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Slot allocation for a Multiple Airport System: Equity and Efficiency

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***Abstract*—Airport slot allocation aims to distribute airport slots to the airlines under given procedures and rules. The objectives of slot allocation are to minimize the total displacements of slot requests, and/or to maximize airline's preferences. Research efforts have been devoted to slot allocation in a single airport over decades. Slot allocation for a Multiple-Airport System (MAS) has been less addressed. In a single airport slot allocation, airport capacity is the only resource that airlines compete for; while in an MAS, there are several resources that should be considered: airport capacity, terminal airspace and fixes capacity. This paper proposes an MAS slot allocation model that incorporates airline fairness. The objective of the model is to minimize the total slot displacements of the MAS, subject to airport capacity constraints, fixes capacity constraints, turnaround time constraints, and fairness constraints. An MAS comprehensive fairness indicator is developed. The trade-off between efficiency and fairness in an MAS slot allocation problem is explored. The model is tested using the data from the MAS of Guangdong-Hong Kong-Macao Greater Bay Area. Computational results show that our model can effectively allocate the airport and airspace capacity of the MAS while considering airline fairness.**

***Keywords*—demand and capacity management; airport slot allocation; multi-airport system; slot-scheduling fairness; mixed integer programming;**

I. INTRODUCTION

Air transportation has always been a significant contribution to a nation's modern transportation system and economy. In 2019, the global air traffic passenger reached 4.54 billion [1]. Despite being disrupted by the COVID-19 pandemic, the industry is recovering quickly and is expected to fully recover by 2024 [2]. However, the growth of demand has placed pressure on the limited infrastructure capacity of airports, leading to congestion and flight delays worldwide. Increasing capacity through the development of physical infrastructures, such as runways and terminals, takes too much time and cost. Feasible options in the short term to mitigate the congestion have been considered: demand and capacity management at the strategic level or air traffic flow management at the tactical level. This paper focuses on airport slot allocation, which is an administrative-based mechanism for demand and capacity management. It proposes a slot allocation model for a Multiple Airport System (MAS) that considers the balance of operation efficiency and airlines' fairness.

Airport slot allocation is a challenging resource allocation problem, which aims to allocate airport capacity to the airlines that are operating or plan to operate in the airport. Previous studies have mainly focused on single airport or airport network slot allocation problems [3], [4]. The Worldwide Airport Slot Guidelines (WASG), endorsed by the International Air Transport Association (IATA), play an important role in the allocation process. In 2012, Zografos, Salouras and Madas developed a slot allocation model with a single objective under rules in the Worldwide Slot Guidelines (WSG), which is the old version of WASG. The model allocates slot request series over a scheduling season at a slot-coordinated airport [6]. Zografos, Androutsopoulos and Madas (2018) proposed two bio-objective slot allocation models according to the WASG [5], [7]. The results show that a flight schedule with better acceptance can be achieved by scarifying certain displacements. Further, Ribeiro et al. incorporated more details of the IATA guidelines into their model. The model fully complied with the slot priority classes specified in the IATA guidelines [8]. Pyrgiotis and Odoni proposed a demand smoothing optimization model to solve a hub airport slot scheduling problem [9]. Jacquillat and Odoni then presented a model that achieves synergistic optimization of slot scheduling at the tactical and strategic levels by controlling runway configuration and flight service rates. The results show that the method can significantly alleviate airport congestion [10].

There are other works that study slot allocation problems in airport networks. Castelli et al. developed a bi-objective model that considers the capacity constraints of all airports within the airport network. However, due to a large amount of data and computational complexities, it failed to achieve an exact solution [11]. Then, a meta-heuristic algorithm has been used by Pellegrini et al., which solved the problem successfully [12]. Later, Pellegrini et al. developed an integer planning model for the simultaneous optimization of European network-wide slot allocation [13]. Corolli et al. extended the single-airport slot allocation model to the airport networks level by developing a two-stage stochastic programming model [14].

However, in an MAS, there are multiple airports and dozens of operating airlines, which makes the interactions between neighboring airports and their shared airspace complex. The slot allocation problem for an MAS differs from that for a single airport in resource types and users. An MAS involves multiple types of resources, such as airport capacity, terminal airspace, and fixes capacity, while airlines in the MAS may not necessarily operate at all member airports. The heterogeneity in airlines' demands further increases the complexity of the slot allocation problem.

A key challenge of the slot allocation process is how to define the objectives that trade off the demands and preferences of different stakeholders. Several studies have attempted to balance efficiency, equity and other objectives. In a slot allocation problem, 'efficiency' often refers to the total schedule displacements; while fairness is usually defined as the distribution of displacements among airlines [15]. Nash proposed and explored the efficiency-fairness equilibrium in the game problem for the first time [16]. In terms of fairness in slot allocation, fairness exists in several dimensions: fairness between flights, fairness between airlines, fairness between airports, etc. The minimization of maximum displacement for single slot requests is used widely in the existing slot allocation model, which can be considered as the pursuit of inter-flight fairness [10]. Inter-airline equity has been taken into account by Zografos and Jiang (2016, 2019) in their single-airport slot allocation model. They construct fairness indicators depending on a flight proportionality principle [17], [18]. Fairbrother et al. propose a "peak request proportionality principle" by defining peak-period requests. Subsequently, Zografos and Jiang diversified the construction of fairness indicators by proposing three different fairness metrics [18]. A common conclusion from the above studies is that a fairer flight schedule can be achieved by increasing a certain amount of schedule displacements. It can be found that most of the existing studies about fairness have been given to airlines within a single airport. While in an MAS, consisting of multiple types of resources, the airlines have heterogeneous demands at airports and fixes [19]. This further adds to the complexity of the slot allocation.

This paper contributes to the literature through the introduction of an MAS slot allocation model that (a) considers multiple types of resource including the airport capacity and fixes capacity; (b) explore the comprehensive fairness between airlines in the MAS; and (c) optimize the multiple capacity allocation to users in a systematic way.

