# An Air Traffic Control Model Based Local Optimization over the Airways Network

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Abstract-The introduction of a new SESAR scenario in the European Airspace will impact the functioning and the performances of the current Air Traffic Management (ATM) System. The understanding of the features and the limits of the current system could be crucial in order to improve and design the structure of the future ATM. In this paper we present some results of the "Assessment of Critical Delay Patterns and Avalanche Dynamics" PhD project from the ComplexWorld Network. During this project we developed a model of Air Traffic Control (ATC) based on Complex Network theory capable of reproducing the features of the real ATC in three European National Airspaces. We then developed an optimization algorithm based on "Extremal Optimization" in order to build efficient and globally optimized planned trajectories. The ATC model is applied in order to study the efficiency of this new planned trajectories when subject to external perturbations and to compare them to the current situation.

#### I. INTRODUCTION

In the forthcoming year the Air Traffic Management (ATM) system is expected to manage an increase of traffic load. Despite the fact that in Europe this increase will be due mainly to the economic growth of extra- European countries, it is possible that the actual system will not be able to function efficiently, i.e. it could not be able to provide safety standards and performances to every aircraft crossing its airspace. For this reason a new possible future scenario has been developed within the SESAR Project [1]. In this future SESAR scenario each aircraft will be allowed to fly in a less structured airspace, following trajectories that meet their business needs in terms of airspace costs and fuel consumption. Moreover such future scenario should be more "safe", meaning that the system will be expected to be more resilient to adverse occurrences and easier to manage for the Air Traffic Control (ATC). In this paper we present some of the results of the "Assessment of Critical Delay Patterns and Avalanche Dynamics" PhD project from the ComplexWorld Network [2] within the WpE of SESAR . The aim of this project was to analyze and model the current ATM system in the framework of Complex Systems Theory [3], in order to gain a better understanding on the capacity limits of the system and of the possible future scenario. Part of the project has been devoted to the development of an Air Traffic Control Model using a datadriven approach in which the activity of the controllers has been modeled as a local optimization process on a network of navigation points. In the first part we present some major details of the model and its application to simulations within the Italian Airspace. We then proceed to introduce a stochastic optimization algorithm called "Extremal Optimization" [6] and apply it to the optimization of aircraft trajectories during the planning process. We will then proceed to use the ATC model in order to test the efficiency of the new globally optimized airspace and compare it to the current one. The paper is structured as follows: in section II we present the ATC model applied and its capacity limits when applied to a realistic airspace built using historical data. The validation of the model using historical data is also briefly described; in section III we introduce the Extremal Optimization Algorithm applied to the trajectory optimization problem and then we proceed testing its resilience to disturbances such as adverse conditions and departure delays; in section IV we will draw our conclusions.

#### II. THE MODEL

The agents of our model are the air traffic controllers whose duty is to provide separation for the aircraft that are crossing the sector under their responsibility. Each aircraft is supposed to fly according to a certain flight plan which is a sequence of navigation points that it has to cross from its origin to its destination. Whenever a controller spots a conflict, his duty is to apply the necessary rerouting and deviations in order to prevent the conflict from happening. The flight plan of an aircraft will be then a certain path  $\{n_1, \ldots, n_l\}$  over a network made of navigation points that represents the airways structure. For sake of simplicity, we assume that every aircraft flies at a constant speed of 800 km/h that is the average speed of a scheduled flight. Moreover at this stage we assume that the system is bi-dimensional, i.e. that every aircraft is flying at the same flight level and that vertical deviations are not used for conflict resolution. Note that both these two hypotheses can be relaxed in order to improve the realism of the model. Every time an aircraft is crossing one of the navigation points in its flight plan, the controller in its current sector looks for conflicts in the segment connecting its next node. Considering a couple of aircraft flying M and N on two different segments, say  $(n_1, n_2)$  and  $(m_1, m_2)$  respectively, we indicate with  $t_n$ 



and  $t_m$  the times at which they will cross the  $m_2$  and  $n_2$ . The conflict between these two aircraft is computed geometrically, requesting that at every point on the segment over which they are flying their relative distance is always larger than 5 NM (assuming that every aircraft has the same speed this spatial threshold can be translated into a temporal one,  $\delta t = 40$  seconds). There are 4 possible situations, corresponding to different patterns of the segments over which the aircraft are flying:

•  $n_2 = m_2$ , i.e. the aircraft are flying towards the same node (Fig. 1 panel a). Assuming that  $t_m > t_n$  the condition for the occurrence of a conflict is

$$|t_m - t_n| < max\left(\frac{\sqrt{2}\ \overline{\delta t}}{\sqrt{1 + \cos\alpha}}, \overline{\delta t}\right),$$
 (1)

where  $\alpha$  is the angle between  $(m_1, m_2)$  and the segment over which the aircraft N is supposed to fly after having crossed  $n_2$ .

•  $n_1 = m_2$  (as well as  $n_2 = m_1$ ), i.e. an aircraft is flying towards the last navpoint that the other aircraft has crossed (Fig. 1 panel b). In this case the condition for the occurrence of a conflict is

$$|t_m + \bar{t}_n| < max\left(\frac{\sqrt{2}\ \overline{\delta t}}{\sqrt{1 - \cos\beta}}, \delta t\right),\tag{2}$$

where  $\bar{t}_n$  is the time at which the aircraft N has crossed  $n_1$  and  $\beta$  is the angle between  $(m_1, m_2)$  and  $(n_1, n_2)$ .

- The segments  $(m_1, m_2)$  and  $(n_1, n_2)$  intersects each other (Fig. 1 panel c). In this case the previous conditions (1) and (2) still hold considering the intersection as a node and interpolating the corresponding crossing times.
- The segments  $(m_1, m_2)$  and  $(n_1, n_2)$  do not intersect each other (Fig. 1 panel d). Also in this case (1) and (2), but now the intersection of the extensions of the segments has to be considered as a node and the crossing times has to be interpolated as the two aircraft fly on a straight line. Note that even though all the conditions of conflict occurrence are satisfied it has to be checked that the conflict occur when both of the aircraft are on the segments and not when one of them is on the extensions.

Equations (1) and (2) can be derived considering the instantaneous relative distance  $d_{mn}(t)$  depending just on the crossing time and the angle between the considered segments and solve the inequality  $d_{mn}(t) < \overline{\delta t}$ . Moreover if the inequality has a certain solution it must be required that such solution overlaps with the time ranges at which the aircraft are on the considered segments. Whenever a conflict has been spotted the controllers must look for possible redirections to apply to the aircraft and thus sending it from  $n_1$  to another  $n_r \neq n_2$ . This process has been modeled as a local search between the nodes of the network so that the controller looks for all the possible nodes toward which redirect the aircraft within a certain subset of nodes of the network and then choses the one that minimize a certain cost function. The two basic strategies that a controller can apply are the redirection within its sector (IN strategy,



Fig. 1: Geometries used to spot the conflict of a couple of aircraft.

fig. 2 panel a) and into a nearby one (OUT strategy, fig. 2 panel a). In both case the controller is applying directs to solve conflicts, trying to speed up the traffic. Note that to do so, the controller can disregard the airways structure and create a new link connecting the nodes of the network. It is also possible a third strategy which corresponds to the "vectoring" of an aircraft (figure 2 panel c), that corresponds to the generation of new navigation points in the sector to be used for rerouting. Once a certain set  $\{n_r\}_{r=1...k}$  of navigation points has been found, the controller choses the one that minimize a cost function representing the efficiency of the flight. In the simple bi-dimensional case this cost function is just the temporal distance to the arrival:

$$C_0(n_1, n_r, s) = d_{n_1, n_r} + d_{n_r, s}^{sp},$$
(3)

where  $d_{n_1,n_r}$  is the distance between  $n_1$  and  $n_r$  and  $d_{n_r,s}^{sp}$  is the length of the shortest-path on the navigation point network between  $n_r$  and the arrival node of the aircraft s.

In the following we will consider mainly an IN-OUT protocol for conflict resolution without vectorization, meaning that a controller will try at first to solve the conflict using a direct towards a node inside its sector and, if that fails, will try a redirection towards a nearby sector. Note that if both these strategies fail, no redirection is applied and the conflict simply occur.

