On Runway Capacity and Wake Vortex Safe Separation Distance (SSD)

L.M.B.C. Campos
CCTAE (Center for Aeronautical and Space Science and Tecnology),
Instituto Superior Tecnico, Lisbon Technical University.
Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal.
luis.campos@istl.utl.pt

J.M.G. Marques
Universidade Lusófona de Humanidades e Tecnologias (ULHT),
Campo Grande, Lisboa, Portugal, and CCTAE.
jmgmarques@ulusofona.pt

***Abstract* — The capacity of ATM is limited by the runway availability at airports. The number of take-off and landings from the same runway is limited by wake vortex separation distances. Currently the safe separation distances (SSD) are established by empirical rules using as sole criterion the classification of the leading and following aircraft into three weight categories "light", "medium" and "heavy". The Boeing B757 has been the subject of an exception to the rules increasing the separation distance on the basis of flight incidents; the Airbus A380 has been subject of an extended separation distance for "super heavy" aircraft as a safety precaution. The purpose of the present paper is to obtain a formula for wake vortex separation distances that involves a number of assumptions and simplifications, but does include more parameters rather than just the weight of the leading and following aircraft. The predicted SSD for pairs of typical light (Citation 500), medium (B737-300) and heavy (B747-400) aircraft are comparable to the ICAO minimum separation rules, suggesting that the latter are moderately conservative on the safe side. The special case of B757-200 is considered justifying the increased separation. In the case of A380-100, the special "ultra-heavy" rules appear to be excessively conservative highlighting the need for a more scientific approach to the subject, especially in cases where a long experience from the past is not available.**

Keywords- capacity, separation, safety, wake hazard.

I. INTRODUCTION

The ICAO rules on minimum separation distances between aircraft at take-off and landing determine the capacity of runways, airports and the whole terminal area air traffic. These empirical rules have proved to be generally safe in a long past experience and should not be changed lightly. However the lack of a scientific basis begs some questions:

Are current separations rules too conservative, unnecessarily limiting capacity?

Is there any reliable way a smaller separation could be introduced without reducing safety?

Not too much can be expect from empirical rules using as sole criterion three weight categories for the leading and following aircraft. This begs another set of questions:

What other parameters besides weight affect the safe separation distance (SSD)?

Can we have some idea of whether current SSD are either (a) too conservative or (h) safe without excessive margin?

Two particular aircraft highlight the issues with current separation rules:

After several incidents the separation distance for the B757 has been increased beyond its "weight category";

On the A380, the first super-heavy aircraft, have been imposed much larger separation distance than for a "heavy" B747, apparently more as a safety precaution than on the basis of clear knowledge due to the lack of precedent.

The present paper does not claim to provide definitive answers to any of these questions, but rather to introduce a more scientific approach that can be compared with empirical separation rules. The method adopted derives an analytical formula for the SSD that shows more clearly the parameters influencing the result than the numerical approaches in the literature [1-7].

A simple analytical formula for the SSD between aircraft is obtained in three steps with some approximations. First, the wake of a aircraft and its evolution in the atmosphere, in the presence of wind, shear and ground effects is a complex fluid phenomenon [8]. Rather than use CFD with turbulence models [9], in order to arrive at a simple formula for the SSD, the wake vortex strength of the leading aircraft is assumed to decay [10] according to a diffusion equation with convection (step 1: section II). The worst case scenario is considered (step 2: section III) where the following aircraft flies straight into the axis of the vortex system of the leading aircraft, so that the induced rolling moment is maximum. It is assumed (step 3: section IV) that only a fraction f of the maximum roll control power of the following aircraft is used to counter the rolling moment induced by the wake of the leading aircraft. This leads to a formula for the SSD separation distance that has two solutions (section V), namely an unsafe close distance and a safer larger distance. The latter is found to agree broadly with the ICAO separation rules, that are found to be moderately conservative for typical aircraft pairs in the "light", "medium" and "heavy" categories. The special case of larger separation distance for the B757 is also explained (section VI), whereas in the case of the A380 as sole representative of the new superheavy class the extra separation distance relative to the “heavy” class appears excessive. In conclusion (section VII) a simple formula has been obtained for the SSD due to wake vortex effects, that takes into account more than just the weight of the leading and following aircraft, and may represent a small step forward in a scientific approach to the subject.