The remainder is organized as follows. In section II, we give the definition of slot allocation problem for an MAS and the definition of fairness. Section III formulates the MAS slot allocation baseline model and fairness constraints. In Section IV, we test our model with data from the MAS of Guangdong - Hong Kong -Macao Greater Bay area. Finally, the paper concludes in Section V.

II. Problem Discription

A. Brief discription

There are three main factors affecting the operation of airports in an MAS: capacity constraints on shared arrival and departure fixes, limited airspace resources within the terminal airspace, and constraints imposed by neighbor airports within the MAS. Previous studies have focused on the identification of traffic patterns of the MAS, improving airspace operational efficiency, and optimizing arrival and departure traffic at the tactical level [20]–[23]. As the MAS resource users, airlines are most concerned with the fair and reasonable allocation of resources. However, the capacity of an MAS relies on many resources, including runways, airport terminals, available airspace, and more. Therefore, fairness in the allocation of resources within an MAS should take into account multiple factors.

This paper integrates fairness into the allocation of airport capacity and fix capacity among the airlines operating within the MAS. The overall objective is to achieve the optimal allocation of airport slots in the MAS. The problem can be briefly described as follows: *Given airport capacities, critical terminal airspace capacities (i.e. fix capacities) of an MAS, as well as operational constraints such as flying time between the airport and fix, and minimum and maximum flight turnaround time, the multiple objectives of slot allocation for an MAS are (i) to allocate airport slots to the airlines that minimize total displacements of airlines' slot requests; (ii) to optimize airline's fairness in the MAS throughout the whole allocating process.*

B. Assumptions

The following assumptions are made to simplify the problem:

1) We only consider regular flights, excluding special circumstances such as extra flights, charter flights, and cancellations.
2) The term "slot" refers to a specific time interval with a minimum length of 5 minutes, rather than a precise time point.
3) The definition of declared capacity is the number of slots that can be allocated to users per unit time in a coordinated airport [24].
4) The constant maximum and minimum turnaround time is set in this paper.
5) Only the flights operating within the MAS are considered.
6) We assume that the flight time for each flight from the airport to the same fix remains constant within the MAS.

C. Fairness Definition

In this section, we introduce the concept of fairness in an MAS. It is widely recognized that fairness is an important factor that affects the results of resource allocation problems. The WASG emphasizes that "To ensure that slots are allocated at congested airports in an open, fair, transparent and non-discriminatory manner by a slot coordinator acting independently". Also, the slot regulation issued by the Civil Aviation Administration of China (CAAC) clearly points out that "the purpose of regulating slot management of civil flights, promoting the fairness, efficiency, competition, and integrity of the allocation of slot resources". Interestingly, a clear definition and measurement of fairness are absent from the guidelines provided by IATA, CAAC, and other authorities. We refer to the existing research on slot allocation for a single airport while considering the airline's fairness [25]–[27]. Generally, fairness is defined as "to balance the displacement of flights from their requested times fairly among the airlines".

TABLE I
EXAMPLES OF FAIR SLOT ALLOCATION IN AN MAS

Arr/Dep	Airline	Airport	Fix	Request	Allocation scheme		
					I	II	III
Arr	AL_1	a	P_1	0800	0815	0815	0815
Dep	AL_2	a	P_2	0800	0800	0800	0800
Dep	AL_1	a	P_1	0805	0805	0805	0805
Arr	AL_2	a	P_2	0805	0820	0820	0820
Dep	AL_1	a	P_2	0810	0825	0825	0830
Arr	AL_2	a	P_2	0810	0810	0810	0810
-	-	-	-	-	-	-	-
Dep	AL_1	b	P_1	0915	0915	0930	0930
Dep	AL_2	b	P_2	0915	0930	0915	0915
Arr	AL_1	b	P_1	0920	0905	0905	0900
Arr	AL_1	b	P_1	0920	0920	0920	0920
Arr	AL_1	b	P_2	0925	0925	0925	0925
Dep	AL_2	b	P_1	0925	0910	0910	0910

To illustrate the differences in fairness between slot allocation for a single airport and for an MAS, Table I shows an example of three allocation schemes. In the first two schemes, we only consider the airport capacity. Fix capacity is taken into account in scheme 3. Two airlines AL_1 and AL_2 submit a total of 12 slot requests at two airports a and b during two periods 08:00-08:30 and 09:00-09:30, with 8 requests from AL_1 and 4 requests from AL_2. The 15min declared capacity at each airport is 3. Therefore, 3 slot requests have to be displaced for each period. Each airline has to modify 3 requests in Scheme 1. It seems to be a fair allocation from the perspective of total displacements to each airline. However, AL_1 requests for 8 slots, while AL_2 only requests 4 slots. It is unfair to AL_2 if one considers the proportion of displacements to the requests. Scheme 2 gives a more fair allocating result. 4 slot requests of AL_1 and 2 slot requests from AL_2 are adjusted, which aligns with the propotional principle. In an MAS slot allocation problem, shared fix resources are one of the critical airspace resources. So in scheme 3, the fix capacity is taken into account. The flights pass two fixes, both of which provide a 15min capacity of 3. The number of adjusted requests in scheme 3 is the same as that in Scheme 2, but the displacement minutes are increased in order to satisfy the fix capacity. It can be seen that the complexity would be much increased if the shared fix resources are considered during allocating airport slots. The problem is extended to the fairness among various stakeholders in the allocating of multiple resources.

