Aircraft Trajectory Prediction in Random Atmosphere. Mathematical background and application to a locally uniform academic case

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***Abstract*—We are interested in aircraft trajectories seen as stochastic processes. These processes evolve in an unknown atmospheric random environment. As several aircraft parameters are unknown such as true airspeed (TAS) and wind, we have to estimate them.**

To this end, we suggest to use ensemble weather forecasts, which give different scenarios for the atmosphere, with a system of trajectory predictions. In this way using the air-traffic data, we evaluate the likelihood of each element and we construct a random weather environment organized by the element weight.

To get this result, we use sequential Monte Carlo methods (SMC) in the special context of random environment. The algorithm called island particle filter allow to estimate both the likelihood of the meteorological forecasts and the aircraft parameters.

***Index Terms*—Trajectory Prediction, Random Environment, Ensemble Meteorological Forecast, Stochastic Process, Particle Filter**

INTRODUCTION

To satisfy the future demand in terms of air transportation, the present air-traffic management system needs to be improved. To this end two projects, NEXTGen in the United-States and SESAR in Europe, have been launched. In both cases, the selected appproach consists in constraining in time and space the aircraft position (4D-trajectory) [3], [4]. Moreover, the SESAR project aims to ensure free flights avoiding any delaying tactics. Therefore trajectory predictors have to be accurate and reliable. In that way, the workload of air-traffic controllers can be reduced using decision support tools. Moreover the capacity of the airspace can be used to its maximal capacities. To explore more innovative techniques, the SESAR JU has developed the WP-E long term research program. This work contains the fundamental methods intend to be use in the WP-E IMET program (http://www.imet.pro). This program investigate the optimal approach for future trajectory prediction systems to use Meteorological uncertainty information.

To compute aircraft trajectories in advance, trajectory predictors need different information. Some concern the flight intent, others are directly related to the aircraft and finally some are environmental parameters, such as wind and temperature. An important source of uncertainty in aircraft trajectory prediction concerns the meteorological parameters. Indeed a part of the along track error in predicting the aircraft trajectories is due to the weather forecasting error [5].

Up to now, aircraft trajectory predictors use only one weather deterministic forecast. However, in the case of the airspace capacity reduction probability in relation with available slots linked to the Met, statistical information about weather forecasts are needed (see CATS report D1.1 [12]). To take into account this Met uncertainty, a proposed solution was to use statistical errors on weather forecast [6]. Some research works use Gaussian assumptions and prescribed correlation functions [6] for the Met uncertainties. European programs have investigated this type of uncertainty modelling: HYBRIDGE [11], ERASMUS [13]. However the wind uncertainties are time or space dependant and are not Gaussian distributed [9]. Recently in the scope of the SESAR program, the TESA project [14] intends to address the question of the statistical envelope of the trajectories according to given Met uncertainties with a priori probability distributions (Gaussian, Poisson, exponential) of the Met entries. It do not take into account the real distribution of the weather forecast errors.

Our work is totally new and uses a different technical direction. It aims to give a solution to the TP Met uncertainties by using ensemble weather forecasts. Indeed national meteorological center are able to provide them. These forecasts give several atmospheric evolution scenarios which reflects the lack of knowledge about the initial state [7]. These scenarios enable to explore the uncertainties about the state of the atmosphere [7]. Another fact at this point is that ensemble forecasts are not delivered with a probability distribution [8]. This problem can be tackled using stochastic methods to weight the elements of the ensemble weather forecasts regarding to air-traffic observations.

In this work we suppose that we have air-traffic observations and an aircraft trajectory predictor. Each aircraft trajectory prediction has an error part and all the aircrafts trajectories in the same area are sharing the same meteorological situation. Now, considering we have a set of weather forecasts, we can evaluate a performing score regards to trajectory prediction errors over the last minutes.

In order to formalise these two ideas, the first part is dedicated to give the formal framework of this problem. Then the ensuing algorithms are explained and finally we give some numerical results on an academic example.

I. Formalism

To get the likelihood of wind proposals with respect to air-traffic radar observations, a mathematical modelling has to be done. We choose to modelize aircraft trajectories as stochastic processes evolving in a random meteorological environment. This modelling is natural according to the physical question of a mobile evolving in a forecasted medium with random errors. As we suggest to use an ensemble of forecasts, classical filters such as Kalman Filters, Particle Filters and their different versions are unsuited. We propose to use a specific algorithm called Island Particle Filter [1]. This filter is very general and requires no special assumptions like linearity or Gaussianity of the parameters. Moreover, the Island algorithms are reputed to learn unknown parameters [1].