II. DECAY OF WAKE VORTEX STRENGTH WITH DISTANCE

The wake vortex system of the leading aircraft is represented in the simplest possible form by the vortex circulation strength [1] of the wing of the leading aircraft (index 1):

$$\Gamma_1 = \frac{c_{r_1} W_1}{\rho U_1 S_1} \tag{1}$$

where W_1 is the weight, c_{r_1} the root chord, ρ the air density, S_1 the wing area and U_1 the airspeed. Note that the wake vortex strength is proportional to the wing loading W_1/S_1 of the leading aircraft. The wake vortex decays in the atmosphere due to turbulent diffusion as it is convected by ambient flows. The simplest representation is an equation for the vorticity Ω :

$$\frac{\partial\Omega}{\partial t} + \frac{\partial(w\Omega)}{\partial z} - \eta\left(\frac{\partial^2\Omega}{\partial x^2} + \frac{\partial^2\Omega}{\partial y^2}\right) = \Gamma_1 \delta(y)\delta(z)\delta(x). \tag{2}$$

including [2] diffusion by a turbulent viscosity η and convection by downwash velocity w of the leading aircraft. The vorticity is generated by the wake vortex strength (1) of the leading aircraft, and t is time and x is the longitudinal coordinate in the direction of the flight path, and y, z transverse Cartesian coordinates.

The forced solution of (2) specifies the evolution of the vorticity as a function of time and position:

$$\Omega(t,x,y,z) = \frac{\Gamma_1}{2\pi\eta(t - z/w)} \exp\left\{-\frac{b^2 + z^2}{2\eta(t - z/w)}\right\}. \tag{3}$$

This formula is simplifies further assuming that: (i) the transverse coordinate is the core radius a_1 related to the span b_1 of the leading aircraft:

$$\left|y^2 + z^2\right|^{1/2} = a_1 = b_1 / 20\,; \tag{4}$$

(ii) the downwash velocity is small relative to the airspeed of the leading aircraft:

$$zU_1 / w \ll x = U_1 t\,. \tag{5}$$

Substituting (4), (5) and (1) in (3) specifies the vorticity as a function of position behind the leading aircraft:

$$\Omega(x) = \frac{c_{r_1} W_1}{2\pi\eta\rho S_1 x} \exp\left(-\frac{(b_1)^2 U_1}{800\eta x}\right). \tag{6}$$

The vorticity (Figure 1) initially increases in the wake of the leading aircraft, goes though a peak Ω_m at a distance x_m :

$$\Omega_m = \frac{W_1 c_{r1}}{\pi e \rho U (a_1)^2}, \quad x_m = \frac{(b_1)^2 U_1}{800\eta}, \tag{7a,b}$$

and then decays. This specifies the wake encountered by the following aircraft, and the rolling moment it induces (section III).

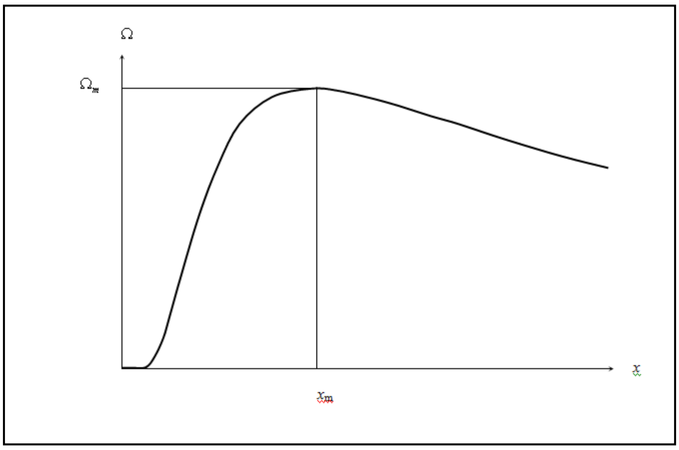

Figure 1. The vorticity in the wake of an aircraft initially increases with distance from the trailing edge of wing tip and ultimately decays due to vortex breakdown, merging and turbulent diffusion, implying that it has a maximum at intermediate distance.