III. Model

A. Baseline model

Decision variables:

$$x_m^t = \begin{cases} 1 & \text{if flight } m \text{ is scheduled to slot } t \\ 0 & \text{otherwise;} \end{cases} \quad (1)$$

TABLE II
MODEL INPUTS

Notation	Description
$S = \{1, 2, ..., \lvert S\rvert + 1\}$:	S represents the set of airports in the MAS. $\lvert S\rvert$ is the total number of airports in the MAS, and the airports outside are referred to as $\lvert S\rvert + 1$;
$P = \{1, 2, ..., \lvert P\rvert\}$:	P represents the all fixes within the MAS. $\lvert P\rvert$ is the total number of fixes;
$T = \{1, 2, ..., \lvert T\rvert\}$:	T is the set of time intervals t, the length of which is 5 minutes;
$A = \{1, 2, ..., \lvert A\rvert\}$:	A represents the set of total airlines operating in the MAS, and $\lvert A\rvert$ is the total number of them;
M:	M refers to the set of total slot requests in the MAS;

The objective of the model is to minimize total displacements of all slot requests in the MAS.

$$d_m = \sum_t |t - t_m| \cdot x_m^t \quad (2)$$

$$D_a = \sum_{m \in M_a} d_m = \sum_{m \in M_a} \sum_t |t - t_m| \cdot x_m^t \quad (3)$$

$$D = \sum_{a \in A} D_a \quad (4)$$

$$\min D \quad (5)$$

where t_m is the slot that flight m requests, while t is the slot that m has been allocated. Thus d_m is the number of displacements that are made to request m. The displacements that airline a obtained are given by D_a, and the total displacements for the MAS are D.

Constraints:

(1) Each flight can only be allocated one slot:

$$\sum_{t \in T} x_m^t = 1 \quad (6)$$

(2) The maximum displacements of a single flight request: The airline may reject the allocated slot if the displacements to the requested slot were too large. To ensure the acceptance of allocating result, the maximum displacements of a single flight are set to be t_{max}:

$$d_m = \sum_t |t - t_m| \cdot x_m^t \le t_{max} \quad (7)$$

(3) Turnaround time constraints: When an arrival flight landed at the destination airport, there will be time requirements for turnaround processes. These processes include passengers and cargo disembarking, refueling, cleaning, and passengers and cargo boarding, etc. A minimum time constraint is required to ensure the completion of all the processes, while a maximum time constraint is to improve the utilization of the gate or apron. Therefore, we have

$$\underline{f}_{(m_1,m_2)} \le \sum_{t \in T} t x_{m_1}^t - \sum_{t \in T} t x_{m_2}^t \le \bar{f}_{(m_1,m_2)} \quad (8)$$

where (m_1, m_2) is a pair of slot requests that are operated by the same aircraft. m_1 is the preceding flight, while m_2 is the succeeding flight. $\underline{f}_{(m_1,m_2)}$ is the minimum turnaround time, while $\bar{f}_{(m_1,m_2)}$ is the maximum turnaround time.

(4) Airport capacity constraints: Airport capacity include departure capacity, arrival capacity, and total airport capacity.

$$\sum_{m \in M^k} \sum_{t}^{t+2} x_m^t \leq CQ_S^k, \quad t = 3(t_q - 1) + 1 \quad, \forall k \in K, \quad t_q \in T_Q \tag{9}$$

$$\sum_{m \in M^k} \sum_{t}^{t+11} x_m^t \leq CH_S^k, \quad t = 12(t_h - 1) + 1 \quad, \forall k \in K, \quad t_h \in T_H \tag{10}$$

Equations 9 and 10 are the capacity constraints for 15min and 60min respectively. $K = \{Arr, Dep, Total\}$ stands for arrival capacity, departure capacity and total. M^k denotes the set of flights whose operation type is k. CQ_S^k and CH_S^k are the capacity of airport S during operation scenario k.

(5) Fix capacity constraint: Fix can be used only for arrival flights or departure flights, or for both arrival and departure flights. Similar to airport capacity, we have 15min capacity and 60min capacity:

Fix only serves for arrival flights:

$$\sum_{m \in M_p^{Arr}} \sum_{t - l_{m,p}^{Arr}}^{t - l_{m,p}^{Arr} + 2} x_m^t \leq CQ_p^{Arr}, \quad t = 3(t_q - 1) + 1, \quad t_q \in T_Q \tag{11}$$

$$\sum_{m \in M_p^{Arr}} \sum_{t - l_{m,p}^{Arr}}^{t - l_{m,p}^{Arr} + 11} x_m^t \leq CH_p^{Arr}, \quad t = 12(t_h - 1) + 1, \quad t_h \in T_H \tag{12}$$

Fix only serves for departure flights:

$$\sum_{m \in M_p^{Dep}} \sum_{t + l_{m,p}^{Dep}}^{t + l_{m,p}^{Dep} + 2} x_m^t \leq CQ_p^{Dep}, \quad t = 3(t_q - 1) + 1, \quad t_q \in T_Q \tag{13}$$

$$\sum_{m \in M_p^{Dep}} \sum_{t + l_{m,p}^{Dep}}^{t + l_{m,p}^{Dep} + 11} x_m^t \leq CH_p^{Dep}, \quad t = 12(t_h - 1) + 1, \quad t_h \in T_H \tag{14}$$

Fix that can be used for both arrival and departure flights:

$$\sum_{m \in M_p^{Arr}} \sum_{t - l_{m,s,p}^{Arr}}^{t - l_{m,s,p}^{Arr} + 2} x_m^t + \sum_{m \in M_p^{Dep}} \sum_{t + l_{m,s,p}^{Dep}}^{t + l_{m,s,p}^{Dep} + 2} x_m^t \leq CQ_p^{Total}, \quad t = 3(t_q - 1) + 1, \quad t_q \in T_Q \tag{15}$$

$$\sum_{m \in M_p^{Arr}} \sum_{t - l_{m,s,p}^{Arr}}^{t - l_{m,s,p}^{Arr} + 11} x_m^t + \sum_{m \in M_p^{Dep}} \sum_{t + l_{m,s,p}^{Dep}}^{t + l_{m,s,p}^{Dep} + 11} x_m^t \leq CH_p^{Total}, \quad t = 12(t_h - 1) + 1, \quad t_h \in T_H \tag{16}$$

$P = \{1, 2, ..., |P|\}$ is the set of all fixes that are considered in the slot allocation model. M_p^{Arr} is the set of arrival flights that fly through p, while M_p^{Dep} is the set of departure flights that fly through fix p. $l_{m,s,p}^{Arr}$is the time for arrival flight m flying from fix p to airport s, while $l_{m,s,p}^{Dep}$is the time for departure flight m flying from airport s to fix p.