Before going deeper into the mathematical formalism, we adopt the following notations. The ensemble of probability measures on a space E is denoted $\mathcal{P}(E)$. For a probability measure μ and a measurable function f, $\mu(f)$ is the expectation of the function f for the measure μ. For a probability operator $Q(x, dy)$ giving the probability to arrive in the element dy starting from x, $\mu Q(dy) = \int \mu(dx)Q(x, dy)$ is the probability of the event dy for the operator Q averaged by the measure μ. Finally $\mu Q(f) = \int \mu(dx)Q(x, dy)f(y)$ is the expectation of the function f for the operator Q through the measure μ.

A. *Definitions of the involved stochastic processes*

Before considering aircraft trajectories, we decompose the real wind at time n, W_n^r into two parts, the forecasted part W_n^f and the forecasting error part X_n^1. The state parameters of an aircraft are denoted by the process X_n^2. X_n^2 may contain the Mode-S information such as the location, ground speed, TAS, etc. The process X_n^2 is directly influenced by the atmosphere and in our computation by the Met prediction errors X_n^1. In this study we intend to evaluate the likelihood of the pair (X_n^1, X_n^2) according to radar observations Y_n. The Mode-S observations Y_n include the aircraft positions, ground speed, TAS, etc and are assumed to be imperfect.

In our study, we have splitted a control area (En-Route or TMA) in sub-domain D_l where the Met errors are spatially uniform. Our interest concerns the definition of the trajectories inside the subdomain D_l and we have to manage the entries and the exits of aircrafts. The modelling presented below corresponds to this locally uniform case.

For any $n \geq 0$ we consider $E_n^{(0)} \subset \mathbb{R}^2$ the location space. Let $X_{n,x}^1$ be a $E_n^{(1)}$-valued random homogeneous environment, *i.e.* a random field, where $n \geq 0$ and $x \in E_n^{(0)}$. $(E_n^{(1)}, \mathcal{E}_n^{(1)})$ is a collection of measurable spaces. In the sequel as far as there is no possible misunderstood, $X_{n,x}^1$ is denoted X_n^1. Let X_n^2 be a $E_n^{(2)}$-valued process. $(E_n^{(2)}, \mathcal{E}_n^{(2)})$ is a collection of measurable spaces such that for any time step $n \geq 0$, $E_n^{(2)}$ encapsulates the location of the aircrafts which are in $E_n^{(0)}$ but also the aircrafts' kinematic parameters for example. It means that some coordinates of the process X_n^2 are locations in the space $E_n^{(0)}$. Let Y_n be a F_n-valued process where $(F_n, \mathcal{F}_n)$ is a collection of measurable spaces.

X_n^1 is supposed to be a Markov chain of transition kernel $M_n^{(1)}$ and initial distribution $\eta_0^{(1)}(dx_0^{(1)})$. X_n^2 is also a Markov process of transition kernel $M_{x_n^{(1)},n}^{(2)}$ and initial distribution $\eta_{x_0^{(1)},0}^{(2)}$. The transition kernel family depends on the evolution of the random medium X_n^1.

Using these notations, the aircraft position process model is

$$X_{n+1}^0 = X_n^0 + S_n(X_n^2) + W_n^f(X_n^0)\Delta t + \Delta X_n^1(X_n^0)$$

where S_n is the flight strategy in a time step $n\Delta t$.

Let $N_n^l > 0$ be an integer denoting the number of aircrafts present in a sub-domain D_l. An air-traffic is N_n^l duplications of the process X_n^2. Moreover we consider that there are no interactions between the aircrafts, for instance no conflict avoidance. It means that the aircrafts $(X_n^{2,j})_{1\leq j\leq N_n^l}$ are independant. The traffic processes $(X_n^{2,j})_{1\leq j\leq N_n^l}$ are living in $E_{n,l}^2 = \otimes_{j=1}^{N_n^l} E_n^{2,j}$. For the sake of simplicity, the family of aircrafts $(X_n^{2,j})_{1\leq j\leq N_n^l}$ is also denoted by X_n^2. The process Y_n is a partial observation of the Markov chain $\left(X_n^1, X_n^2\right)_{n\geq 0}$.

B. *Learning the Trajectory Processes in a Random Environment when the environment is decomposed in several domains*

We first deal with the quenched process, which corresponds to the case where the evolution of the random environment is assumed to be fixed by the Met forecasts. In the next section, we will treat the case where the environment is regarded as a random process.

1) Quenched restricted process: In order to manage the subdomain exit of the aircrafts, we create a specific point called cimetery point, denoted $\mho_n^l$, where the aircrafts are affected as they go outside the subdomain.