III. EFFECT OF THE WAKE OF THE LEADING ON THE FOLLOWING AIRCRAFT

The worst case scenario is considered in which following aircraft flies straight into the axis of the vortex wake of the following aircraft thus being subject to a vertical velocity:

$$w_2(x,y) = (y - y_r)\Omega_r(x) - (y - y_\ell)\Omega_\ell(x), \tag{8}$$

where Ω_r, Ω_ℓ are the vorticities of the right and left wing tip vorticities of the leading aircraft, whose axis are respectively at positions y_r and y_ℓ. The vertical velocity (8) has a constant term that does not affect the rolling moment, and a term proportional to the spanwise coordinate

$$w_2(x,y) = y\Omega(x), \tag{9}$$

involving the difference of vorticities due to the right- and left-wing vortices:

$$\Omega(x) \equiv \Omega_r(x) - \Omega_\ell(x), \tag{10}$$

that is given by the wake (6) of the leading aircraft.

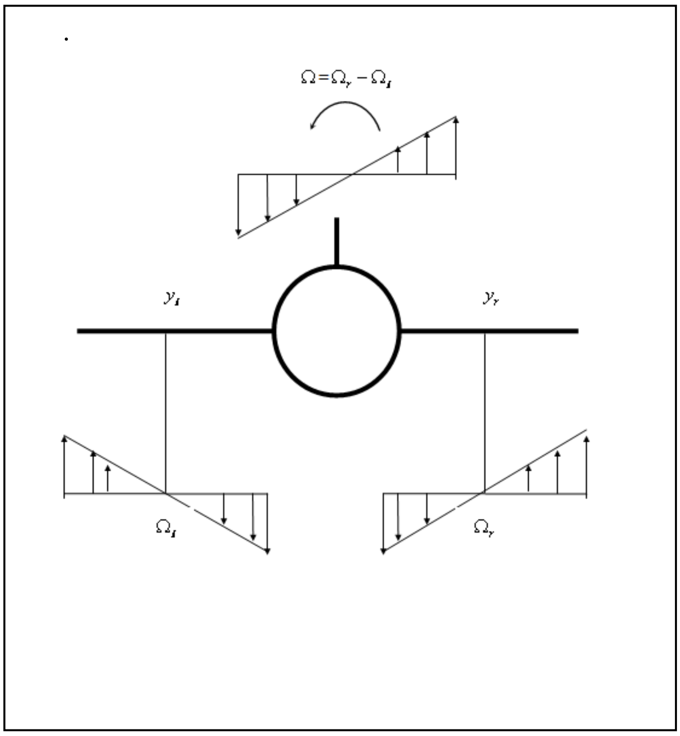

Figure 2. The worst case scenario of a wake vortex encounter is when the following aircraft flies straight into the axis of the vortex wake of the leading aircraft catching the two wing tip vortices. The difference of vorticities induces a vertical velocity upward on one wing and downward on the other wing causing to a rolling moment.

The lift on a section of the wing of the following aircraft is given [3] by:

$$\ell(y) = \pi\rho\, c_2(y) U_2 \left(U_2\alpha_2 + \Omega y\right) \tag{11}$$

where $c(y)$ is the chord, U_2 is the airspeed, α_2 the small angle-of-attack in the first term; the vorticity in the second term is positive $\Omega > 0$ increases lift on the port wing and decreases lift on the starboard wing seen from the front in the Figure 2 causing a rolling moment:

$$R_2 = \int_{-b/2}^{+b/2} y\, l(y)\, dy, \tag{12}$$

where b_2 is the span of the following aircraft. If the aircraft has a symmetrical wing (all current airlines do), only the second term of (11) contributes to the rolling moment induced on the following aircraft by the wake of the leading aircraft:

$$R_2(x) = \frac{\pi}{12} \rho h_2 U_2 \bar{c}_2 (b_2)^3 \Omega(x). \tag{13}$$

where $\bar{c}$ is the mean geometric chord and h a dimensionless parameter:

$$h = \frac{12}{\bar{c} b^3} \int_{-b/2}^{+b/2} y^2\, c(y)\, dy, \tag{14}$$

specified by the shape of the wing. It is unity $h = 1$ for rectangular wing $c(y) = \bar{c}$. The rolling moment on the following aircraft is proportional to: (i) the vorticity induced by the leading aircraft; (ii) the airspeed U_2 and wing area $S_2 = \bar{c}_2 b_2$ of the following aircraft; (iii) the span of the following aircraft appears to the square $(b_2)^2$ (or cube $(b_2)^3$ including S_2) because an aircraft with larger span catches more of the vortex further outboard contributing more to the rolling moment.

