The impact of 4D trajectories on arrival delays in mixed traffic scenarios

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Abstract—In this paper we present a mathematical model devoted to the description of the delays in an ATM context. Our objective is to analyse the benefits of an increased arrival predictability on ATM performance in terms of delays. We show that our model gives a description of the delays that is in good agreement with the actual data, collected in a large European airport. Then we study with our model some variations of a target SESAR scenario, taking into account the possibility that a finite fraction of the aircraft is equipped with 4D-trajectory technology. We show, in the first scenario, that if the traffic is managed on a *first-come, first-served* basis, in order to have a sensible reduction of the congestion the fraction of the aircraft equipped with 4D-trajectory technology has to be quite high. On the other side, in the second scenario, if the discipline becomes *best-equipped, best-served*, i.e. if the aircraft equipped with 4D-trajectory technology are served with priority, the tail of the distribution of the time spent in queue for the aircraft without 4D trajectory becomes sensibly fatter, giving a non vanishing probability to have for them very high queueing delays.

Keywords: predictability, delays, 4D trajectory, queueing theory, stochastic processes.

I. INTRODUCTION

One of the key innovation drivers of the SESAR concept is the shift to 4D trajectory management [1]. The capability of aircraft to flight increasingly accurate 4D trajectories will be a stepping stone to conflict-free planning, de-confliction by precise navigation, fine-tuned planning of take-off and arrivals slots. 4D trajectories will feed ground-based systems with accurate data reflecting both future positions and intentions, thus informing better decision making that takes into account Air Traffic Management (ATM) constraints and users preferences.

Among the key benefits that are expected from 4D trajectories, the SESAR Target concept mentions the achievement of Capacity targets, in the order of a "3-fold increase in capacity which will also reduce delays, both on the ground and in the air" [2]. Similarly, the Predictability KPA will extensively rely on 4D trajectories, to minimise the variability in departure and arrival punctuality, and in flight phases duration. It is expected that a more controlled variability will avoid delays and prevent knock-on effects on other flights. The target goal for predictability is of 95% of flights arriving as planned (3 min tolerance), while the other flights should have a delay of less than 10 minutes (on average).

The SESAR Performance Target acknowledges how such sharp performance increases are severely constrained by the current airport infrastructures (i.e. runways). However, it does not go as far as to say that capacity increases will not be possible unless new runways are built. On the contrary, the capacity increase will come from a more efficient use of not-yet-congested airports and from a more efficient and smooth airport management of the congested airports.

The objective of this contribution is to try and quantify which benefits can be gained by an increased predictability of aircraft arrivals, to see whether this can be considered the leverage via on which more capacity and less delays could be achieved. In details, we will analyse the causal link between the increased predictability of aircraft arrivals and the delays (in total and per aircraft):

- To what extent is the reduced variability of arrival times going to benefit the ATM performance in terms of delays?

Other related research questions are the following two:

- What is the impact of a mixed traffic arrival flow (i.e. with varying percentage of 4D aircraft) on the delays distribution?

- What is the impact of a *best-equipped, best-served* principle on the delays distribution?

II. OPERATIONAL SCENARIOS

In order to address our research question, we will analyse the delays distribution in different scenarios, obtained by combining two conditions.

One condition relates to the percentage of 4D equipped aircraft, which we will vary from zero (current scenario) to 100% (SESAR target). The second condition concerns the underlying principle behind the ATM service, whether it is going to apply a *first-come, first-served* principle, or a more controversial *best-equipped, best-served* (BEBS) principle. The latter principle is currently being debated in the SESAR community with a large variety of positions and no emerging consensus on any of them. Under a BEBS principle, early adopters of SESAR avionics will receive a "preferential service" over non-equipped aircraft. Our contribution does not intend to side any of the current opinions, which would be

clearly beyond the scope of this work, but we would indeed provide data to inform the discussion.

