performed using (13).

$$\begin{cases} FES_j = (0, 0, 0, 0), & j = 1\\ FES_j = max\{FES_i(+)FET_{ij} | i \in Pred_j\}, & j \neq 1\\ (13) \end{cases}$$

Then, we set $FLF_n = FES_n$, by the definition that the fuzzy earliest time of the last task is equivalent to the fuzzy latest time of the last task. In the backward process, FLF_j should be calculated from the end node to the start node in a reverse order (j = n - 1, n - 2, ..., 2, 1). Then, the fuzzy latest time of the successor subtracts fuzzy duration FET_{ij} of this task by trapezoidal fuzzy subtraction (8) to obtain fuzzy latest time FLF_j of this task. Reference to (14), there must be the smallest FLF_j of all possible paths.

$$\begin{cases} FLF_n = FES_n, & j = n\\ FLF_j = min\{FLF_k(-)FET_{jk} | k \in Succ_j\}, & j \neq n\\ (14) \end{cases}$$

After that, the task slack time FTS_{ij} is the allowable delay in the task implementation because it does not cause any delay in the entire project duration. Therefore, there is no fuzzy slack time in the critical path [30]. The fuzzy slack time FTS_{ij} could be represented as (15).

$$FTS_{ij} = FLF_j - (FES_i + FET_{ij}), 1 \le i \le j \le n; |i, j \in N$$
(15)

The fuzzy duration of the path P_i in the project network is the sum of all fuzzy slack time in all possible paths, which is formulated by (16).

$$FCPM(P_i) = \sum_{1 \le i \le j \le n, |i,j \in P_i} FTS_{ij}, \ P_i \in P$$
(16)

The fuzzy critical path refers to the minimal fuzzy set of all possible paths in the project depicted by (17).

$$FCPM(P_C) = min \ FCPM(P_i) \mid P_i \in P \tag{17}$$

The fuzzy critical path method calculation includes the forward-and backward processes, which possess consistent procedures with the CPM technique. The fuzzy task duration FET_{ij} is obtained from the goodness of fit result from Table II.

Table IV shows Fuzzy Earliest Time (FET) and Fuzzy Latest Time (FLT) of each process. The results of calculation are obtained using (13) and (14).

Regarding FES_j and FLF_j , now the fuzzy slack time, FTS_{ij} , can be calculated using (15) and the results are depicted in Table V.

Reference [35] proposed a decision-aid method concerning fuzzy sets to identify the fuzzy critical paths as well as the fuzzy duration for the network by calculating the risk index

TABLE IV. Fuzzy Earliest Time (FET) and Fuzzy Latest Time (FLT)

Task	Fuzzy Earliest Start	Fuzzy Latest Finish
IB	(0.00, 0.0.00, 0.0.00, 0.0.00)	(-76.99, -7.04, 7.04, 76.99)
ACC	(0.36, 0.72, 0.96, 6.00)	(-70.99, -6.08, 7.76, 77.35)
DEB	(4.20, 4.20, 6.20, 21.00)	(-49.99, 0.12, 11.96, 81.55)
CLE	(1.76, 2.20, 3.08, 22.00)	(14.56, 37.36, 45.4, 100.35)
CAT	(3.40, 4.08, 5.34, 34.68)	(-21.44, 8.40, 19.24, 88.83)
FUE	(7.28, 7.28, 8.28, 28.55)	(-21.44, 8.40, 19.24, 88.83)
UNL	(7.00, 7.00, 9.00, 25.00)	(-21.44, 8.40, 19.24, 88.83)
LOA	(10.88, 10.88, 11.88, 32.00)	(24.76, 46.36, 53.4, 101.75)
BOA	(11.52, 26.16, 28.96, 36.00)	(14.56, 37.36, 45.40, 100.35)
FIN	(1.40, 8.00, 9,00,10.20)	(24.76, 46.36, 53.4, 101.75)