IV. INDUCED ROLLING MOMENT VERSUS CONTROL POWER

The rolling moment induced by the wake of the leading on the following aircraft must be compensated by the available control power of the following aircraft that is given by:

$$R_a = \frac{1}{2} C_{A\max}\, \rho (U_2)^2 S_a b_a, \tag{15}$$

where $C_{A\max}$ is the maximum lift coefficient of the ailerons, S_a their area and b_a the moment arm. The following aircraft should keep a safety margin by using only a fraction $0 < f < 1$ of the maximum roll control power to compensate the rolling moment induced by the leading aircraft:

$$R_a f_2 \geq R_2(x). \tag{16}$$

Substituting the induced (13) and control (15) rolling moments in (16) specifies

$$\Omega \leq (6/\pi)(f_2/h_2)(S_a b_a / S_2 b_2) C_{A\max} (U_2/b_2), \tag{17}$$

as the maximum wake vorticity of the leading aircraft that the following aircraft can cope with.

The maximum wake vorticity from the leading aircraft that the following aircraft can compensate (17) is: (i) proportional to the fraction of the maximum available roll control power that is used; (ii) proportional to the roll control effectiveness defined as the product of the ratios of the aileron to wing area and aileron moment arm to wing span:

$$\mu = b_a S_a / (b_2 S_2), \tag{18}$$

because the wing is affected by the wake vortex and the aileron counter this effect; (iii) proportional to the maximum lift coefficient of the ailerons, that determine the maximum available roll control power. Substituting the weight of the following aircraft:

$$W_2 = 0.5\rho C_L\left(\alpha_2\right) S_2 \left(U_2\right)^2 , \qquad (19)$$

in the maximum vorticity (17) that can be compensated:

$$\Omega \leq \frac{12}{\pi}\frac{f_2}{h_2}\frac{S_a b_a}{S_2 b_2}\frac{C_{A\max}}{C_L\left(\alpha_2\right)}\frac{W_2 / S_2}{\rho b_2 U_2} \equiv \Omega_2 , \qquad (20)$$

it follows that it is: (i) proportional to the ratio of aileron to wing lift coefficients, because the wing reacts to the wake vortex and the aileron opposes it; (ii) proportional to the wing loading, so that a higher wing loading opposes a stronger vorticity; (iv) inversely proportional to air density, airspeed and wing span, that is larger values lead to smaller vorticity that can be countered.

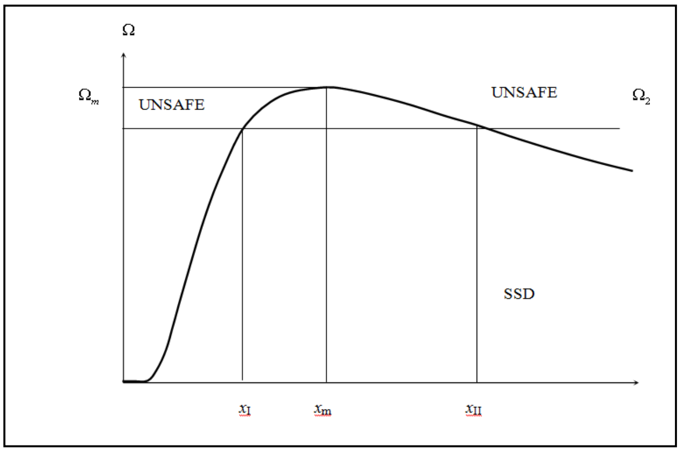

Figure 3. The following aircraft can compensate with aileron deflection the rolling moment induced by the wake of the leading aircraft (Figure 2) at two positions with equal vorticity (Figure 1). The closest position is unsafe because increasing the separation leads to a larger rolling moment. The farthest position is the safe separation distance such that as increase in separation reduces the rolling moment.