The resulting scenarios are the following ones:

- baseline scenario: *first-come, first-served*, no 4D aircraft;
- initial 4D scenario: *first-come, first-served*, 33% of 4D aircraft;
- advanced 4D scenario: *first-come, first-served*, 66% of 4D aircraft;
- target 4D scenario: *first-come, first-served*, 100% of 4D aircraft;
- initial best equipped 4D scenario: *best-equipped, best-served*, 33% of 4D aircraft;
- advanced best equipped 4D scenario: *best-equipped, best-served*, 66% of 4D aircraft;

The target best equipped 4D scenario is obviously the same that target 4D scenario: *first-come, first-served*, 100% of 4D aircraft is like *best-equipped, best-served*, 100% of 4D aircraft, because in both cases no delays occur.

The baseline scenario is obtained by analysing one days data from a large European airport and comparing it with our theoretical model, see below, while the other scenarios are simulated.

III. Scope and granularity of the model

To simulate the SESAR scenario, we will focus on the approach phase, assuming that the aircraft have already received their Controlled Time of Arrival (CTA) for a specific merging point. We will analyse the slot allocation on that merging point, assuming that the trajectories will be stable after that point (i.e. in the final approach and landing phase) in order to calculate delays. This means that the aircraft will not accumulate other delays, once the CTA is met.

The model also assumes that the trajectories to reach the merging fix are conflict-free (any tactical sequencing is also ignored) and that no other action is taken by controllers if the aircraft can comply with the assigned CTA. In case the aircraft cannot meet its CTA, the controllers will reposition it in the next available slot (under the *first-come, first-served* discipline). Under the *best-equipped, best-served* discipline the slot will also be considered free if occupied by a non-4D aircraft (which will be moved to the next free slot). This is consistent with the idea that the Controlled Time of Arrival is considered as a coordination means between ground and airborne, to be renegotiated as soon as the aircraft cannot comply with it.

In practical terms, this means that the relevant elements of our model are (i) the slot allocation, (ii) the probability for the aircraft to be 4D, (iii) the probability for the aircraft to be late and miss the assigned slot (calibrated on the current situation, reduced to zero for 4D aircraft). If the aircraft misses the slot, we apply one of the two disciplines explained above.

IV. Modeling queues and delays: the standard approach

The natural tool, from a mathematical point of view, to describe the models listed above would be the classical queueing theory. The assumption of the classical queueing theory are:

- the arrival of the clients to the system are described in terms of random interarrival times, that are supposed to be independent realizations of a single random variable
- the service times are also independent realizations of a single random variable
- when a client arrive to the system finding the server(s) busy, it has to wait in a queue.

The aim of the theory is to describe the probability distribution of the queue and of the time spent in queue (queueing delay). The theory is quite complete if the three assumption above are satisfied, see e.g. [3].

In this context, however, the first and the third assumption are questionable: as far as the distribution of the interarrival times is concerned, sophisticated statistical studies (see e.g. [5]) showed that the hypothesis of the independence of the actual interarrival times seems to be quite close to the observed data, even if the calculated arrival times are scheduled in advance. Moreover the interarrival times seem to be exponential, giving rise to an arrival process completely memoryless. This would encourage the use of the classical queueing theory in order to describe the queueing delays of the aircraft. Many recent studies are based on this assumption, see for instance [6] [7], [8] and [9] and references therein. As it will be presented below, however, although such studies capture many features of the system, in general the previsions of the classical queueing theory differs from the observed data most of all if one is interested in the evaluation of the small probabilities to have a very large degree of congestion. Hence we can say that while the arrivals to the system seem to be very well modeled by a memoryless process, the resulting queue is quite different from the one that one would forecast using classical queueing theory. Note, moreover, that it is difficult in this context to define the instant queue of a system in which the clients can not wait in a static queue.

V. Modeling queues and delays: our approach

We will present in the rest of the paper a new method for the evaluation and the forecasting of the queueing delays. More precisely, we will show the following facts:

- It is possible to introduce a mathematical model different from the standard queueing model in order to describe arrival process of the aircraft.
- It is possible to give an approximate but reasonable evaluation of the time spent in queue by the aircraft starting from the actual flight data.
- A queueing model based on the classical theory is unfit to describe the actual data (baseline scenario).
- Our model fits much better with the actual data, giving a good description of the baseline scenario.

- Our model is suitable to be generalized to the other scenarios listed above, and it seems to be able to catch some non completely intuitive features of the system.