TABLE V. Fuzzy slack time in the turnaround process

Nr.	Task	FTS
(1,2)	(IB, ACC)	(-76.99, -7.04, 7.04, 76.99)
(2,3)	(ACC, DEB)	(-76.99, -7.04, 7.04, 76.99)
(2,7)	(ACC, UNL)	(-48.44, 15.52, 26.80, 82.11)
(3,6)	(DEB, FUE)	(-76.99, -7.04, 7.04, 76.99)
(3,5)	(DEB, CAT)	(-83.12, -4.10, 10.24, 80.87)
(3,4)	(DEB, CLE)	(-70.44, -1.84, 12.12, 82.51)
(7,8)	(UNL, LOA)	(-48.44, 15.52, 26.80, 82.11)
(8,10)	(LOA, FIN)	(-48.44, 15.52, 26.80, 82.11)
(6,9)	(FUE, BOA)	(-76.99, -7.04, 7.04, 76.99)
(5,9)	(CAT, BOA)	(-83.12, -4.10, 10.24, 80.87)
(4,9)	(CLE, BOA)	(-70.44, -1.84, 12.12, 82.51)
(9,10)	(BOA, FIN)	(-76.99, -7.04, 7.04, 76.99)
(10,11)	(FIN, OB)	(-76.99, -7.04, 7.04, 76.99)

to rank the fuzzy numbers of all possible paths in the project, here turnaround. The ranking of the fuzzy number is achieved through the risk index β ($0 \le \beta \le 1$), which reflects the approach towards the risk of the decision-maker. $\beta > 0.5$ indicates an optimistic approach toward the risk, while $\beta <$ 0.5 illustrates a pessimistic approach towards the risk. The ranking index β reflects the approach towards the risk of the decision-maker, which is calculated by (18).

$$\beta = \left[\sum_{i} \sum_{j} \frac{b_{ij} - a_{ij}}{(b_{ij} - a_{ij}) + (d_{ij} - c_{ij})}\right] / n \qquad (18)$$

The fuzzy number of all possible paths in the project can be ranked with the help of the risk index β using (19).

$$R\left(\widetilde{A}_{i}\right) = \beta \left[\frac{d_{i} - x_{1}}{x_{2} - x_{1} - b_{i} + d_{i}}\right] + (1 - \beta) \left[1 - \frac{x_{2} - c_{i}}{x_{2} - x_{1} + a_{i} - c_{i}}\right]$$
(19)

The ranking value for each path in the project is determined through the risk index β , according to this principle, the path with the minimum risk index is the critical path in the entire turnaround. According to Table VI the path containing processes (IB, ACC, DEB, FUE, BOA, FIN, OB) is the critical path, as expected from literature review mentioned in Section I, and deviation in each of the processes of this path can lead to a delay in the whole turnaround.

TABLE VI. FCPM ranking values

Rank	Path <i>i</i>	FCPM (P _i)	RFCPM (P _i)
1	(1,2,3,6,9,10,11)	(-461.94,-42.24,42.24,461.94)	0.425
2	(1,2,3,5,9,10,11)	(-474.20,-36.36,48.64,469.70)	0.426
3	(1,2,3,4,9,10,11)	(-448.84,-31.84,51.40,472.98)	0.433
4	(1,2,7,8,10,11)	(-299.30,32.48,94.48,400.31)	0.485

IV. APPLICATION ANALYSIS

In this section, a comparative verification and analysis of the obtained results are discussed. Furthermore, a comparison between the FCPM-based turnaround time and historical data is introduced, the delay influences on the turnaround process are discussed using linear regression analysis.

A. Description of Historical Data

The historical data for this paper is derived from over 50,000 Lufthansa A320 flights from March to October 2017, including scheduled and actual turnaround time (TA) for all flights.

Historical data show a wide range of turnaround times for A320 daily flights. In addition to delays caused by bad weather or air traffic control, the flight scheduling plan also affects the aircraft turnaround. In order to increase the validity of the statistics, the historical data of turnaround time should be filtered appropriately. In this paper, flights with turnaround times more significant than 10 hours will be excluded. Then the schedule- and actual TA will be sorted in ascending order to remove further 5% of the historical data from the head and tail, respectively, due to not ensnaring in unreasonable and extreme values.

To evaluate the tendency of the data in order to find the behavior of the actual data, the mean value, median value, standard deviation, and quartile deviation are calculated. The mean value of schedule and actual TA are 69.62 j *min* and 75.90 *min*, respectively. The median value of the schedule and actual TA are 60.00 *min* and 67.00 *min*, respectively. The mean value of the schedule and actual TA are 34.87 *min* and 33.98 *min*, respectively.