## A. Model On Realistic Airspace

Using data coming from the Demand Data Repository (DDR) and gathered together into the ELSA Project ([4], [5]) it is possible to gain information on both the flight plans and the actual trajectories of the aircraft within the European Airspace and in a certain time frame (from the  $9^{th}$  to the  $21^{st}$  of June 2011). Here we used the flights in 3 different National Airspaces (Italian, Greek and Estonian) in a week





Fig. 2: Strategies of conflict resolution: (a) IN strategy, (b) OUT strategy, (c) Vectoring strategy. Areas of different colors are different sectors. Star-shaped points in (c) are geographical references that are not part of the navigation points network.

in June 2011 to reconstruct the structure of the airways using their flight plans. Assuming that the planned trajectories follow the structure of the airways, the navigation point network of an airspace can be built using all the navpoint crossed by the flight plans in the dataset and then linking a couple of navpoint if there is at least an aircraft that has flown from one to another in the considered time frame. Counting the number of aircraft that have traveled over every link it is also possible to weight the links of the network and thus to study the deployment of the traffic over the airspace.

Moreover information regarding the sectoral structure of each airspace is present, so that it is possible to define a static and simplified structure to be used in the simulations. The features of the used airspaces are presented in table I.

For each airspace, we can build a simulation in order to understand the limits of the model, i.e. what happens when the density of aircraft within an airspace is too high and the optimization algorithm starts to fail. Thus we choose a certain number of aircraft, say  $N_{aircraft}$ , and we randomly assign to each one of them a flight plan from the dataset. Since with this procedure the most frequent flight plans are the most likely to be chosen, we are sure that the most trafficked routes in the real system will also be the most trafficked in the simulation. Then we assign to each flight a departure time from a uniform distribution in [0 hours, 2 hours]. In order to prevent the generation of conflicts around the airports due to close departure times of different aircraft, no conflict can occur within a radius of 21 NM around the airports.

As often happens when dealing with physical systems, the finite size of a system can highly influence the results of a numerical or analytical computation. This is usually due to the effects of the "border" of the system that are known as "boundary effects". In our case these effects can lead to a



Fig. 3: Sectors structure of the Greek Airspace. Blue areas are sectors within the airspace, while red areas are the external sectors used to compensate boundary effects.

spurious generation of conflicts since an aircraft flying close to the border will have less possibilities for rerouting than an aircraft in the inside of the airspace. The problem of the boundary conditions is solved here considering also the external part of each airspace, so that aircraft close to the border can be rerouted outside assuming that conflicts can only occur inside the airspace. Although, to prevent all the aircraft to escape in this "safe" region, redirections outside the airspace are allowed only if the next sector that the aircraft is about to exit the airspace, that is if its next sector according to its flight plan lies in another national airspace. Figure 3 shows the sectors within the Greek airspace as well as all the exterior sectors considered during the simulations. Note that

Name	N	Nairports	N <sub>sectors</sub>	Surface $(NM^2)$
Estonia	51	4	3	285540
Grece	371	45	9	852729
Italy	725	43	18	1299055

TABLE I: Number of nodes, airports, sectors and surface area of the national arispaces used in the simulations.

this problem can be also solved using "periodic airspaces", but this possibility will not be considered in this work.

For every value of N, we simulate various realizations corresponding to different initial conditions. Since with the same value of N but different sizes of the airspace the density of aircraft flying in the system may vary,  $N_{aircraft}$  it is not a good parameter itself to compare the results of the simulation as the size of the system grows. Although the average number of flying aircraft  $\overline{N_f(t)}$  per sector grows linearly with  $N_{aircraft}$ in every case and thus it can be used as the free parameter of the model.