As previously stated, the discussion of the results of the last item is beyond the aims of this paper. However it is clear that the intermediate scenarios have to be investigated and eventually controlled carefully, maybe with some more complex choice of queueing discipline.

VI. A MODEL FOR THE BASELINE SCENARIO: PSRA ARRIVALS

The crucial question in order to make previsions on the scenarios mentioned above is the distribution of the actual arrivals of the aircraft. Let us introduce an arrival process, which has been presented in [4]. We will call such process *pre-scheduled random arrivals* (PSRA) process and we will study its main features. The PSRA process is defined as follows. Let $\frac{1}{\mu}$ be the time needed for a landing, that we suppose deterministic. We define $t_i \in \mathbf{R}$ the actual arrival time of the i-th aircraft by

$$t_i = \frac{i}{\mu} + \xi_i, \qquad i \in \mathbf{Z}, \tag{1}$$

where ξ_i's are i.i.d. random variables. Moreover we delete with independent probability a fraction $1 - \varrho$ of arrival times. In this way we assume an intensity of traffic, i.e. an utilization factor of the runway capacity, equal to ϱ.

The idea is therefore the following: the aircraft are scheduled in such a way that, in absence of random delay, each of them would arrive to the runway and find it available. Clearly, the presence of the random delay ξ implies that an additional delay due to the conflict with other aircraft is often imposed by the ATC procedures. We will call this additional delay *time spent in queue*. The initial random delay, which has a standard deviation σ much bigger that $\frac{1}{\mu}$, tends to destroy the prescheduling, giving an arrival process that, for σ tending to infinity, tends to be completely memoryless. This is the reason of the problem desribed in the previous section: from a statistical point of view (see e.g. [5]) the arrival process to a congested airport tends to be very similar to a memoryless (poissonian) distribution.

A realistic choice of the parameters of the PSRA arrivals, however, has a large but finite value of σ. To be concrete, the landing time is around one minute and half, while σ is around 15 minutes. This means that, with a realistic choice of the parameters, the observation of the arrival process fits reasonably with the observed results: the arrival process to the system is very close to a memoryless one. However, the resulting time spent in queue keep some memory of the original prescheduling. This is easy to understand from an intuitive point of view: since the aircraft are prescheduled, an interval in which there are less arrivals than expected is easily followed or preceded by an interval in which the arrivals are more frequent. This self correlation is quantitatively small

(see [4] for its explicit expression). It turns out that a small self correlations between the arrivals, that is difficult to detect from a statistical point of view, has a great influence on the resulting congestion giving a queue that is considerably different from what one would have assuming arrivals that are truly memoryless. This difference is particularly evident when the system is congested, i.e., in the case of hubs in which the capacity of the runways is almost completely exploited (ϱ close to 1).

The mathematical description of the congestion in a queueing system in which the arrival process is given by (1) with thinning (deletion) probability $1 - \varrho$ it is not completely standard, due to the circumstance that this self correlation of the arrival process does not satisfy the independence assumption of the interarrival times as in classical queueing theory. However in [4] it has been proved that the system can be completely described splitting its fluctuations on two different time-scales. On a time scale of the order of $\frac{1}{\mu}$ the system has very rapid and irregular fluctuations of the number of aircraft in queue. On a time scale of the order of $\frac{1}{\mu}\frac{1}{1-\varrho}$, there's a quantity, that we will call α, that is roughly conserved. This quantity is defined in terms of

- the number n_i of aircraft in queue at time i
- the number d_i of aircraft that were supposed to be arrived to the airport at time i but are not yet arrived
- the number a_i of aircraft that were supposed to be not yet arrived to the airport at time i but are actually arrived.

It turns out that the quantity

$$\alpha_i = n_i - a_i + d_i \tag{2}$$

decreases by one if at time i the corresponding aircraft is deleted, it increases by one if at time i the runway remains unused, and it remains constant in all the other cases. For highly congested hubs this means that α remains constant on time interval of the order of $\frac{1}{\mu}\frac{1}{1-\varrho}$, which is a time considerably large compared to $\frac{1}{\mu}$. It turns out also that actually the expected value of α is very close to the expected value of n. Moreover the distribution of the actual number of aircraft in queue is easy to describe as the superposition of the processes defines on the two time scales, and it gives a tail of the distribution of the number in queue that is considerably thinner than the tail of the distribution assuming memoryless arrivals.