CQ_p^k and CH_p^k refer to the capacity of fix p within 15 minutes and 60 minutes respectively, where $k \in K$, $K = \{Arr, Dep, Total\}$.

B. Airline's fairness constraints

The baseline model developed in the previous section outputs an optimized flight schedule for the MAS. However, it does not take into account of airline's fairness. To investigate how to allocate both airport capacity and fix capacity to the airlines in a more equal and efficient way, we have to develop fairness indicators for airlines in the processes of airport slot allocation and fix capacity allocation, respectively. Then by combining the two fairness indicators, we build a comprehensive fairness indicator for airlines in the MAS.

According to the "proportion principle" in section II-C, the fairness indicators of an airline at the airport and at the fix are defined as follows:

(1) Airline fairness index at the airport $\rho_{a,s}$:

$$\rho_{a,s} = \begin{cases} \frac{\frac{D_{a,s}}{D_s}}{r_{a,s}} & r_{a,s} \neq 0 \\ 1 & r_{a,s} = 0 \end{cases} \tag{17}$$

$D_{a,s}$ is the total displacements of airline a at airport s. D_s is the total displacements to all the flights in airport s. $r_{a,s}$ is the proportion of slot requests of airline a at airport s. When $r_{a,s} = 0$, then airline a has no slot request at airport s. In this case, $D_{a,s} = 0$, and $\rho_{a,s} = 1$. This is absolutely fair to the airline a.

(2) Airline's fairness index at fix $\rho_{a,p}$:

$$\rho_{a,p} = \begin{cases} \frac{\frac{D_{a,p}}{D_p}}{r_{a,p}} & r_{a,p} \neq 0 \\ 1 & r_{a,p} = 0 \end{cases} \tag{18}$$

$D_{a,p}$ is the total displacements of airline a at fix p. D_p is the total displacements to all the flights through fix p. $r_{a,p}$ is the proportion of slot requests of airline a flying through fix p. Again, when $r_{a,p} = 0$, airline a has no slot request using fix p. In this case, $D_{a,p} = 0$, and $\rho_{a,p} = 1$. This is absolutely fair to the airline a.

There are three possible scenarios for $\rho_{a,s}$ and $\rho_{a,p}$:

$$\rho = \begin{cases} [0, 1) & \text{the airline is favoured} \\ 1 & \text{the airline is fairly treated} \\ (1, \infty) & \text{the airline is treated unfairly} \end{cases} \tag{19}$$

(3) The comprehensive airline fairness indicator in the MAS ρ_a:

$$\rho_a = \sum_s w_s \rho_{a,s} + \sum_p \rho_{a,p} \tag{20}$$

$$\sum_s w_s + \sum_p w_p = 1 \tag{21}$$

where w_s and w_p are the weights of fairness at airport s and fix p, which can be adjusted based on the preferences of relevant authorities.

To measure the fairness of the total resource allocation of the MAS, the Gini index is introduced. Based on the fairness indices developed for the airlines, we can evaluate the overall fairness of slot allocation of an MAS.

$$\text{Gini}_{\rho} = \frac{\sum_{i\in A}\sum_{j\in A}|\rho_i - \rho_j|}{2 \times |A| \times \sum_{i\in A}\rho_i} \tag{22}$$

where ρ_i, ρ_j are the comprehensive fairness indices of airline i and j. $|A|$ is the number of airlines operating in the MAS. Gini_p is the Gini index of the MAS. The Gini index is closer to 0, the allocation scheme is more fair. The detail implication of Gini index is given in Table III.

TABLE III
RANGE OF GINI COEFFICIENT AND ITS MEANING

Range	Meaning
< 0.2	Perfect equity
0.2-0.3	Good equity
0.3-0.4	Average equity
0.4-0.5	Poor equity
> 0.5	Absolute inequity

By the leverage of ϵ constraint, we add the fairness into the baseline model, where $\epsilon \in [0, 1]$. Until here, one can trade off the fairness and efficiency of slot allocation by setting ϵ in the appropriate range.

$$\varepsilon_1 \leq \text{Gini}_{\rho} \leq \varepsilon_2 \Rightarrow \varepsilon_1 \leq \frac{\sum_{i\in A}\sum_{j\in A}|\rho_i - \rho_j|}{2 \times |A| \times \sum_{i\in A}\rho_i} \leq \varepsilon_2 \tag{23}$$

IV. Results

We use the data from the MAS of the Guangdong-Hong Kong-Macao Greater Bay Area (GBA) to validate and examine the performance of the proposed model. Five airports are considered in the MAS: Guangzhou Baiyun Airport (ICAO code: ZGGG), Shenzhen Bao'an Airport (ICAO code: ZGSZ), Zhuhai Jinwan Airport (ICAO code: ZGSD), Macao Airport (ICAO code: VMMC), and Huizhou Pingtan Airport (ICAO code: ZGHZ). Hong Kong Airport (ICAO code: VHHH) is excluded from this study because its arrival traffic and departure traffic is separated from the other airports. In other words, there is no conflict between the traffic of VHHH and of other airports within the MAS. The throughput of the five airports in 2019 is presented in Table IV. We can see that the throughput of ZGGG and ZGSZ are significantly higher than that of the other three airports, accounting for more than 80% of the MAS.