V. FORMULA FOR THE SAFE SEPARATION DISTANCE

The wake vorticity (6) depends on the distance behind the leading aircraft (Figure 1). For a given following aircraft (Figure 2) the induced wake rolling moment (13) and the roll control moment (15) balance when (20) is a equality:

$$\frac{c_{r_1} W_1}{\eta S_1 x}\exp\left[-\frac{\left(b_1\right)^2 U}{200\eta x}\right] = \frac{24}{\pi}\frac{f_2}{h_2}\frac{S_a b_a}{S_2 b_2}\frac{C_{A\max}}{C_L\left(\alpha_2\right)}\frac{W_2 / S_2}{b_2 U_2} ; \qquad (21)$$

Equation (21) has two roots (Figure 3). The first root corresponds to the following aircraft close to the leading aircraft in the region of increasing vorticity before the peak x_I. This is an unsafe position because if the following aircraft loses power or falls behind for any reason $x > x_I$ the vorticity increases beyond the available roll control power; in the unsafe forward position the following aircraft has only one option: keep distance or come closer to the leading aircraft. The second root of (21) corresponds to the region of decaying vorticity after the peak vorticity $x_{II} > x_m$ that is safe: if the wake vorticity of the leading aircraft is too strong the following aircraft should increase the separation distance until the available roll control power is sufficient.

If the second root of (21) satisfies (22a)

$$\left(b_1\right)^2 U \gg 200\eta x_{II} :$$

$$x_{II} = \frac{\pi}{24}\frac{h_2}{f_2}\frac{S_2 b_2}{S_a b_a}\frac{W_1 / S_1}{W_2 / S_2}\frac{C_L\left(\alpha_2\right)}{C_{A\max}}\frac{c_{r_1} b_2 U_2}{\eta S_1} , \qquad (22a,b)$$

the exponential in (21) is approximately unity and the safe separation distance (SSD) is given by (22b). The formula (22b) for the SSD shows that it increases if: (i) the leading aircraft has a higher wing loading, because it causes a stronger vortex wake; (ii) the following aircraft has a lower wing loading because it is more susceptible to the wake of the leading aircraft; (iii) for smaller aileron area and moment arm that reduces the roll control power; (iv) larger wing area of the following aircraft that catches more of the wake vortex; (v) larger wing span of the following aircraft to the square because it catches more of wake vortex further outboard; (vi) larger root chord of the leading aircraft that increases the wake vortex strength; (vii) larger airspeed of the following aircraft that increases the rolling moment induced by the wake of the leading aircraft; (viii) larger ratio of wing to the aileron lift coefficient of the following aircraft; (ix) smaller fraction of the maximum available roll control power used; (x) smaller turbulent diffusivity implying that the wake vortex decays more slowly with time and distance; (xi) larger wing geometry parameter (14), e.g. an unswept wing $h_2 = 1$ is more affected than a delta wing $h_2 = 1/2$ because it has more outboard area affected by the wake vortex.

TABLE I.
AIRCRAFT WEIGHT CLASSIFICATION

ICAO classification		Example chosen	
Designation	Weight – W	Aircraft	Maximum Landing weight W (kg)
Heavy (h)	W > 136 t	Boeing 747-400	W =396 893
Medium (m)	7 t < W < 136 t	Boeing 737-700	W =70 080
Light (l)	W< 7 t	Cessna Citation 525	W =5 375