For each size of the system, we find a transition from a phase in which the optimization process solve all the conflict to a phase in which many are not. This can be seen looking at the average number of unresolved conflicts ( $\overline{N_{conflicts}}$ ) as a function of  $\overline{N_f(t)}$  in figure 4. For small values of the free parameter we have  $\overline{N_{conflicts}} = 0$ , while above a certain threshold depending on the size of the system  $\overline{N_{conflicts}} = 0$ grows with  $\overline{N_f(t)}$  in power-law fashion with exponent ~ 4.5. The transition presents also a scaling property with the system





Fig. 4:  $\overline{N_{conflicts}}$  as a function of  $\overline{N_f(t)}$  for the Estonian, Greek and Italian Airspaces. (Inset)  $\overline{N_{conflicts}}$  as a function of  $\overline{N_f(t)}$  rescaled with  $N^{0.43}$  for the Estonian, Greek and Italian Airspaces.

size. Assuming that the size is represented by the number of navigation points N within the airspace, all the shown curves collapse into one if  $\overline{N_f(t)}$  is rescaled with  $N^a$  with  $a \simeq 0.43$  (figure 4 inset). This transition represent the validity limit of the model and the boundary in which it performs "safely". Every simulation of realistic situation must take into account the possibility of conflict generation, so that it must be assured that the density of aircraft must always lie below the transition point.

## B. Model Validation

The validation of the model can be performed using the historical data used to build the previous simulations. As well as we can use the flight plans to build the structure of the airways and study the deployment of the traffic over them, we can use the real trajectories to build a similar network. Such network will be similar to the previous one with a different pattern of traffic and a different topology. All these changes are the results of the activity of the air traffic control and its management activity over the planned routes. Thus one part of the validation has been performed checking if the model is able to reproduce such topological changes in the structure of the airspace, performing simulation of one day schedules in the previously introduced airspaces. The metrics on which we will focus our attention are well-known in the field of Complex Networks [7], [8]:

- Degree k of a node: the number of other nodes that are linked to the considered one.
- Strength *s* of a node: the sum of the weights of all the links connected to the considered node. In our case this metric represents the traffic load of a navigation point.
- Betweenness Centrality *b* of a node: this metric, introduce initially for social networks [9], measures the "importance" of a node in the network, i.e. how likely is to cross it if one wants to travel from a node to another following the shortest-paths of the networks.

Together with this "macroscopic" validation, it is also possible to compare the distribution of the changes in length, crossed navigation points and en-route delay of each flight in order to see if the ones generated by our model are compatible with those found in the data.

The setup for the validation activity is slightly different with respect to that presented when we studied the capacity limits of the model. In this case the protocol used by the controllers is:

- IN-OUT protocol for conflict resolution.
- The flight levels are taken into account and each aircraft is supposed to fly at the required flight level indicated in the dataset. We will consider just the en-route phase, so the ascend and descend part will not be simulated.
- Small flight level changes of  $\pm 10$  FL from the required flight level are allowed in order to solve a conflict.
- Capacity constraints are enforced: each sector has a maximum number C of aircraft per hour that can manage. Every redirection that generates a path violating this constraint in one sector is not allowed.
- Controllers can assign directs, sending an aircraft to its next sector with a probability  $p_{direct}$  that is checked just before the conflict resolution procedure presented before. If the direct is assigned, the controller tries to send the aircraft in a nearby sector using the OUT strategy, but the redirection is applied only if it shortens the length of the flight.

The capacities of each sector are measured from the dataset as the maximum number of aircraft that has traveled over the sector in an hour. The probability  $p_{direct}$  of a sector has been measured counting the fraction of aircraft that has been rerouted from a node in the sector to a node in another sector with a segment long at least 80 NM.

We simulated the schedule of flights on the  $9^{th}$  of June 2011 in three distinct airspaces, the Italian, the Greek and the Estonian. Table II shows the correlations coefficients between the variations of the networks metrics for the navigation point network measured using the real trajectories and the output trajectories of the simulations. These variations are computed in both cases with respect to the corresponding navigation point network built with planned trajectories. The positive values of these coefficients indicate that the simulation is able to reproduce the variations correctly, in particular the correlations between the variations of strength  $\delta s$  indicates that the modifications in the deployment of the traffic performed by the ATC are similar to those obtained with the simulations, while  $\delta d$  and  $\delta b$  are respectively the variations in degree and betweenness. Concerning metrics regarding the variations over the single trajectory, the distribution of the variation in number of crossed way points and in the traveled distance during the flight are reproduced correctly (Fig. 5 panel a and b). On the other hand the distribution of the en-route delays are not coincident since many small positive delays are not reproduced (Fig. 5 panel c). To increase the agreement with the dataset, we introduce disturbances in the system which represents every kind of occurrence that can happened during the flight (e.g. bad weather conditions). These disturbances are