TABLE IV
AIRPORT THROUGHPUT IN THE MAS OF GBA

Airport	Pax(M)	Cargo(M tons)	Movements(M)
ZGGG	73.378	1.920	0.491
ZGSZ	52.932	1.283	0.356
ZGSD	12.283	0.051	0.085
VMMC	9.611	0.042	0.077
ZGHZ	2.554	0.009	0.020

M: million;

A. Problem setting

1) Model input: The main inputs into the model include the capacity of each member airport in the MAS, fixes capacity, the flying time between each airport and each fix, and the maximum displacements to a single flight.

Among the five airports in the MAS of GBA used in the example, there are two coordinated/Level 3 airports (Guangzhou Baiyun Airport and Shenzhen Bao'an Airport), one Level 2 airport (Zhuhai Jinwan Airport) and one non-coordinated airport (Huizhou Pingtan Airport). To simplify the expression, we use declared capacity as the airport capacity for all five airports of the MAS. The capacity data provided by the air traffic control authorities are shown in the table V.

TABLE V
DECLARED CAPACITY OF FIVE AIRPORTS (NUM. OF MOVEMENTS)

Airport	Dep. (15min)	Arr. (15min)	Total (15min)	Total (1hr)
ZGGG	20	19	32	67
ZGSZ	15	15	20	55
ZGSD	5	5	8	20
VMMC	5	5	8	22
ZGHZ	5	5	8	18

Dep.:Departure; Arr.: Arrival;

Fix *GYA, LMN, YIN* are the fixes that we focus on in this paper. Their capacity is given by the air traffic authorities, as shown in the table VI.

TABLE VI
FIX CAPACITY (NUM. OF MOVEMENTS)

Fix	Capacity (15min)	Capacity (1hr)
GYA	8	27
LMN	8	32
YIN	9	36

Flying time between an airport and a fix is affected by multiple factors such as weather, air traffic control strategies, or aircraft performance. Therefore, the flying time is stochastic actually. The model in this paper does not consider the uncertainty of flying times. The median of historical flying time during 2018-2019 between an airport and a fix is selected as the flying time between the airport and the fix in the model.

2) Slot requests: Due to data confidentiality, we are not able to obtain airlines' slot request data. To validate the model and examine its performance, we use the flight schedules of a typical day in 2019 as the slot request data. Follow-up research can use actual slot request data. The presented work uses a total of 3055 slot requests, including 1160 connecting flights.

TABLE VII
SLOT REQUESTS OF THE AIRLINES AT THE AIRPORT AND FIX

Rank	Airlines	ICAO Code	Total requests	Sot requests at the airport					Slot requests at the fix		
				ZGGG	ZGSZ	ZGSD	VMMC	ZGHZ	YIN	GYA	LMN
1	China Southern Airlines	CSN	1061	757	241	57	4	2	161	128	166
2	Shenzhen Airlines	CSZ	403	111	270	12	10	0	49	48	42
3	China Eastern	CES	210	124	68	12	0	6	17	37	29
4	Air China	CCA	202	119	61	16	4	2	60	13	37
5	Hainan Airlines	CHH	180	86	88	4	0	2	38	14	30
6	Xiamen Air	CXA	68	24	28	8	0	8	6	11	3
7	Spring Airlines	CQH	63	24	31	2	0	6	7	7	7
8	Air Macau	AMU	60	0	0	0	0	60	11	0	5
9	SF Airlines	CSS	59	8	51	0	0	0	9	2	4
10	Shandong Airlines	CDG	50	18	12	20	0	0	4	9	1
11	All other airlines	\	699	370	197	73	29	30	81	173	2
Total			3055	1641	1047	204	47	116	443	442	326

A total of 100 airlines operated flights at the airports in the MAS. Among them, the top 10 airlines in terms of the number of flights requested 2,356 slot requests, accounting for 41.2% of the total request. Thus, we consider 100 airlines when testing the model but give our focus on the top 10 airlines when analyzing the results. The information on these 10 airlines, as well as the number of slot requests of each airline at the five airports, is shown in table VII. It can be found that slot requests at ZGGG and ZGSZ are significantly higher than that of the other three airports, while slot requests at ZGHZ are the lowest. China Southern Airlines (CSN) and Shenzhen Airlines (CSZ) have their main bases at ZGGG and ZGSZ respectively, thus the slot requests of these two airlines are obviously large; While at VMMC, Air Macau (AMU) is the main slot user. The number of slots requested by airlines at the three fixes is given in Table VII. It can be seen that almost all airlines have flights passing through three fixes, except that AMU at LMN. According to the proportion of slot requests, ten airlines can be simply divided into three categories: a large amount (CSN), a medium amount (CSZ, CES, CCA, CHH), a small amount (CXA, CQH, AMU, CSS, CDG).

The model in this paper is a mixed integer programming(MIP) model, which involves a huge amount of flight data at five airports and is complex to solve. The model is solved using Gurobi 9.5, and the computer used is equipped with eight-core Intel(R) Core(TM) i7-10700 CPU, 2.90GHz.

B. Results

1) Trade-off between slot displacements and Gini-based fairness:

In this section, we analyze the trade-off between slot displacements and fairness from three aspects: the MAS, the airlines and airports.

