Pre-Tactical Time Window Assignment: Runway Utilization and the Impact of Uncertainties

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***Abstract*—Efficient planning of runway utilization is one of the main challenges in Air Traffic Management (ATM). It is important because runway is the combining element between airside and groundside. Furthermore, it is a bottleneck in many cases. In this paper, we develop a specific optimization approach for the pre-tactical planning phase that reduces complexity by omitting unnecessary information. Instead of determining arrival/departure times to the minute in this phase yet, we assign *several* aircraft to the same time window of a given size. The exact orders within those time windows can be decided later in tactical planning. Mathematically, we solve a generalized assignment problem on a bipartite graph. To know how many aircraft can be assigned to one time window, we consider distance requirements for consecutive aircraft types. We develop an optimization model, which can be solved fast in practice but may provide unnecessary large time buffers, and extend it to a model that solves the problem to global optimality.**

In reality, uncertainty and inaccuracy almost always lead to deviations from the actual plan or schedule. We present computational results concerning the abovementioned optimization approaches and investigate the impact of disturbances on our deterministic solutions. As a next step, we will incorporate uncertainties directly in our model. Therefore, we analyze real-world data from a large German airport in order to obtain realistic delay distributions and describe a simulation environment to test current and future solution methods.

I. Introduction

ATM systems are driven by economic interests of the participating stakeholders and, therefore, are performance oriented. As possibilities of enlarging airport capacities are limited, one has to enhance the utilization of existing capacities to meet the continuous growth of traffic demand. The runway system is the main element that combines airside and groundside of the ATM System. Therefore, it is crucial for the performance of the whole ATM System that the traffic on a runway is planned efficiently. Such planning is one of the main challenges in ATM. Uncertainty, inaccuracy and non-determinism almost always lead to deviations from the actual plan or schedule. A typical strategy to deal with these changes is a regular re-computation or update of the schedule. These adjustments are performed in hindsight, i.e. after the actual change in the data occurred. The challenge is to incorporate uncertainty into the initial computation of the plans so that these plans are robust with respect to changes in the data, leading to a better utilization of resources, more stable plans and a more efficient support for ATM controllers and stakeholders. Incorporating uncertainty into the ATM planning procedures further makes the total ATM System more resilient, because the impact of disturbances and the propagation of this impact through the system is reduced.

In this paper, we investigate the deterministic problem of optimizing runway utilization and the effect of disturbances on our solutions. As a next step, we will incorporate uncertainties into the initial plan in order to retain its feasibility despite changes in the data.

We focus on the pre-tactical planning phase, i.e. we assume the actual planning time to be several hours, or at least 30 minutes, prior to actual arrival/departure times. We develop an appropriate mathematical optimization model for this particular planning phase. The basic idea is that in pre-tactical planning we can reduce the complexity of the problem by not determining an exact arrival/departure sequence in terms of exact landing/take-off times for each aircraft, as we do later in tactical planning. Instead, we answer the question of how many aircraft can be scheduled to one time window of a given size without violating distance requirements. (For example, it is definitely possible to assign more than one aircraft to a time window from 12:00 pm to 12:10 pm.) Then, we consider a discretized time horizon consisting of such time windows and assign each aircraft to one of them.

This paper is organized as follows: In Section II, we give an overview over the literature related to runway optimization and explain why our approach is different. We develop a mixed integer program (MIP) for the pre-tactical optimization of runway utilization in Section III. Actually, we describe two models: The first one is difficult in theory but can be solved very fast in practice, the second one is an extension which solves the problem to global optimality at the expense of longer runtimes. In Section IV we show computational results and analyze the impact of disturbances on our solution methods. In order to be able to test our optimization approach in a more realistic setting, we analyze real-world delay data from a large German airport in Section V and describe a simulation environment to test current and future optimization approaches. We conclude in Section VI.

II. Related Work

There are many different approaches that deal with the optimization of runway utilization in the literature. Most of them treat the runway scheduling problem in the tactical planning phase.

A. Deterministic Approaches

The most cited MIP model in this context is probably the one introduced by Beasley et al. [5]. Their linear objective function minimizes delay, the constraints come from the aircraft dependent separation times. They also present an integer program (IP) formulation where time is discretized, but they don't explore it further because of disappointing computational experiences. Soomer and Franx [17] consider the problem from an airline point of view. They use Beasley's MIP but allowing airlines to define their own cost functions for each flight. Bertsimas et al. [7] develop a comprehensive IP for Air Traffic Flow Management which integrates all phases of a flight, different costs for ground and air delays, rerouting, continued flights and cancellations. Kjenstad et al. [14] state a time-discretized model. They assign an aircraft to a time window and claim that a number of subsequent time windows (dependent on the aircraft type) remains unassigned. In their model, they also consider minimal taxiways and the possibility to drop departures. Their linear objective function minimizes delay and the number of dropped departures.