Nation	Estonia	Greece	Italy
$n_{links}$ (data)	90	1022	3056
$n_{links}$ (simulation)	90	1284	3160
$\delta d$ corr. coeff.	0.55	0.61	0.60
$\delta s$ corr. coeff.	0.61	0.82	0.81
$\delta b$ corr. coeff.	0.70	0.85	0.87

TABLE II: Comparison between the real navigation points networks built with the data and with the trajectories generated by simulations for each national airspace used in the validation. The variations of each metrics are computed with respect to the navigation point network built with the planned trajectories used for the validation.

modeled as delay generating perturbed areas with a radius  $r_{dist}$  randomly chose from a uniform distribution in [1 NM, 15 NM]. If an aircraft crosses one of this areas it gains a random penalty delay from a uniform distribution in a range between 1 min. and 0.14  $NM^{-1}$  min  $r_{dist}$  (0.14  $NM^{-1}$  min.range between 1 min. and 0.14  $NM^{-1}$  min  $r_{dist}$  (0.14  $NM^{-1}$  min. The position of these areas are completely random in the airspace and their distribution is reassigned every 60 minutes of simulation time. As we vary the number of disturbances  $n_{ext}$  (the only free-parameter of the model), the agreement between the simulated and real en-route delays distribution starts to improve until  $n_{ext} \approx 200$ . Above this value the generated delays is too much and the agreement is completely destroyed (Fig. 5 panel d and e).

## III. GLOBAL OPTIMIZATION

In the future SESAR scenario the structure of the airspace will be particularly different from the current one, not just because of different regulations, but also because a more flexible planning of the trajectories of the flights will surely result in a new airways topology. With this perspective it is reasonable to ask if the new structure will effectively be more efficient than the current one, in terms of performances for the flights and in manageability for the controllers. Since our model is able to reproduce the action of the ATC in a normal situation, one of its possible applications is the study of new configurations and its response to the typical disturbances that can occur during the operations.

Since we do not have information about possible future scenarios, we built a new solution for the planned trajectories in which besides the usual capacity constraints of the sectors also the conflicts between the trajectories are taken into account. In other words, if the controllers act as local optimizer, this solution is a globally optimized one computed with a stochastic optimization algorithm well-known in complex systems physics.

#### A. Extremal Optimization Algorithm

In complex systems physics a lot of optimization algorithm have been developed in order to solve heuristically a wide variety of problems. Despite the fact that many of these algorithms have been developed to solve specific problems, they can be applied to situations far from the framework of



Fig. 5: (a) Distribution of the variation in length ( $\delta l$ ) of the real (black) and simulated (red) trajectories. (b) Distribution of the variation of the number of crossed navigation points ( $\delta n$ ) of the real (black) and simulated (red) trajectories. (c,d,e) Distribution of the en-route delays ( $\delta t_{enr}$ ) of the the real (black) and simulated (red) trajectories with 0, 200 and 2000 external disturbances.

physics. Some notable examples are the "simulated annealing"



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techniques [10] and the "genetic algorithms" [11]. In relatively recent times a new algorithm has been proposed, the "Extremal Optimization" (EO) algorithm, based on the avalanche phenomenon present in systems with "Self-Organized Criticality" (SOC). One of the most notable example of systems with SOC is the Bak-Sneppen model [12], which modeled the evolution of interrelated animal species. At each time step the species with the smallest fitness is updated and its fitness is randomly reassigned (representing the death of such species and its substitution with a new one). This change in fitness has an impact also on the neighbors, so that their fitness is also randomly reassigned. The system rapidly reaches a state called SOC, in which all the fitnesses are above a certain value and avalanches take place. These are chain reactions leading to large fluctuations that make every possible configuration of the system virtually accessible. Extremal optimization is based on the same principles of the Bak-Sneppen model. The optimization is performed updating at each step the element of the system with the lowest "fitness" (or cost in this case). The generation of fluctuations through avalanches in this optimization process, make the system visit much of its accessible configuration preventing it from being trapped in a local minimum (or maximum) of its cost function. Considering a certain set of variables  $x_i$  each one with an assigned fitness  $\gamma_i$  so that each fitness contributes linearly to the cost function  $C(\gamma)$  defined as

$$C(\gamma) = \sum_{i} \gamma_i. \tag{4}$$

Indicating with S a generic configuration of the variables  $x_i$ ,  $\Omega(S)$  will be the set of neighbor configurations of S, i.e. a set of configurations that are close and accessible from the configuration S. Note that the definition of  $\Omega(S)$  is completely arbitrary. The algorithm proceeds as follows:

- Choose a starting configuration and set  $S_{best} := S$
- For the configuration S:
  - Evaluate the fitness  $\gamma_i$  of each variable
  - Find the variable with the highest fitness (indicated by the letter j)
  - Choose a new configuration  $S' \in \Omega(S)$  so that the variable j changes its value
  - Accept S := S' unconditionally
  - If  $C(S_{best}) \ge C(S)$ , set  $S_{best} = S$ .
- Repeat the previous point as long as desired.
- At the end the sub-optimal configuration will be  $S_{best}$  and the sub-optimal cost will be  $C(S_{best})$ .

## B. Application to Trajectory Optimization

In our case the variables to be optimized are the trajectories of the aircraft. In order to simplify a bit the problem we will disregard the departure and arrival part of the flight and will focus on the en-route phase, meaning that the within a certain radius to the airports ( $\approx 20$  NM) all the conflicts between the aircraft are disregarded. We will also assume that each aircraft fly at its required flight level and that height changes are not possible in the optimization process. Moreover we can assume that the optimization has to be performed using fixed geographical references, that we assume to be the same navigation points existing today. Thus each variable  $x_i$ , corresponding to the trajectory of the  $i^{th}$  aircraft is

$$x_i = \{ (n_{start}, t_{start}), (n_1, t_1), \dots, (n_{stop}, t_{stop}) \}, \quad (5)$$

where  $n_k$  is the  $k^{th}$  crossed navigation point and  $t_k$  is the corresponding crossing time. Note that  $t_{start}$  is fixed, while  $t_{stop}$  depends on the path. As in the ATC model, we will assume that each aircraft flies at the same constant speed of 800 km/h. All the previous simplifications, i.e. constant speed, constant flight level and fixed departure time could be relaxed leading to better optimal solutions, but the behavior of the algorithm will not be affected. Finally we assume that capacity constraints must be satisfied, i.e. configurations S in which the number of aircraft per hour inside a certain sector is above the threshold must be rejected during the optimization. Given a certain configuration of trajectories  $S = \{x_1, \ldots, x_N\}$ , its cost function will be

$$C(S) = \sum_{i=1}^{N} l(x_i) + \frac{\epsilon}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} m(x_i, x_j), \qquad (6)$$

where  $l(x_i)$  is the length of the path  $x_i$  divided by the length of the straight line connecting it departure and arrival node and  $m(x_i, x_j)$  is the number of conflicts between the aircraft *i* and the aircraft *j*. The parameter  $\epsilon$  is a positive real number that quantifies the weight of conflicts in the optimization process. If  $\epsilon = 0$  then the optimal solution is when all the trajectories are a straight line from departure to arrival and  $l(x_i) = 1$ for every  $x_i$ . On the other hand if  $\epsilon \gg 1$ , we will have a conflict-free solution. In order to apply the EO algorithm to this problem, the cost function must be in the form (4). This can easily achieved by defining the fitness of each trajectory  $x_i$  as

$$\gamma_i = l(x_i) + \frac{\epsilon}{2} \sum_{j=1, j \neq i}^{N} m(x_i, x_j).$$
 (7)