(1) The MAS total displacements and Gini-based fairness

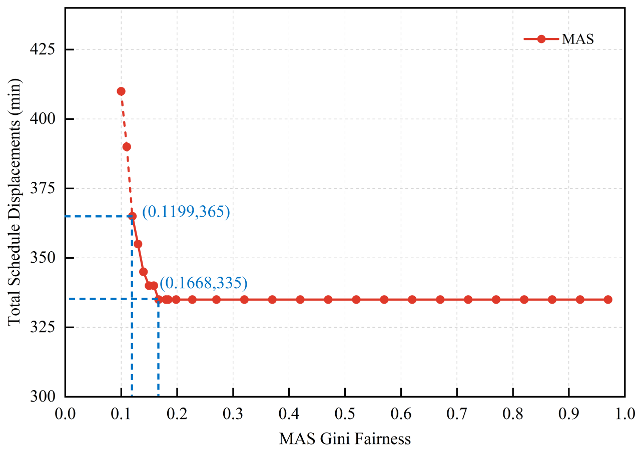


Figure 1. MAS total displacements and Gini-based fairness

Figure 1 plots the efficient frontier for MAS slot allocation. We use solid and dashed lines to distinguish the feasible and optimal solutions. When the Gini-based fairness is constrained below 0.1199, only feasible solutions can be obtained due to the time limits, with the Gurobi gap value of around 0.08. Recall that the Gini coefficient is defined to measure the fairness of resource allocation among the users. It commonly employs 0.4 as a threshold for quantifying fairness. When the Gini coefficient is less than 0.4, the allocating result is considered to be relatively fair to all the users. Otherwise, it is considered to be unfair to some of the users. In addition, the smaller the Gini fairness, the more equitable the slot allocation of the MAS. From figure 1, it can be observed that the total displacements increase with the decrease of Gini-based fairness below 0.1668. The result suggests that achieving a highly fair flight schedule would require more adjustments to airlines' requests. When the Gini-based fairness of the MAS is greater than 0.1668, the total displacements remain at

335 minutes. By comparing their specific optimized schedule, we found that there are differences in their adjustment. For example, one slot request from airline CSC is shifted when Gini-based fairness is 0.1668 but has not been shifted when Gini-based fairness is 0.2701. This observation suggests the existence of multiple solutions, or flight schedules, with the same total displacements. Different flight schedules lead to different Gini fairness. However, when the Gini-based fairness is set over 0.3200, the optimized schedule for the MAS remains the same. In other words, the slot allocation results are no longer influenced by the Gini-based fairness constraint. Overall, it is crucial for the slot coordinator or managing body to balance fairness and efficiency during the MAS slot allocation.

(2) Slot displacements and Gini-based fairness of top ten airlines

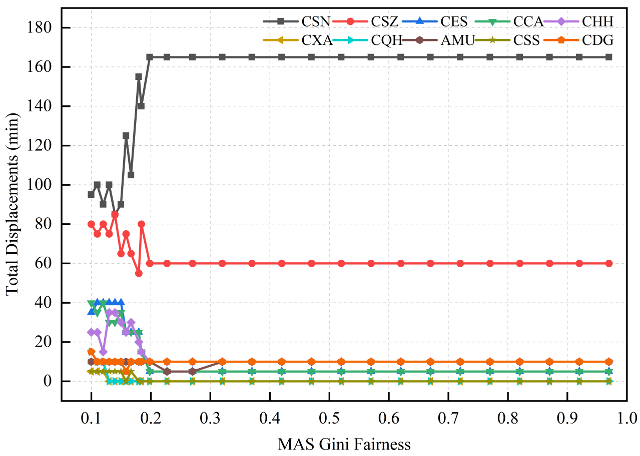
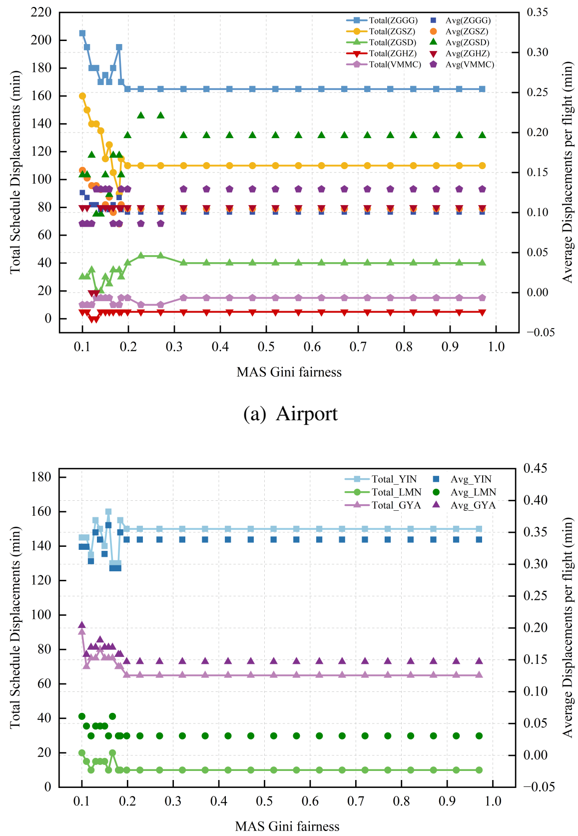


Figure 2. Airline's total displacements and MAS Gini-based fairness

Figure 2 presents the total displacements of the top 10 airlines with the MAS Gini-based fairness. Overall, each airline's slot displacements fluctuate up and down when Gini fairness changes. When the MAS Gini Fairness reaches 0.3200, the airlines' displacements remain stable since the optimized flight schedule are the same. The 10 airlines can be classified into three groups based on their total displacements: (1) Group one: CSN. The total displacements of CSN are always the highest among the 10 airlines for each Gini-based fairness, fluctuating within [90 min, 180 min]. This is reasonable because the slot requests from CSN are far more than other airlines. The total displacements of CSN decrease as the MAS Gini fairness narrows, indicating that the burden of adjusted slots for CSN is shared by other airlines, which improve the fairness between airlines. (2) Group two: CSZ, CCA, CES and CHH. The slot displacements of these four airlines stay at a medium level, with a maximum of 90 minutes and a minimum of 5 minutes. The minimum bound indicates that slot adjustments always occur in these airlines. (3) Group three airlines are CXA, CQH, AMU, CSS and CDG, with the slot displacement remaining at a low level. These five airlines' slot displacement changed in the range of [0,15 min]. It can be found that there is a clear relationship between the airlines' slot displacement and their slot request number. In general, the more slot requests, the more slot displacement. This finding is consistent with the "proportional principle" used in the model.