Many authors use heuristic methods aiming to provide solutions in close to real-time. To schedule aircraft in a first-come first-served order (FCFS) seems to be fair and also reduces the work of traffic controllers. However, such an approach doesn't provide maximal throughput or minimal delay in general (Bennell et al. [6]). Dear [9] developed the concept of Constrained Position Shifting, were each aircraft can be scheduled only a limited number of steps away from the FCFS sequence. Balakrishnan and Chandran [3] solved this problem as a shortest path problem on a special network.

Anagnostakis and Clarke [2] formulate a two-stage heuristic algorithm for the outbound runway scheduling problem. In the first stage, candidate weight class sequences are determined w.r.t. distance requirements, ordered by the corresponding throughput. In the second stage individual aircraft are assigned using operational constraints (e.g. earliest and latest departure times of aircraft).

As mentioned, in our optimization model we allocate time windows to aircraft. However, though many papers about runway optimization deal with "slot allocation", this term is used to describe different problems. Often, it is associated with the Ground-Holding Problem (GHP), where "slot" means a certain departure time which is assigned to an aircraft. Ball et al. [4] also address the GHP, but they assign arrival slots to aircraft which provide the corresponding departure delay in hindsight. They consider matchings in a bipartite graph which they call the "flight allocation graph". The main focus in this paper lies on the graph structure and matching algorithms.

TABLE I
MINIMUM SEPARATION TIMES (IN SECONDS)

Predecessor \ Successor	Heavy	Medium	Light
Heavy	100	125	150
Medium	75	75	125
Light	75	75	75

None of the approaches above deal with "slots" as time windows to which *several* aircraft can be assigned. Thus, to the best of our knowledge there is no approach similar to ours in which the pre-tactical planning phase is modelled by assigning such time windows to aircraft.

B. Approaches that Incorporate Uncertainties

All runway optimization approaches presented above assume that all parameters are known with certainty. We found few works where uncertainties are incorporated. Chandran and Balakrishnan [8], e.g., develop an algorithm with Constrained Position Shifting that handles uncertainty in the estimated time of arrival. Hu and Paolo [12] formulate a genetic algorithm and compute solutions disturbing the estimated arrival time of 20% of the aircraft. Sölveling [16] presents a two-stage stochastic program for solving the mixed-mode runway scheduling problem with uncertain earliest times. In the first stage he determines the weight class sequence. An exact sequence of individual aircraft follows in the second stage.

III. The Modeling

As mentioned above, we model the problem of optimizing runway utilization in the pre-tactical planning phase by assigning time windows to aircraft. Throughout this paper, we consider single-mode runways with only arriving aircraft. In the future, we will adjust our models to mixed-mode runways. But since the single-mode problem is already quite complex from a mathematical point of view, we decided to focus on arrivals for now. In our modeling approach we claim that each aircraft has to receive exactly one time window as each aircraft has to be scheduled. On the other hand, the number of aircraft that can be assigned to one time window depends on its size and the weight classes of the aircraft. The underlying idea is that, contrary to tactical planning, we don't need to determine arrival times to the minute yet, because we are several hours (or at least 30 minutes) prior to the first scheduled time. Thus, the exact arrival sequences within the time windows can be decided later.

In this section, we develop a MIP for the described problem. The objective is the maximization of punctuality. In other words, the deviation from scheduled times in both directions shall be minimized. The MIP constraints consist of general assignment constraints and the modeling of minimal time distance requirement. Those minimum separation times between two consecutive aircraft depend on their corresponding weight classes. Hereof, we consider three different aircraft categories (*Light*, *Medium* and *Heavy*) and use Table I ([13]).

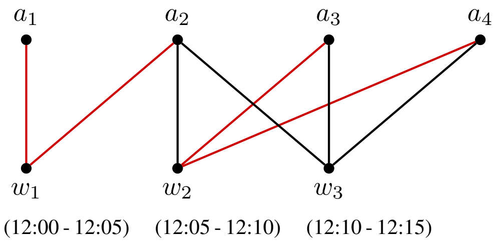

Fig. 1. Bipartite assignment graph. Red edges show a possible assignment: aircraft a_1 and a_2 are assigned to time window w_1, a_3 and a_4 are assigned to w_2.

Before we can state our model, we have to analyze the underlying problem structure more precisely.

For each aircraft, we consider several corresponding times:

- *Scheduled time of arrival* (ST): a fix time that yields a benchmark to identify delay and earliness of the aircraft. This may be the time the passenger finds on his flight ticket.
- *Earliest time of arrival* (ET): depends on operational conditions (and on the impact of disturbances).
- *Latest time of arrival* (LT): latest time the aircraft can land without holdings. It depends on the earliest time ET and on the actual planning time (or start time, respectively, if the aircraft is still on the ground), denoted by t. In the pre-tactical planning phase, we use the following formula (because ET$-t \geq 20$ min, see Figure 3):

$$\mathrm{LT} = \mathrm{ET} + \frac{\mathrm{ET} - t}{2} - 120\,\mathrm{sec.} \qquad (1)$$

- *Maximal latest time of arrival* (maxLT): a hard condition for landing which is calculated with respect to physical, operational and other relevant conditions (for instance, amount of fuel, prioritization, etc.).