Considering that $X_{n,l}^1 = x_{n,l}^{(1)}$, let denote $(X_{n,l}^{2,i}(X_{n,l}^0))_{1\leq i\leq N_n^l}$ the aircraft state where $X_{n,l}^0$ is the location process. The aircrafts evolve with the transition kernel $M_{x_{n,l}^{(1)},n,l}^{(2)}$ for any x_{n-1} to the target dy according to:

$$\begin{aligned} M_{x_{n,l}^{(1)},n,l}^{(2)}(x_{n-1}, dy) &= \mathbb{1}_{D_l}(y) M_{x_{n,l}^{(1)},n}^{(2)}(x_{n-1}, dy) + \\ &\quad (1 - \mathbb{1}_{D_l(y)})\delta_{\{\mho_n^l\}}(dy) \end{aligned}$$

Therefore the mutation kernel $M^{(2)}_{x^{(1)}_{n,l},n,l}$ corresponds to a survival process where the aircraft goes to the cemetery $\mho^l_n$ if it exits the domain D_l. After this transition step in D_l there are $\tilde{N}^l_n$ remaining aircrafts.

Then $N'_{n,l}$ new aircrafts are added, that means that some aircrafts are entering into the domain D_l. This step is modelled by the kernel transition $P_{n,l}$ which is defined for any probability measure η by :

$$\eta P_{n,l} = \eta \otimes \eta * \mu^l_n$$

where μ^l_n is the new aicraft reallocation measure. $P_{n,l}$ can be written in the preeceding form because the mutation kernel $M^{(2)}_{x^{(1)}_{n,l},n,l}$ does not account any interaction process for instance without any conflict avoidance scheme.

Each aircraft generates an observation $Y^i_{n,l}$, with probability density function $G_{x^{(1)}_{n,l},n,l}(X^{(2),i}_{n,l})$. This density function corresponds to the likelihood of the radar observation (wind or flight parameters) with respect to the process restricted to the uniform domain D_l.

The Trajectory Prediction (TP) distribution with respect to the Met environment and the observations is denoted :

$$\begin{aligned}\eta^{(2)}_{x^1_{[0,n],l},n,l} &= \mathbb{P}(X^2_{n,l} \in dx^2_{n,l} | Y_{[0,n-1],l} = (y_{0,l}, \ldots, y_{n-1,l}),\\ &\qquad X^1_{[0,n],l} = (x^{(1)}_{0,l}, \ldots, x^{(1)}_{n,l}))\end{aligned}$$

The updated version of this distribution using the new observation and corresponding to the optimal TP is denoted by :

$$\begin{aligned}\hat{\eta}^{(2)}_{x^1_{[0,n],l},n,l} &= \mathbb{P}(X^2_{n,l} \in dx^2_{n,l} | Y_{[0,n],l} = (y_{0,l}, \ldots, y_{n,l}),\\ &\qquad X^1_{[0,n],l} = (x^{(1)}_{0,l}, \ldots, x^{(1)}_{n,l}))\end{aligned}$$

As it was proved in [2], $\eta^{(2)}_{x^1_{[0,n],l},n,l}$ satisfies the following non-linear equation :

$$\eta^{(2)}_{x^{(1)}_{[0,n],l},n,l} = \phi^{(2)}_{x^{(1)}_{[0,n],l},n,l}\left(\eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l}\right)$$

with

$$\begin{aligned}&\phi^{(2)}_{x^{(1)}_{[0,n],l},n,l}(\eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l})(dx^{(2)}_{n,l}) =\\ &\int_{E^{(2)}_{n-1,l}} \psi_{x^{(1)}_{[0,n-1],l},n-1,l}(\eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l})(dx^{(2)}_{n-1,l})\\ &\qquad M^{(2)}_{x^1_{n,l},n,l}(x^{(2)}_{n-1,l}, dx^{(2)}_{n,l}) P_{n,l}(x^{(2)}_{n,l})\end{aligned}$$

where

$$\begin{aligned}&\psi_{x^{(1)}_{n-1,l},n-1,l}(\eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l}) = \mathbb{1}_{E^2_{n-1,l}} \eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l}\\ \otimes \quad &(1 - \mathbb{1}_{E^2_{n-1,l}}) \frac{G_{x^{(1)}_{n-1,l},n-1,l}\eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l}}{\eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l}(G_{x^{(1)}_{n-1,l},n-1,l})}\end{aligned}$$

This complex nonlinear system gives the sequential evolution of the TP distribution. It has no analytical solution and we have to use a Monte-Carlo algorithm to compute an approximate solution. Finally we summarize the evolution scheme of the TP distributions by the following scheme :

$$\begin{array}{ccc}\hat{X}^2_{n-1,l} \sim \hat{\eta}^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l} & \xrightarrow{M^{(2)}_{x^{(1)}_{n,l},n,l}} & \tilde{X}^2_{n,l} \sim \tilde{\eta}^{(2)}_{x^{(1)}_{[0,n],l},n,l}\\ & & \big\downarrow P_{n,l}\\ \hat{X}^2_{n,l} \sim \hat{\eta}^{(2)}_{x^{(1)}_{[0,n],l},n,l} & \xleftarrow{S_{n,\tilde{\eta}^{(2)}_{x^{(1)}_{[0,n]},n,l}}} & X^2_{n,l} \sim \eta^{(2)}_{x^{(1)}_{[0,n],l},n,l}\end{array}$$