(a) Airport

(b) Fix

Figure 3. **Total displacements and MAS Gini fairness**

(3) Slot displacements at the airports & fixes, and MAS Gini-based fairness

Figures 3(a) and 3(b) depict the changes in slot displacements for the five airports and three fixes in the MAS, respectively. In Figure 3(a), the ranking of the slot displacements of the airport aligns with the ranking of airport slot demand. The slot displacement of ZGGG and ZGSZ is significantly higher than that of the other three airports. Besides, the slot displacements of ZGSD, ZGHZ and VMMC are always small, never exceeding 50 minutes. Finally, ZGHZ has the fewest slot displacements, only changing between 0 and 10 minutes. Compared with other airports, ZGHZ has the fewest slot requests and few flights passing through congested fixes, so the slot requests of it rarely need to be shifted. In terms of fixes, figure 3(b) shows that the slot displacements of the three fixes also fluctuate in their ranges. The YIN fix has the largest slot displacements, ranging between [120 min, 160 min]. The GYA fix follows with a range of [60 min, 100 min], and the LMN is the smallest, almost staying at 0. It is true that when the Gini-based fairness of the MAS is less than 0.1668 and continues to decrease, the total slot displacements of the MAS as a whole increase significantly. The obvious raise also happen to airports or fixes, with some fluctuation to some extent. This may be due to the definition of the MAS Gini-based fairness, which is aimed to measure whether the resources are allocated fairly among the airlines. The 10 airlines in the case study have slot applications in 5 airports, and most of them pass through YIN, GYA and LMN fixes. So when the Gini-based fairness narrows, the increased slot displacement will be distributed to each airport and each fix.

In general, a fairer flight schedule of an MAS can be achieved with a slight increase in the number of total displacements. The pursuit of "extremely fair" requires a greater sacrifice of total displacements. The results are consistent with current research on single airport slot allocation. The amount of displacement obtained by the airline is related to the slot requests of the airline itself operating in the MAS. A similar conclusion can be made from the perspective of airports and fixes. Generally, the more slot requests, the greater the possibility of slots being shifted.

2) Sensitivity analysis: When building the comprehensive fairness indicator of airlines, we used two parameters w_s and w_p. The parameters respectively represent the weight of the airport s in the MAS and the weight of the fix p. According to the theory of fair distribution of multiple types of resources, the weight reflects the preference of resource allocation subject for the fair allocation of this resource. When handling slot allocation, the weight reflects the preference of the slot coordinator for the fair allocation of airport capacity and fix capacity. For airlines, different airports and fixes imply different importance to the company's efficiency and benefits. In an MAS, airports and fixes have their specific functioning position. Therefore, how to measure the preference is worth studying but it is not the focus of this paper. Considering the influence of weights on the slot allocation results, we define λ as the ratio of the two weight parameters, namely

$$\lambda = \frac{w_s}{w_p} \tag{24}$$

Under the same ϵ constraint, we change the value of $\lambda(\lambda \in \{0.1, 0.5, 1, 2, 10\})$ to initially explore the influence of different weight ratios on the MAS Gini fairness and the total slot displacements.

Figure 4 displays the change curve of the total slot displacements of the MAS as the value of λ varies. The graph illustrates that the value of λ has an impact on the problem of MAS slot allocation. Specifically, when the value of λ decreases, a final schedule with larger total displacements will be obtained. This means that when the weight of fixes in the fairness indicator is greater than that of airports, the slots required to be adjusted are fewer. We surmise this may be because the capacity and demand conflict of fixes is more prominent than that of airports within the MAS. This conclusion also highlights the need to consider the allocation of fixes capacity within the MAS. For different MAS with specific congestion characteristics, different weight values should be set. In future research, it is necessary to develop rules for setting appropriate weight values.

3) Capacity allocation optmization: A major difference in MAS slot allocation is the consideration of critical fix capacity. This section analyses the changes in the traffic flow

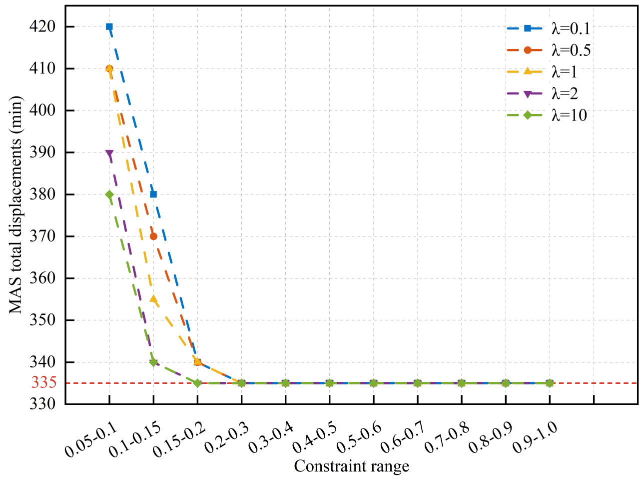


Figure 4. Total displacements of the MAS under different λ

of three fixes before and after optimization to verify the effectiveness of the model in optimizing the allocation of MAS capacity resources.

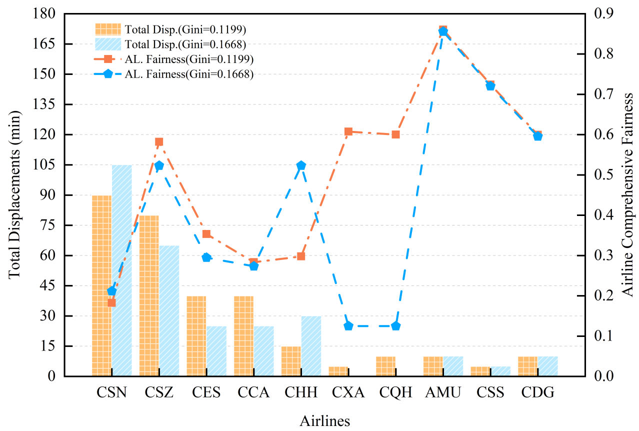


Figure 5. Airline's displacements and fairness indicators in different Gini-based fairness