Finally we must define  $\Omega(S)$  and the process to go from S to  $S' \in \Omega(S)$ . At the beginning of the optimization process every trajectory is composed by one long segment going from  $n_{start}$  to  $n_{stop}$ , i.e. in the optimal configuration if  $\epsilon = 0$ . At each optimization step, the path of the less fit aircraft is built using a biased random walk process starting from  $n_{start}$ and ending in  $n_{stop}$ . Note that the interaction term in (7) guarantees that changing  $x_i$  will modify all the fitnesses of the others trajectories. The new random path is built starting from  $(n_0, t_0) = (n_{start}, t_{start})$ . At each step  $(n_k, t_k)$ , the next node  $n_{k+1}$  is randomly chose from the set of all the other nodes with a probability

$$p(n_{k+1}) \propto \left(\frac{d_{n_k, n_{k+1}} + d_{n_{k+1}, n_{stop}}}{d_{n_k, n_{stop}}}\right)^{\alpha},$$
 (8)

where  $\alpha < 0$  and  $d_{n,m}$  is the geographical distance between the navigation points n and m. The process ends when  $n_{k+1} = n_{stop}$  and it is restarted whenever a step violates the capacity





Fig. 6:  $\langle l(x_i) \rangle$  (a) and  $N_{conflicts}$  (b) as functions of the optimization step for  $\epsilon = 2$ .

constraints of the sectors. Note that (8) disadvantages steps that are far from being on the straight line connecting the previous step with the destination. In the following we will fix the parameter  $\alpha = -4$  arbitrarily. Other choices may fail to converge (e.g. if  $\alpha$  is too close to 0) or may lead to solutions too close to the straight line (e.g. if  $\alpha$  is too large).

#### C. The Optimal Solution

We apply the algorithm to the Greek National Airspace in order to optimize all the trajectories of the flight that took place on the  $9^{th}$  of June 2011. We assign to each flight its real departure time, i.e. considering the departure delay acquired that day and we do not put any constraint on the arrival time. As it is shown in figure 6, initially the algorithm generates deviation for many trajectories in order to solve some or all the conflicts and thus increasing the values of  $l(x_i)$ . As we vary the parameter  $\epsilon$ , we modulate from a trivial solution in which all the aircraft fly on a straight line to a conflictfree solution. Figure 7 shows the sub-optimal values of the average  $\langle l(x_i) \rangle$  over all the trajectories and the total number of unresolved conflicts  $N_{conflicts} = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} m(x_i, x_j)$ as  $\epsilon$  varies. Starting from the trivial case  $\dot{\epsilon} = 0$ , a conflictfree solution is obtained for  $\epsilon \geq 2$ . Note that the  $\langle l(x_i) \rangle$ is close to 1 for every value of  $\epsilon$ , so every solution is a set of trajectories that are slightly deviated from the straight case. In every considered case the resulting structure of the airspace is very different from the current one. Since building a simple navigation point network as no meaning in this new optimal case (navigation points are just arbitrary points in space used for the optimization, they could have been substituted by any other set of geographical references), we proceeded considering all the intersection points between



Fig. 7:  $\langle l(x_i) \rangle$  (a) and  $N_{conflicts}$  of the sub-optimal solution as functions of  $\epsilon$ .

the trajectories and applying a clustering algorithm, so that all the close intersections are clustered together into one single node. Then two nodes are linked if at least two intersections in each cluster belong to the same trajectory, i.e. if an aircraft has flown from a node to the other. We applied the same procedure also to the real historical trajectories in the same day of the dataset in order to obtain two comparable networks. Figure 8 shows the inverse cumulative distribution of the traffic load over the nodes in the two networks. The number of highly trafficked nodes is considerably reduced in the optimal case and the network is more homogeneous due to the fact that more of the available spaces has been used by the flight plans.

## D. Efficiency of the Optimal Solution

The topological properties by themselves are not sufficient in order to understand if a different kind of planning is wellperforming. In fact despite the fact that a global sub-optimal solution might be conflict-free, such solution is built in a situation in which many external factors are not taken into account. For example it is not possible to forecast departure delays due to for example to reactionary delays or the occurrence of every adverse weather condition. In particular in our case none of this conditions have been considered in the optimization process and the solution found is valid in a idealized situation in which everything goes just as planned. Thus it is important to test how the planned routes are working when such external occurrences are taking place and the ATC model we developed is suited to give useful insights about this issue.

In particular we built a new simulation setup similar to that presented in II-B for the validation of the ATC model. In particular:





Fig. 8: Inverse cumulative distribution of the strength of the nodes for the real and optimal Greek Navigation Point Networks.