A. Baseline scenario: a comparison with real data

The computation of the actual time spent in queue by each aircraft is a problem in itself. It is clear that some congestion is present, because many aircraft are forced to follow holding trajectory, but an exact evaluation of the queue is complicated. We used an approximated procedure on a small set of flights on a large European airport. We considered the STAR around the airport, and we defined the *approaching time* as the time elapsed from the passage on the STAR to the landing. Comparing the approaching times with their minimum value we obtained an estimate of the actual time spent in
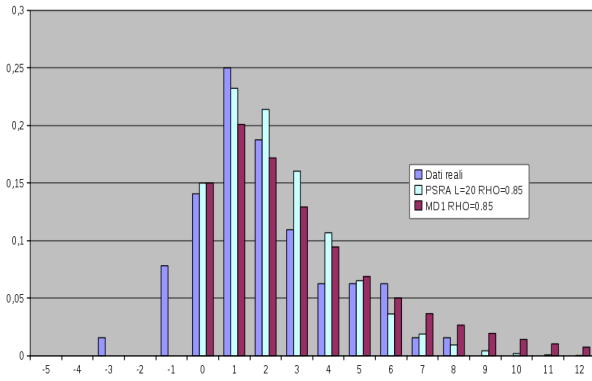
Figure 1

Figure 2

Figure 3

queue. We actually decided to correct slightly this procedure, assuming that some approaching time particularly small may be considered as a trajectory faster than the standard one, and therefore we accepted the idea that for some particularly small approaching time our computation leads to a negative time spent in queue. As a matter of fact we are interested in the tail of the distribution of the time spent in queue, since it express the probability small but non zero to have a very high degree of congestion. In figure 1 are shown the actual distribution of the time spent in queue (in darker blue), its evaluation assuming memoryless arrivals (in red) and its evaluation assuming PSRA (in lighter blue). It is evident that the tail of the distribution of the time spent in queue is grossly overestimated by the memoryless assumption, and it is much more consistent with the PSRA assumption. It is also evident, on the other side, that in order to increase the representativeness of our model it is necessary to incorporate in the model the details of the approaching procedure when the degree of congestion is low.

VII. PSRA ARRIVALS WITH 4D TRAJECTORIES

We want now use the PSRA process, that we consider to be a reasonable tool to provide a forecast of the delays, assuming the scenarios in which a finite fraction p of aircraft is equipped with 4D trajectory technology, while the rest of the aircraft behaves as in the baseline scenario. We assume, in this scenarios, a *first-come, first-served* discipline of the service. This implies that the distribution of the random delay ξ appearing in (1) is as before with probability $1 - p$, while ξ is identically 0 with probability p. This circumstance tends to decrease the variance of ξ, and consequently also the congestion decrease. As the analytical study of the PSRA process shows clearly, however, a small change of the variance of ξ is not very effective. In the baseline scenario the standard deviation of ξ is about 15 minutes, and in order to have a sensible decrease of the congestion its value has to arrive close to 5 minutes. As it can be seen in the figure 2, a fraction of 4D trajectory aircraft of the order of 33% gives a decrease of the distribution of the queue that is not very high. The average value of the queue is 2.6 without 4D trajectories and 2.1 with 33% of 4D trajectories. Even in an advanced 4D scenario
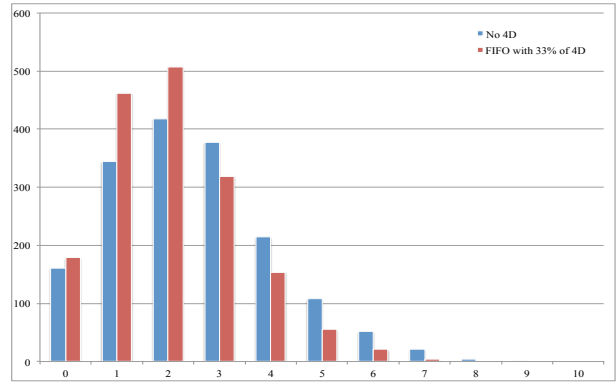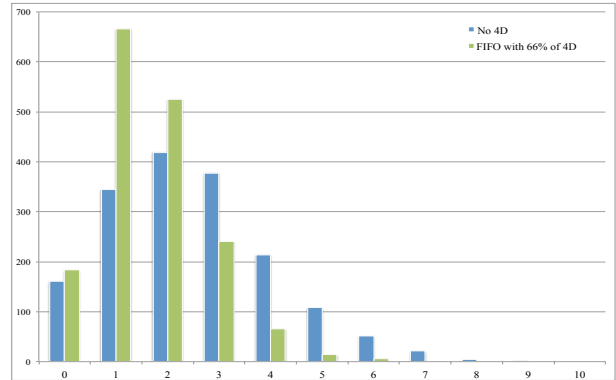
(66% of 4D aircraft, figure 3) the decrease of the queue is more relevant (the average value of the queue is 1.7) but there is still a non vanishing probability to have a certain degree of congestion. The queue obviously vanishes in the target 4D scenario. This seems to suggest that the introduction of the 4D technology tends to keep a finite queue as long as the process to provide 4D technology to all aircraft is still uncomplete.