Figure 5 shows the slot displacements and comprehensive fairness indicator for ten airlines, under two different MAS Gini-based fairness values. The figure contains two scenarios: (1) Gini-based fairness of 0.1668, which is the best Gini fairness the model can achieve with the smallest total slot displacements (335 minutes); (2) Gini-based fairness of 0.1199, which is the smallest Gini-based fairness value for which the model can obtain an optimal solution within the time limit. It corresponds to total slot displacements of 365 minutes. Firstly, we compare the MAS total slot displacements under the two Gini-based fairness values. It can be concluded that the model can achieve better Gini-based fairness by sacrificing a certain amount of slot displacements. When the Gini fairness is 0.1668, none of the two airlines (CXA, CQH) have slots to be displaced. The airline with the most slot requests, CSN, receives a total of over 100 minutes of slot displacements, while the other airlines only have no more than 75 minutes. In contrast, when the Gini fairness is

0.1199, the airlines have a smaller gap in the number of slot displacements and slot displacements occur to all airlines. In terms of the comprehensive fairness indicator, when the Gini-based fairness is 0.1668, the comprehensive fairness indicator varies more across airlines but is still less than 1.0 (airlines are favored). At a Gini fairness of 0.1199, the differences between airlines' fairness indicators reduce obviously. Thus, a lower Gini fairness allows for a more even distribution of slot displacements between airlines. Finally, the airlines' fairness indicators reach a better balance as well.

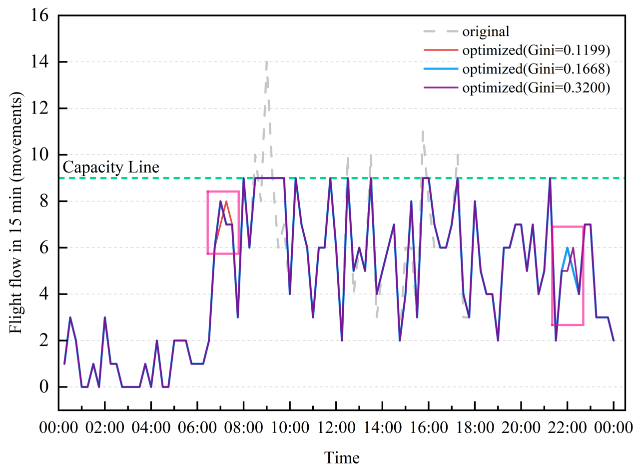


Figure 6. The 15min traffic flow at YIN when Gini-based fairness is 0.1199, 0.1668 and 0.3200

After the data analysis, we found that the optimization pattern of capacity allocation was similar for the three fixes. Thus, we selected the fix YIN as the representative to analyze the optimization effect of the model, whose capacity conflict is more intense. Figure 6 shows the 15-minute traffic flow of fix YIN for three different MAS Gini fairness. The green line in the figure is the fix capacity line with a value of 9, indicating a 15-minute capacity constraint of 9 flights at the fix. As can be seen from the graph, the model can achieve a reasonable allocation of capacity resources at the fix. The result satisfies the capacity limit of the fix and optimizes the flight distribution. Taking the fix YIN as an example, slots are adjusted around 09:00, 12:00-14:00, 16:00 and 17:00. Clearly, the scheduled slots during these periods exceed the fix capacity. They are considered the peak periods for YIN that highly require to mitigate demand-capacity conflicts. Figure 6 also shows a comparison between three different Gini fairness. The optimization effect under them is almost the same with tiny differences as highlighted in red boxes. The smaller Gini fairness optimization solution achieves a gentler flight distribution curve, indicating a more even distribution of flights. Interestingly, the difference between 0.1668 and 0.3200 only happened during the period of 6:00-8:00 in the morning. However, the MAS fairness experienced an obvious improvement. This may indicate that the adjustment of a specific period of slots could improve the whole fairness. Overall, our approach has the advantage of smoothing out the flight distribution during the periods, "reducing" the busyness of the busy periods and increasing the workload in the relatively "free" periods. Finally, it realizes demand management at a strategic level.

Overall, the model is effective in achieving the optimal allocation of multiple resources in an MAS. When the MAS Gini-based fairness is reduced, the additional displacements from a fairer flight schedule can be more evenly distributed across the airlines. The adjusted flight schedules result in a more even and smooth distribution of traffic. By coordinating flights from busy to idle periods, the model alleviates the busyness of busy periods to some extent.

V. Conclusions

A multiple airport system consists of several airports that provide air transport service to the metropolitan area. To improve the utilization of resources of an MAS, this paper aims to allocate slots for all the airlines in the MAS considering trading off equity and efficiency. A baseline model is first proposed with the objective of minimizing the total slot displacements of the MAS. The baseline model considers airport capacity constraints, fixes capacity constraints, flight turnaround time constraints, and the maximum adjustments for a single flight. Then, comprehensive fairness indicators for airlines are developed to measure airlines' fairness in resource allocation at airports and fixes. A new slot allocation model for an MAS considering the airline's fairness is then formulated by adding fairness constraints into the baseline model. The models are validated and tested using flight schedule data of the MAS in the Guangdong-Hong Kong-Macao Greater Bay Area. The results demonstrate that our model can improve the balance of efficiency and fairness to some extent during the MAS slot allocation. In general, the more the airline's demand, the greater the possibility of the slot being displaced. The model is shown to be an effective tool for optimally allocating the airport and airspace capacity of the MAS.

The findings of this study have a number of important implications for future practice. First, the type of airlines and operations can be taken into account to enhance the validity of the fairness indicator. Second, this paper focuses on fairness among airlines, the airport is yet another important stakeholder in the MAS. Continued efforts are needed to investigate the impact of the scheduling results on fairness among the member airports. In addition, the priority rules on slot requests that are listed in WASG, for example, historic flights enjoy the first priority, should be included in the model. Finally, both the MAS and airport networks involve multiple airports, thus the slot allocation model proposed in this paper can be extended for the slot allocation problem of airport networks.

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