- Every aircraft flies according to a flight plan obtained with the EO algorithm for various values of  $\epsilon$ .
- The structure of the sectors is unvaried with respect to the one used in II-B, but the airways structure is not considered. After every redirection and aircraft is sent directly to its destination following a straight line.
- Since the trajectories are already as straight as possible, directs in order to speed up the traffic are not considered.
- As in the previous case, controllers solve conflicts using the IN-OUT protocol.
- Capacity constraints are considered as in the previous case.

We tested the solution considering two kinds of external stochastic disturbances, i.e. the delays generating perturbed areas presented in II-B and random departure delays. In the second case we apply to each flight a departure delay from a uniform distribution in a range  $[-\tau, \tau]$ , where  $\tau$  is a free parameter. For both cases we studied the response of the system in terms of generated delays and number of actions performed by the controllers as functions of  $n_{ext}$  (the number of perturbed areas) and  $\tau$  for different sub-optimal solutions obtained with different values of  $\epsilon$ . Moreover we performed the same studies, using the validation setup in II-B in order to compare such results to a case close to the actual situation. The most natural way in order to compare these different situation is, according to our model, the number of actions that have to be performed to manage the traffic, despite the fact that some actions may require more effort for the ATC with respect to others. Figure 9 shows the number of actions performed by the ATC (considering also the directs) as a function of the parameter  $\tau$  of the delay distribution. Each value of the curve is averaged over many realizations of the same initial conditions. It is evident that such number is constant and it is not affected by the intensity of departure delays in the current situation. This fact is not surprising since nowadays, the ATC



Fig. 9: Number of actions as a function of the departure delay distribution amplitude  $\tau$  for the current system and for the sub-optimal planning in the Greek Airspace.



Fig. 10: Number of actions as a function of the number of external disturbances  $n_{ext}$  for the current system and for the sub-optimal planning in the Greek Airspace.

does not care about the delay of an aircraft crossing a sector as long that capacity limits are not overcome. On the other hand with the sub-optimal planning this number is independent from  $\tau$  only for  $\epsilon = 0$ . As  $\epsilon$  grows instead the curve presents a clear trend, growing from small values to a constant value for large  $\tau$ . Note that the constant value for large  $\tau$  depends on  $\epsilon$  and it is lower as the planning approaches the conflictfree initial situation. Moreover, for any value of  $\tau$  or  $\epsilon$ , the number of actions in the sub-optimal case is always lower than the constant value for the system in the current situation.

Considering the other kind of disturbances, the system in the current situation is not able to function in the same way independently from their number (Fig. 10). In this case the number of actions grows as the number of disturbances grows and it does not seem to approach any constant value. The behavior is the same also considering the sub-optimal solutions. For every value of  $\epsilon$ , the number of actions grows with the number of disturbances and it does not approach a constant value. Although, as it happens for the departure delays case, the actions required to manage the sub-optimal cases are always less than those needed in the current situation and their number gets smaller as  $\epsilon$  grows.



#### **IV.** CONCLUSIONS

In this paper we presented a model of Air Traffic Control based on a local optimization dynamics on the complex network of navigation points within a national airspace. The model has been built and validated using historical data about flight plans and real trajectories covering a time frame of a couple of weeks in June 2011. Setting up simulation of a one day schedule in the Italian, Greek and Estonian airspaces the model is capable of reproducing the statistics of the single trajectories of the aircraft as well as the topological modifications of the navigation point network of the considered nations performed by the action of the ATC in its traffic management activity.

By simulating an increase of traffic load with a simplified setup, the model shows a transition to a state in which the optimization process is not able to solve many of the occurring conflicts. This transition represents the theoretical limit of the model, indicating the maximum density of aircraft that the model can simulate correctly.

We then built a global optimization algorithm based on Extremal Optimization, in which avalanche effects are used in order to reach a local minimum of the cost function of a system. Tuning a parameter of the algorithm, we constructed flight plans ranging from the shortest possibles to completely conflict-free ones. We used the ATC model in order to test the efficiency of these solution in the Greek Airspace, measuring the number of actions performed by the ATC in response to adverse occurrences such as departure delays and delay generating areas. In both cases the solutions are performing better than the current situation, even though their behavior is less stable as the number or the intensity of the disturbances varies.

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