VIII. PSRA ARRIVALS WITH 4D TRAJECTORIES AND *Best-Equipped, Best-Served* (BEBS) DISCIPLINE

The framework of the PSRA process can be used in order to forecast the congestion also in a different, BEBS, scenario. This corresponds to give the priority to the aircraft equipped with 4D trajectory. Each aircraft is 4D equipped with independent probability p. Also in this case the problem is completely solvable, both with numerical and analytical approach. The fact that the 4D aircraft are served with priority corresponds to say that they land exactly when they are prescheduled. The rest of the aircraft, hence, has a fraction p of landing slots that are unavailable. This corresponds to say that the parameter α defined in section VI increases also when the slot is unavailable, i.e. it increases at each time independently with probability p. On the other side, since only the aircraft that are not equipped with 4D technology appears in the model, the intensity of traffic ϱ is decreased of p. In this case there are two

Figure 4

Figure 5

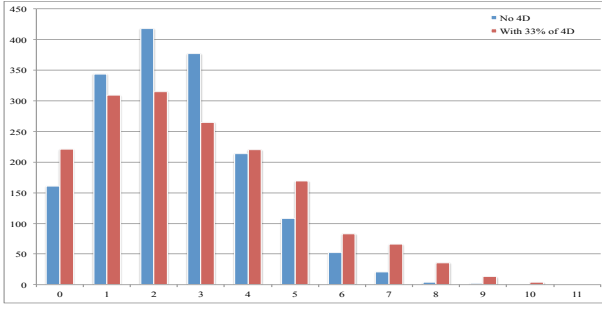| Discipline	Queue	Time in queue
No 4D	2.32	2.58
FIFO with 33% of 4D	2.15	2.38
FIFO with 66% of 4D	1.73	1.92
BEBS with 33% of 4D	3.89	6.47
BEBS with 66% of 4D	2.42	8.06

TABLE I
COMPARISON OF THE QUEUE IN THE VARIOUS SCENARIOS.

different effects that tend to balance: the time independence of the dynamics of the parameter α implies that the tail of the distribution of the length of the queue tends in this case to be much more similar to the memoryless distribution. This causes an increase of the average value of the queue, but most of all gives a much bigger probability to have a large amount of aircraft that are waiting to land, and this implies and increased level of work in terms of ATC, with all the well known negative consequences on the resilience of the system. On the other side, the standard deviation of the delay ξ tends to be smaller with respect to the time between two landings of non 4D aircraft. This tends to decrease the length of the queue. In the latter case, however, one has to keep in mind that the time spent in queue increases, because it has to be multiplied by a factor $\frac{1}{1-p}$.