2) Random restricted distribution process: In this section the environment is not fixed and we take into account its unknown evolution. As we decompose the space E^0_n for each time step $n > 0$ such that the random field X^1_n is uniform in each cell D_l, we have to restrict the random process in distribution space η' on each D_l. To this end, we introduce the stochastic process:

$$X'_{n,l} = (X^1_{n,l}, \eta^{(2)}_{X^{(1)}_{[0,n],l},n,l})$$

This stochastic process takes its values in $E'_{n,l} = E^1_{n,l} \times \mathcal{P}(E^2_{n,l})$. As it was proved in [2], it is a Markov chain with transitions defined for any function $f'_{n,l}$ and for any state $(u, \eta) \in E'_{n,l}$:

$$\begin{aligned}&M'_{n,l} \left((x^1_{n-1,l}, \eta^{(2)}_{x^1_{[0,n-1],l},n-1,l}), d(x^{(1)}_{n,l}, \eta^{(2)}_{x^{(1)}_{[0,n],l},n,l})\right)(f'_{n,l})\\ &= \int_{E^{(1)}_{n,l}} M^{(1)}_{n,l}(x^{(1)}_{n-1,l}, dx^{(1)}_{n,l})\\ &\qquad f'_{n,l}(x^{(1)}_{n,l}, \tilde{\phi}^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l}(x^{(1)}_{n,l}, \eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l}))\end{aligned}$$

with :

$$M^{(1)}_{n,l}(x_{n-1}, dy) = \mathbb{1}_{D_l}(y) M^{(1)}_n(x_{n-1}, dy)$$

and an initial distribution $\eta'_{0,l} \in \mathcal{P}(E'_{0,l}) = \mathcal{P}(E^{(1)}_{0,l} \times \mathcal{P}(E^{(2)}_{0,l}))$ defined by :

$$\eta'_{0,l}(d(x,\nu)) = \eta^{(1)}_{0,l}(dx)\delta_{\eta^{(2)}_{x,0,l}}(d\nu)$$

The application $\tilde{\phi}^{(2)}_{x^{(1)}_{[0,n-1],l},n,l}$ is defined by :

$$\begin{aligned}&\tilde{\phi}^{(2)}_{x^{(1)}_{[0,n-1],l},n,l}(x^{(1)}_{n,l}, \eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l})(dx^{(2)}_{n,l})\\ &= \phi^{(2)}_{x^{(1)}_{[0,n],l},n,l}(\eta^{(2)}_{x^{(1)}_{[0,n-1],l},n-1,l})(dx^{(2)}_{n,l})\end{aligned}$$

The most important point to keep in mind to differentiate ϕ_l from $\tilde{\phi}_l$ is that in the quenched framework we know the environment and its evolution in time whereas in this distribution space X^1_l is a random variable.

Now following the same scheme as before, we define the marginal quantities $\eta'_{n,l}$ in distribution space. We can find here that

$$\eta'_{n,l} = \mathbb{P}(X^1_{n,l} \in dx^1_{n,l}, \eta^{(2)}_{X^1_{[0,n],l},n,l} | Y_{[0,n-1],l} = (y_{0,l}, \dots, y_{n-1,l}))$$

and

$$\hat{\eta}'_{n,l} = \mathbb{P}(X^1_{n,l} \in dx^1_{n,l}, \eta^{(2)}_{X^1_{[0,n]},n,l} | Y_{[0,n],l} = (y_{0,l}, \dots, y_{n,l})).$$

One can check that η'_l is verifying a non-linear equation :

$$\eta'_{n,l} = \phi'_{n,l}(\eta'_{n-1,l})$$

with

$$\psi'_{n-1,l}(\eta')(f'_{n,l}) = \eta'(G'_{n-1,l}f'_{n,l})/\eta'(G'_{n-1,l})$$

where

$$G'_{n,l}(x,\mu) = \int_{E^{(2)}_{n,l}} \mu(dy) G_{n,l}(x,y) = \mu(G_{n,l}(x,.))$$

In other words, $G'_{n,l}$ represents the probability that the observations of $y_{n,l}$ is made given that $Y_{[0,n-1],l} = (y_{0,l}, \dots, y_{n-1,l})$, and $X^1_{n,l} = x^{(1)}_{n,l}$.