TABLE II.
AIRCRAFT DATA

	Index	h-heavy	m	l	s	v
Symbol	Unit	Boeing 747-400	Boeing 737-300	Cessna Citation 500	Boeing 757-200	Airbus 380-100
W	kg	260360	58060	4400	89810	381000
S	m^2	541.16	125.00	22.30	185.25	920*
b	m	64.44	34.31	14.26	38.05	79.80
c_t	m	1.50*	1.00*	0.80*	1.47*	3.87*
S_a	m^2	20.90	2.00*	0.30*	4.46	40.00*
b_a	m	23.00*	11.00*	5.00*	11.00*	34.00*
V_s	$m\ s^{-1}$	60.7	51.5	42.2	54.3	53.8*
$\overline{c}$	m	8.40	3.64	1.56	4.87	11.53
c_r	m	15.30	6.28	2.33	8.27	19.16
h	-	0.615	0.596	0.756	0.651	0.668
Γ_0	$m^2\ s^{-1}$	707	330	63.5	430	859
w	$m\ s^{-1}$	1.12	0.982	0.454	1.15	0.771
a	m	3.22	1.72	0.71	1.90	3.99
U	$m\ s^{-1}$	78.9	66.9	54.9	70.6	70.0*
W/S	$kg.m^{-2}$	481	464	197	484	414
W/(Sb)	$kg.m^{-3}$	7.46	13.54	13.84	12.74	5.19
bS/b_aS_a	-	72.5	195	212	144	54
Ω_m	$m^3\ s^{-1}$	409	99	139.5	1280	557
x_m	m	426	10300	14.5	133	580
x_m / b	-	6.60	3.00	1.02	3.49	7.27

* - estimated

VI. COMPARISON OF THE THEORETICAL SSD WITH EMPIRICAL SEPARATION RULES

The formula (22b) for the safe separation distance is applied to three aircraft typical (Table I) of the "light", "medium" and "heavy" categories, respectively the Cessna Citation 500, the Boeing 737-300 and Boeing 747-400. The data used for these aircraft appears in the Table II. The formula (22b) for the SSD involves an unknown parameter namely the diffusivity of the atmosphere η. Its value was determined $\eta = 0.96\,\mathrm{m}^2\mathrm{s}^{-1}$ by fitting the SSD of two heavy aircraft to the ICAO value $x_{hh} = 4\,\mathrm{nm}$, that is therefore exact in the Table III. The faction of the maximum available roll control power used

$$\overline{f} = f\,\frac{C_{A\max}}{C_L(\alpha_2)}, \tag{23}$$

is indicated at the bottom of the Table III. There are no other free parameters in the comparison of the theoretical SSD with the ICAO separation rules in the Table III. The calculated SSD is always less than the ICAO rule by a small amount, suggesting that the minimum separation distances used are safe without being too conservative. Only in the case of medium leading and following aircraft is the ICAO separation slightly less than the prediction.

This leads to the case of the B757 that is treated as "special" because it has larger separation distance than other "medium" aircraft (Table IV). The calculated SSD for the special aircraft is again lower and close to the FAA separation rules, suggesting that they are safe without being too conservative. The reason why the "special" B757 aircraft has stronger wake effects than other "medium" aircraft is explained by the formula (22b) for the SSD taking into account the data for the Boeing 757-200 in the Table II: (i) the special aircraft has a roll control authority (18) intermediate between that of the much larger aircraft and that of a medium aircraft; (ii) the volume loading defined as the ration of the weight to the product of wing area by span:

$$\vartheta \equiv \frac{W}{bS} = \frac{W}{b^2\overline{c}}, \tag{24}$$

for the special aircraft is comparable to that of medium and light aircraft, and much higher than for the heavy aircraft; (iii) the special aircraft has an approach speed intermediate between those of the medium and heavy aircraft; (iv) the downwash velocity of the special aircraft is larger than for the medium aircraft, and exceeds slightly that of the heavy aircraft, which indicates a strong wake. Although none of the effects (i) to (iii) is dramatic in isolation, they all add in the same direction, of giving the special aircraft a stronger wake than its weight category would suggest as confirmed by its downwash velocity higher than all the others. This is also shown by a peak vorticity $x_m = 133\,m$ behind the aircraft, which in terms of wing spans $x_m / b_1 = 3.49$ is intermediate between medium and heavy aircraft, in Table II.