These features of the BEBS discipline are explained in figure 4 and 5. Here we compare the distribution of the number of aircraft in queue without 4D-trajectory aircraft (in blue) with the distribution (in red and green) of the length of the queue of aircraft that are not equipped with 4D-trajectory technology. The fraction of 4D-trajectory aircraft is 0.33 in figure 4 and 0.66 in figure 5. Some comments are necessary. Observe that in figure 4 the distribution is fatter, as observed above. Observe moreover that the time spent in queue by each aircraft is multiplied by a factor $\frac{1}{1-p}$, and hence the queue in figure 5 seems to be smaller for the effect mentioned above, but it gives a time spent in queue that is much longer (three times) than the corresponding time in queue in the case without 4D-trajectory aircraft. This shows that the effect of the priority in the discipline of the system gives a loss of efficiency for the aircraft worst equipped that can be considered affordable if p is small, while is definitely too big when p approaches 1, due to the rapid increasing of the time spent in queue.

IX. CONCLUSIONS AND FUTURE WORKS

The main aim of this work was to study the effects of an increased arrival punctuality, due to the introduction of 4D technologies, on the ATM system performances. Our work enhanced our understanding of future SESAR scenarios, providing non-trivial insights on new possible prioritisation rules to better manage the transition from the current situation to the 2025+ future ATM system.
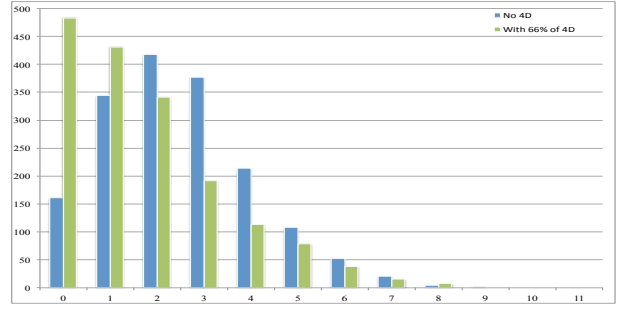
In Table 1 we give a summary of our results in terms of averaged quantities in the various scenarios. We report both the length of the queue and the time spent in queue, since both are relevant quantities. The time spent in queue is expressed in terms of number of average landing time, hence the delays in terms of minutes are longer. Recall that the average time spent in queue in the BEBS scenarios refers to the worst equipped aircraft, since the 4D aircraft do not wait in queue.

Main findings can be summarised as: (1) 4D trajectory management will be effective and will significantly enhance the ATM system overall predictability, only if the adoption of 4D technologies will be widespread all over Europe; (2) Mixed traffic situation will be difficult to manage. The progressive introduction of 4D-equipped aircraft, even with new prioritisation rules actually under discussion, will affect both the efficiency and the fairness of the overall system, due to the effect of the increasing delays of worst equipped aircraft. For the above reasons, technological improvements should be fostered as much as possible to properly reach SESAR objectives. Principles like the best-equipped, best-served may be appropriate to incentivise early adopters, but their drawbacks on the overall efficiency should be analysed (as in our results). Other leverages should then be considered and assessed.

An important open issue is the discussion of the accuracy of the PSRA model with respect to real air traffic data. We obtained promising preliminary results showing that the description of the distribution of the length of the queue using the PSRA as arrival process is more accurate than the description assuming a Poisson process and to describe.

Further works will involve an analysis of wider datasets in order to better substantiate this assumption, eventually refining our baseline model and its parameters.

Other future works will include:

- The introduction of other prioritisation policies, including the refinement and detailing of the existing ones.

- The introduction of time critical activities in the slot allocation, for instance by simulating the short term change of the slot allocation due to the late downlink of an unable to comply message by one (or more) of the 4D aircraft.

- A more radical scenario will involve a drastic restructuring of the whole slot allocation, simulating a sudden (with immediate effect) notification of a new ATM constraint, one that triggers the recalculation of the Target Arrival Time by a large percentage of all the aircraft involved (technical failure leading to sudden unavailability of an arrival procedure might be a candidate scenario for this case).

Another possible subject of further study could be the analysis of the impact of 4D-trajectories introduction on other ATM Key Performance Areas, by enlarging the scope of our model and including other existing models, e.g. on airport runway management, fuel efficiency, and so on. More discussion with operational experts will be carried out to assess the feasibility and interest of going in this direction.

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