As the probability $\eta'_{n,l}$ cannot be calculated analytically, we have to use particle techniques to approximate them. Any traditional particle filters as SMC [10] or Kalman filters cannot be used in this special context. Indeed, the random environment where the stochastic process evolve does not allow the use of these techniques. The Island Particle Filter algorithm presented in the Algorithm 1 is then the only adapted one. In this generic algorithm the hidden state $x'_n = (x^1_n, \eta^2_{n,x^1_n})$ has to be estimate with respect to the observation sequence $y_{1:n}$. To this end there is two nested SMC algorithms. The first one correspond to the environment level, the second one to the trajectory process level. Basically, for each environment proposed we perform the approximation of the trajectory process law. Then, for each environment we compute the likelihood, suming the likelihood of each proposed traffic situation, and we get the likelihood of the environment. That allow us to estimate the environment law, which is exactly the scope of our work.

In few words, we describe this algorithm. It uses N_1 different Met forecasts and the observation of an aircraft traffic. Each weather forecast is considered as an Island. For each Island, for each aircraft the algorithm generates a set of N_2 trajectories by perturbing randomly the flight parameters. Using this set of predicted trajectories (one set per aircraft), a confrontation to the reality is performed and a distance between the prediction and the observation is computed. Then the trajectory predictions receive a weight according to their distance. After this step of trajectory weighting, the Island themselves are assessed by their mean performance to predict the air traffic. Therefore the algorithm compute without any knowledge the likelihood probability function of the trajectories and the Met forecasts. Then, the trajectories are resampled randomly according to their weight. The highly

Algorithm 1 Island Particle Filter - IPF

Require: η'_0, M' et ψ'
Ensure: Particle approximation of $p(x'_n|y_{1:n})$ and $p(x'_n|y_{1:n-1})$

$\boxed{\text{Begin}}$

1. Initialization $p = 0$

for $i = 1, \dots, N_1$ **do**

Sample $\varepsilon^i = (\zeta^i_0, \nu^i_{\zeta^i_0,0}) \sim \eta'_0$,

$\zeta^i_0 \overset{i.i.d}{\sim} \eta^{(1)}_0$, and $\nu^i_{\zeta^i_0,0} = \frac{1}{N_2} \sum_{j=1}^{N_2} \xi^{i,j}_0$ where $\xi^{i,j}_0 \overset{i.i.d}{\sim} \eta^{(2)}_{\zeta^i_0,0}$

end for

$p = 1$

2. Selection of islands

Sample $I_p = (I^i_p)^{N_1}_{i=1}$ multinomially with probability $\propto \left(\frac{1}{N_2} \sum_{j=1}^{N_2} G_p(\zeta^i_p, \xi^{i,j}_p) \right)^{N_1}_{i=1}$

for $i = 1, \dots, N_1$ **do**

3. Selection of particles inside each island

Sample $J^i_k = (J^{i,j}_k)^{N_2}_{j=1}$ multinomially with probability $\propto \left(G_p(\zeta^{I^i_k}, \xi^{I^i_k,j}_k) \right)^{N_2}_{j=1}$

4. Mutation of Island

Sample independantly ζ^i_{p+1} according to $M^{(1)}(\zeta^{I^i_p}_p, .)$

for $j = 1, \dots, N_2$ **do**

5. Mutation of particles

Sample $\xi^{i,j}_{p+1}$ according to $M^{(2)}_{p,\zeta^i_{p+1}}(\xi^{I^i_p,J^i_p}, .)$

end for

end for

$p \longleftarrow p + 1$ go to step 2.

$\boxed{\text{End}}$

probable are more often drawn but the less probable are not systematically dismissed. The new set of trajectories described the envelope of the trajectories according to the observation system and each weather forecast is weighted according to the reality. Then a new prediction is performed and the sequence starts again.

This algorithm, allow us to treat our problem without making assumptions on the linearity of our model nor the Gaussianity of the errors. Then the limitations of this algorithm are not related on which needed assumptions but more in the computational cost. Indeed more the trajectory model is not well known more it needs computational time in order to compensate the lack of knowledge.

II. Numerical results

In order to test our method, we have designed an academic, but realistic, experiment (see Fig. 2). We consider an air traffic sector observed by Mode-S radar with a given meteorological situation. We assume that there is a meteorological perturbation such as a cold front. Then two domains appear, one behind the cold front with a specific wind force and direction, and a second zone with a different wind. The ensemble Met forecasts provide different location of the cold front and different wind forecasts. Moreover we assume that three aircrafts evolve following a straight line in the control sector and only one is crossing the cold front limit. Therefore the flight model is very simple with a null constant acceleration except for random instants. We suppose that the aircraft altitude is constant and their airspeeds are constant piecewise functions with some slight variations. Then the purpose is to evaluate the likelihood of an ensemble of Met predictions and to learn the TAS of the aircrafts present in the traffic.