Quartile deviation is a statistical method that sorts all data in ascending order and then divides the arranged data equally into quartiles to measure the dispersion of the data. The value at the 25% position is called the quartile and is denoted by Q1, Q2 is the median, and Q3 represents the value corresponding to the 75% position. The interquartile range IQR is calculated by (20).

$$IQR = \Delta Q = Q_3 - Q_1 \tag{20}$$

B. Comparison between real data and FCPM result

The optimal parameter combinations of trapezoidal FMFs are identified through optimization and iteration by the Kolmogorov-Smirnov test in Section III, with these parameter combinations, the p-value in the K-S test reaches its peak.

Fig. 6 shows the result of goodness-of-fit of the real data. The blue line represents the real data based on the data set and the red line depicts the trapezoidal FMF. After calculating and updating the optimal parameter

combinations under trapezoidal FMF, schedule turnaround posses (59.64,69.58,69.58,268.38) parameter combination, with a p-value of 0.036, and actual turnaround has (9.92,9.92,9.92,248.00) parameter combination with a p-value of 0.078.



Figure 6: Transformation Result for real data (X-axis shows the duration in Min)

At the time of writing this paper, there is no means to allow direct comparison of several fuzzy sets, especially when they have overlapping parts in the literature [36]. Several fuzzy numbers can only be compared if they are defuzzified to crisp numbers. According to Principle 1 described in Section III, only the non-negative part of a fuzzy number is practically meaning. Therefore, there is a preprocessing procedure for the FCPM-based turnaround time, and only the positive value of the parameters could be retained. The figure of these FMFs' comparison is depicted by Fig. 7.

In this paper, we set up a set of delay scenarios based on historical data to simulate real delays, followed by linear regression models to predict the trend of different delay levels at the "On-Block" point to the FCPM-based turnaround time. The probability-possibility transformation algorithm is proven to be useful also for irregular distribution. Fig. 8 shows the procedure of delay fuzzy sets construction.

Table VI shows the results of defuzzification of different delay fuzzy sets and their FCPM-based turnaround time.

After defuzzification of different delay fuzzy sets and their FCPM-based turnaround time, a Simple Linear Regression (SLR) is used to illustrate the relationship between delay scenarios and FCPM-based turnaround time, which is depicted by Fig. 9 and formulated by (21). The determination coefficient is $R^2 = 0.504$ (the closer the R^2 is to 1, the model fits the data better). From the 21, the delay scenarios are positively related to the FCPM based turnaround time.

$$Y = 231.368 + 2.462x + \epsilon \tag{21}$$

V. CONCLUSIONS

The successful application of the fuzzy critical path method for predicting turnaround target times was demonstrated in this paper and opened a wide range of possibilities for the development of controller decision-support systems. First of all, the new method can calculate the turnaround completion after a chosen data set and risk of decision-making level in real-time and with high mathematical accuracy, thus making common simulation techniques negligible. Furthermore,



Figure 7: Comparison of FMFs of the actual TA, schedule TA and calculated TA (X-axis shows the duration in Min)



Figure 8: Flowchart of Constructing Delay Fuzzy Set

compared to some common approaches such as Monte Carlo simulation, FCPM avoids extensively repetitive calculations [37]. In day-to-day use, due to shortage of handling resources or infrastructure which causes the uncertainty in turnaround time calculation, turnaround time prediction is crucial in order to maintain flight schedule over the day. One of the advantages of this algorithm is that it considers simultaneously all network processes together based on their individual uncertainty parameters, the method is highly adaptive to any operational environment (any airline and any aircraft type) and has no restrictions on the distribution of parameters and can be modeled with any kind of real data sets. A small range



Figure 9: Linear Relationship between Delays and FCPM (Xaxis shows the duration in Min)

of network adaptations in the format of process and sequence alterations was analyzed inside this paper and showcased the time-wise considerations an AOC might undertake when evaluating the effectiveness of potential schedule recovery actions on a disrupted turnaround. More in-depth analyses of the individual processes will further investigate whether the current parameters should be more granularly adapted to actual weather, traffic, or personnel situations. Finally, further research will explore how the current prediction can be translated from a tactical into a real-time support tool so that the TOBT can be estimated once major disruptions hit the ongoing turnaround, such as maintenance problems or missing resources.

ACKNOWLEDGEMENTS

This project has received funding from the German Federal Ministry of Economic Affairs and Energy (BMWI) within the LUFO-V project OPs-TIMAL.

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