TABLE III.
SAFE SEPARARTION DISTANCES IN NAUTICAL MILES ICAO RULES (CALCULATED VALUES IN BRACKETS)

Leading a/c a/c Following	h Heavy (B 747-400)	m Medium (B 737-300)	l Light (Citation 500)
h Heavy (B 747-400)	$\overline{x}_h \equiv \overline{x}_{hh} = 4$ ($x_h \equiv x_{hh} = 4$)	$\overline{x}_{mh} = 3$ ($x_{mh} = 2.66$)	$\overline{x}_{\ell h} = 3$ ($x_{\ell h} = 2.08$)
m Medium (B 737-200)	$\overline{x}_{hm} = 5$ ($x_{hm} = 4.87$)	$\overline{x}_m \equiv \overline{x}_{mm} = 3$ ($x_m \equiv x_{mm} = 3.24$)	$\overline{x}_{\ell m} = 3$ ($x_{\ell m} = 2.54$)
l (Citation 500)	$\overline{x}_{h\ell} = 6$ ($x_{h\ell} = 5.40$)	$\overline{x}_{m\ell} = 4$ ($x_{m\ell} = 3.55$)	$\overline{x}_\ell \equiv \overline{x}_{\ell\ell} = 3$ ($x_\ell \equiv x_{\ell\ell} = 2.81$)
$\overline{f}$	0.5	0.3	0.06

TABLE IV.
CALCULATED (22B) SEPARATION DISTANCES

Following aircraft	Leading aircraft	
	Special B757-200	Ultra-heavy A380-100
Identical	$x_{ss} = 4.00 = \overline{x}_{ss} = 4.00$	$x_{vv} = 2.47 < \overline{x}_{hh} = 4.00$
Heavy (> 115t)	$x_{sh} = 3.63 < \overline{x}_{sh} = 4.00$	$x_{vh} = 3.13 < \overline{x}_{hh} = 4.00$
Large (18.6 t< W< 115t)	$x_{sm} = 4.44 < \overline{x}_{sm} = 5.00$	$x_{vm} = 4.92 < \overline{x}_{hm} = 5.00$
Light (<18.6t)	$x_{s\ell} = 4.91 < \overline{x}_{s\ell} = 6.00$	$x_{v\ell} = 4.49 < \overline{x}_{h\ell} = 6.00$
$\overline{f}$	0.3	0.5

For the ultra heavy aircraft A380-100 the peak vorticity is even farther $x_m = 580m$ or $x_m / b_1 = 7.27$ wing spans than for the special and heavy aircraft; compared with the heavy aircraft, the ultra-heavy has a peak vorticity at a larger absolute distance, and larger relative distance in terms of wing spans. The same Table II suggests that the very large aircraft might not have special wake separation problems, starting with a roll control power higher than that of the heavy aircraft; the volume loading is lower than for a heavy aircraft and much lower than the special aircraft. In addition, the downwash velocity is much lower for the ultra-heavy than for the heavy and special aircraft; in fact the downwash velocity of the ultra-heavy is intermediate between the medium and light aircraft, which is a remarkable result, considering the size difference. Thus the ultra-heavy aircraft may require a separation distance not much larger than the heavy aircraft (Table IV). The ICAO and FAA rules for the minimum separation distance take into account only the weight of the leading and following aircraft. The formula (22b) for the safe separation distance shows that a more relevant parameter is the ratio of wing loadings:

$$\nu = \frac{W_1 / S_1}{W_2 / S_2} . \tag{25}$$

The wing loading of the ultra-heavy aircraft (Table II) is lower than that of the heavy aircraft, that is comparable with the special aircraft. The formula (22b) for the SSD also involves other parameters like the roll control effectiveness that can vary significantly between aircraft.

VII. CONCLUSION

In the cases of the “light”, “medium”, “heavy” and “special” aircraft for which there is a very substantial experience the theoretical formula for the SSD gives results broadly in agreement with the empirical minimum separation rules, suggesting that the latter are safe without being too conservative, or unnecessarily limiting capacity at airports. In contrast the “ultraheavy” aircraft may be penalized with excessive separation distances as a safety precaution against an unprecedented case without the benefit of long experience. The analytical formula does provide useful indications of what parameters have the greatest effect on the SSD. The objective of deriving an analytical formula for the SSD has implied a number of simplifications; the final result also involves two parameters namely the diffusivity that depends on atmospheric conditions and the fraction of maximum roll control power used that is a pilot choice. The evolution of a vortex wake does depend on atmospheric and ground conditions, and thus empirical of theoretical separations are only a guide. The methods used to calculate the SSD can be extended [11] to the response of an aircraft to a wake vortex encounter, including the effect of controls and aerodynamic damping [12].

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