The Island Particle filter is used to learn both the ensemble forecast weights and the aircraft parameters (here only the TAS). The method is described in Fig. 1. For any aircraft present in the traffic, using any Met forecasts, the algorithm generates several trajectories. Then using the radar observations, the trajectories are resampled according to their likelihood. The updated trajectories for all the aircrafts are used to compute the weight of the Met forecasts according to the Mode-S information.

For our different numerical experiments, we model the wind error by stationnary and uniform values in each subdomain. The limit of the domain delimited by a vertical border is not known. The unknown wind error is uniform in direction and strength over each area. Therefore, the forecasting wind error in both areas in terms of strength and direction, and the location of the border have to be estimated. It is known that aircraft compensate lateral wind. It is the reason why we have to use more than one aircraft direction line. The aircraft true airspeed is not exactly observed and we consider that the speed is piecewise constant with some little random jump. In the experiment, the random jump are modeled by a Poissonian process. The configuration of the experiment is resumed in the Fig. 2. The blue line with triangles represents the unknown limit we have to estimate. The arrows in each domain separated by the blue line have the same direction and the same number of dash which means that the wind force is the same overall the area.

The observation process is given by Mode-S radar information. Using the perturbed true airspeed and ground speed, we can deduce a perturbed 2D wind force. In both domains, the wind force is about 40 kt. This can be used as wind observation. The three aircrafts have a true airspeed about 400 kt.

In this academic work, the perfect observations are perturbed with Gaussian centered random noises. We have choosen a white noise with a variance of 0.1 on each aircraft position and for the deduced wind with a variance of $\sqrt{5}$. The period of sampling observations is 15 seconds. In this example the experiment simulates 20 minutes of air-traffic started at 12h00 UTC.

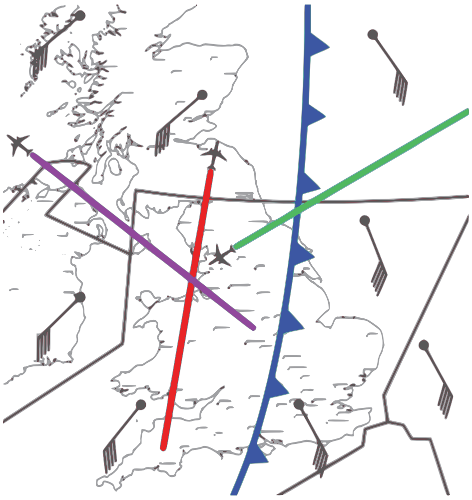

Fig. 2. Example of possible numerical experiment. Crossing a sector control, there are a cold front and three En-Route aircrafts moving with different speeds. The cold front is a limit of two domains with different wind directions. The purpose of the experiment is to estimate the likelihood of the weather and the aircraft airspeed using an ensemble of weather forecasts and radar observations.

Consequently, all the ingredients needed to perform the Island Particle Filter method are available. Concerning the Met proposals, we have designed an ensemble of 3125 forecasts with a combination of 5 different forecasted wind force normally distributed around the true value, 5 different directions and 5 possible border locations uniformly distributed around the true value.

The numerical results are quite good both for the learning of the Met environment and for the TAS of the aircrafts.

First we put our attention to the Met situation. As regards the limit, as soon as an aircraft experiment the limit, the true limit is perfectly determined. The Figure 3 represents the likelihood evolution of the vertical limit proposals over time. First all the limit proposals are equivalent as no aircrafts experiment the limit yet. Therefore the likelihood of all the proposal are the same. In the experiment only one aircraft is crossing the limit from the right to the left. When the aircraft is crossing a wrong proposal, the likelihood of the proposal decreases down to zero and gradually all the wrong limits obtain a weak likelihood. At the end, the highest likelihood limit proposal are concentrated on the left where the real limit is.

Once the limit between the two domains is known, we can put our interest on the meteorological parameters, for instance for the left area. First the likelihood of the wind direction forecasts is examined. As it might be noticed on the Fig. 4, the weight evolution of the direction proposals for the uniform domain on the left is concentrated over one proposition. At the beginning of the experiment the direction weights

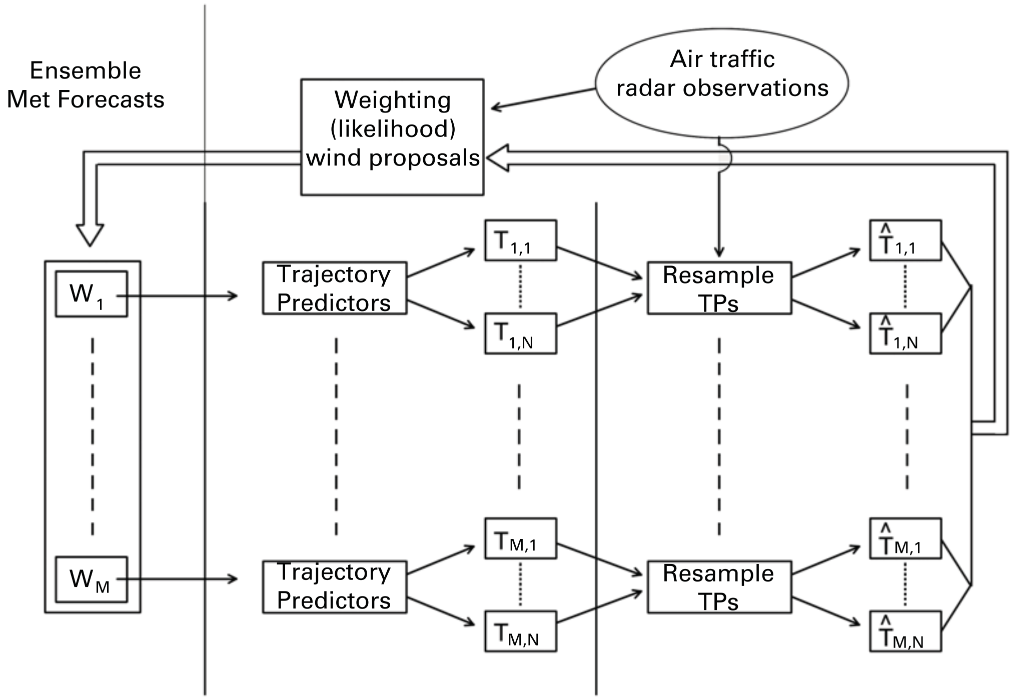

Fig. 1. Sequential estimation of the ensemble weather forecast weights and TAS of the aircraft using the Island Particle Filter. For each forecast, the algorithm generates several TPs which are selected using the radar observations. This step learns the aircraft parameters such as the TAS. Then for each weather forecast the algorithm estimates its likelihood with respect to the radar observations.

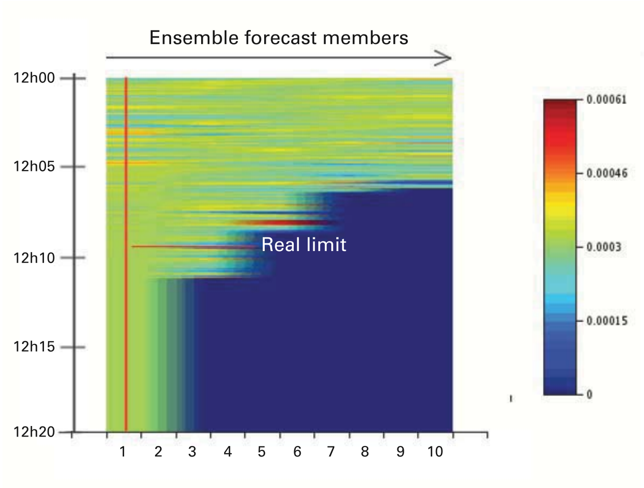

Fig. 3. Time evolution (y-axis from top to bottom) of weight (colorscale) of the different limit proposals (x-axis) between the two domain. The algorithm gives gradually the maximum of likelihood to the forecast which has the most probable limit. The other limit are excluded as soon as an aircraft experiment the border.

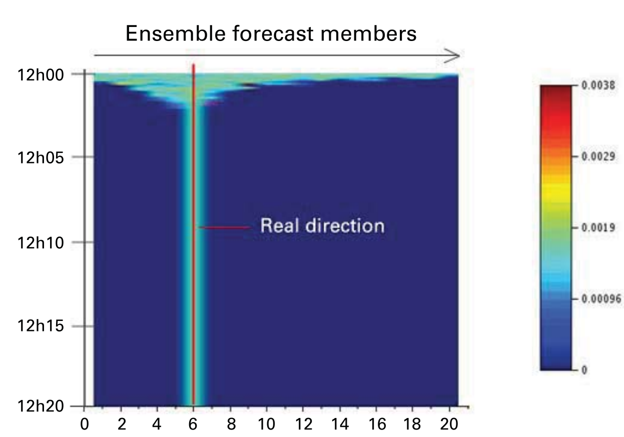

Fig. 4. Likelihood evolution (in color) over time (y-axis from top to bottom) of direction proposals (x-axis) obtained with IPF for the left uniform area. Using the algorithm, the maximum of weight is quickly concentrate over one direction giving the best forecast regarding to the air-traffic radar observations. In this example, the best forecast corresponds to the real direction.

are equidistributed. Then using the Mode-S information, the weight starts to concentrate on only one direction till the end of the experiment. This weight concentration on one direction corresponds to the real direction which has been successfully learned.

The direction of the wind being learned, the figure 5 presents the wind force relative errors. One can see that this relative error is about 2%. Concerning the wind force, it seems to have two periods. The second period and the jump in the

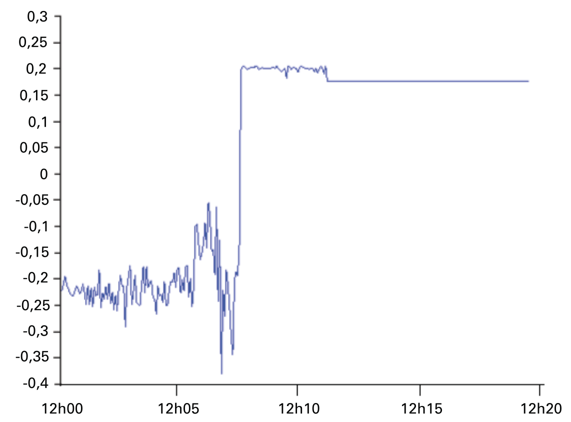

Fig. 5. Evolution in time (x-axis) of the relative error of the IPF estimated wind force for one area to the real wind force. During the first third of the series, the error computed with two aircraft is not very stable. When a new aircraft is entering into the zone, the estimation is better and more stable. The relative errors on the wind force stay about 2%, i-e less than 1 kt.

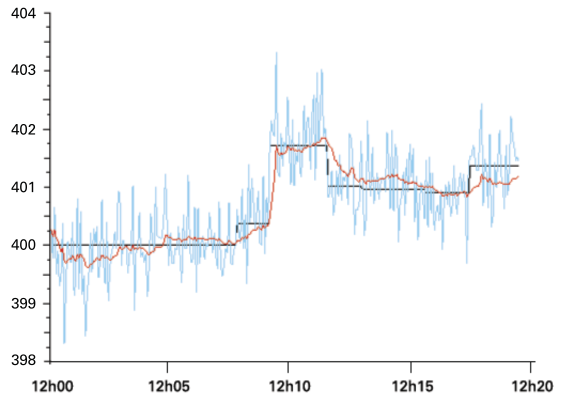

Fig. 6. Estimated aircraft true airspeed by IPF method. In spite of the strong perturbation on the TAS observations, the estimation of the TAS is quite efficient with an absolute error smaller than 0.5 kt.

error values correspond to the entry of the right aircraft in the left area. In the first one, the relative errors are quite unstable showing the learning phase with two aircrafts. While in the second period the relative error is very stable showing the end of the learning process with the three aircrafts in the same domain.

While the environment parameters are learned by the Island Particle system, the aircraft parameters are also estimated. In our experiment we only have to estimate the true airspeed of each aircraft about 400 kt. The airspeed estimation of one of these aircrafts is represented in Figure 6. On this graphic, the black line represents the true airspeed which needs to be estimated (knots), the blue line the mode-S radar observations and the red line the reconstructed signal by the IPF algorithm. Even if the perturbations of the TAS observation are strong, the estimation of the TAS is efficient picking out the Poissonian jump.

In this numerical experiment, we have shown the capability of our method to estimate the likelihood of an ensemble of Met forecasts while learning some aircraft parameters. For further experiments, we intend to work with multiple areas and more realistic meteorological forecasts.

Conclusion and Further Developments

In this study, we have developed a stochastic modeling of the aircraft trajectories in random atmopsheric conditions. The stochastic process is a Markov process in a random environment partially observed by radar. This process can be estimated by a special Particle Filter called Island Particle Filter. Each island corresponds to a weather prediction and is used to evaluate the likelihood of the Met forecasts.

Then the methodology developped in this work allows us to give a weight to each element set of the ensemble weather forecasts regarding to the traffic-observations. That is we can infer the random environment: learning the likelihood of wind proposals while learning some flight parameters such as true airspeed. The numerical experiment have shown the powerfull of our stochastic modelization developed and the capability of the Island Particle Filter.

The next mathematical step of this work is to relax the assumption of uniformity for the Met errors. We are working on this topic using errors which are uniform in probability law (it means that it is the probability distribution which is uniform and not the errors themselves) on sub-domains. Then we intend to deal with real weather ensemble forecasts such as the forecasts provided by European operational meteorological centres. The applicability of the preceding algorithm to real data makes no doubt as far as the algorithm does not have any restricting assumptions. However, this method needs an efficient programming code and parallel computing techniques in order to get real time operational applications.

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