Those times further determine the corresponding time windows ST_W, ET_W, LT_W and maxLT_W for each aircraft.

A. Assignment Graph

To model the problem of assigning aircraft to time windows, we consider a bipartite graph $G = (A \cup W, E)$ consisting of a vertex set A of aircraft and a vertex set W of time windows of a given size in a given time period (ordered chronologically). An edge $(i, j) \in E$ corresponds to a possible assignment of aircraft i to time window j. Possible assignments concerning a certain aircraft are all time windows from ET_W to maxLT_W.

Now, a feasible solution for our assignment problem is a set of edges such that

- every aircraft vertex is linked with exactly one edge from this set, i.e. every aircraft is assigned to exactly one time window,
- every time window vertex is linked with a number of edges from this set, so that no separation time constraints are violated.

In Figure 1 we see a small example of a bipartite graph with a possible assignment of aircraft $a_1, \ldots, a_4 \in A$ to time windows $w_1, w_2, w_3 \in W$.

B. Decision Variables

To solve our assignment problem, we have to decide whether to choose a certain edge or not. To model this decision in our MIP, we introduce a binary variables x_{ij} for each edge $(i, j) \in E$:

$$x_{ij} = \begin{cases} 1, & \text{if aircraft } i \text{ is assigned to time window } j \\ 0, & \text{otherwise} \end{cases}$$

C. Objective Function

Our objective is the minimization of delay and earliness, respectively. We model delay/earliness as edge weights. The weight c_{ij} of an edge $(i, j) \in E$ results from the distance of time window j to the ST_W of aircraft i (counted in number of time windows). Delay is penalized quadratically for reasons of fairness (e.g., a solution in which one aircraft has a delay of six time windows is worse than a solution in which two aircraft have a delay of three time windows each). Earliness is penalized linearly. If the assigned time window is after the LT_W (i.e. between LT and maxLT), we add an extra penalization term, namely the squared distance from LT_W. Assume an aircraft i with ST_W w_5, ET_W w_1, LT_W w_{10} and maxLT_W w_{13}. Then we'd have, e.g., $c_{i\,2} = 3$, $c_{i\,8} = 3^2$ and $c_{i\,12} = 7^2 + 2^2$.

Now the objective function of our optimization model is the following:

$$\min \quad \sum_{(i,j) \in E} c_{ij} x_{ij} \qquad (2)$$

D. Aircraft Constraints

First of all, we have to assert that each aircraft is assigned to exactly one time window. So we claim

$$\sum_{j \in W_i} x_{ij} \;=\; 1 \qquad (3)$$

for each $i \in A$, where $W_i = \{j \in W : (i, j) \in E\}$ describes the set of time windows that aircraft i can be assigned to.

E. Time Window Constraints

Further, we have to determine the number of aircraft that can be assigned to one time window. In order to do so, we need to consider the distance requirements, dependent on the weight classes of consecutive aircraft. We use the minimum separation times shown in Table I[1]. Clearly, the maximum number of aircraft that fit in one time window is reached when a sequence from Light to Heavy is assumed. In more detail, to avoid separation times of 125 and 150 seconds, such a sequence contains sub-sequences of aircraft of the same type. First, all Lights are scheduled, followed by all Mediums, and finally by all Heavies. According to Table I, we therefore need a separation time of 75 seconds after each Light and each Medium, whereas we need 100 seconds after each Heavy

[1] An adaptation of the results in this paper to other minimum separation time tables is possible as well.

except the last one (the needed separation time after the last aircraft in a time window models the distance requirements at the window boundary and is analyzed later in this section).

For each time window we get upper bounds on the number of aircraft by assuming such a sequence from Light to Heavy. Mathematically, it yields the following two constraints for each $j \in W$:

$$75 \sum_{i \in L_j} x_{ij} + 75 \sum_{i \in M_j} x_{ij} + 100 \sum_{i \in H_j} x_{ij} \leq s + 100 \quad (4)$$

$$75 \sum_{i \in L_j} x_{ij} + 75 \sum_{i \in M_j} x_{ij} \leq s + 75 \quad (5)$$

Here, s is the size of the time windows (in seconds). Further, $L_j = \{i \in L : (i,j) \in E\}$ describes the set of Lights that may be assigned to time window j (and thus, the corresponding sum yields the number of Lights that *are* assigned to it). M_j and H_j are defined analogously.

If we assign aircraft to a time window j without exceeding these bounds in (4) and (5), we know that there *exists* a sequence of those aircraft that fits in the time window. However, we do not determine how that sequence looks like exactly in terms of concrete predecessors and successors. We are still flexible in (re)arranging different aircraft of the same type. And if the time window contains enough "empty space", we can even deviate from the Light-Medium-Heavy order without changing the assignment.

In the following we extend (4) and (5), because they do not assert security distances at the time window boundaries yet. This means that the last aircraft in one time window and the first aircraft in the subsequent time window may be planned to land at the very same time. In order to obtain feasible solutions, we describe two different ways to provide appropriate buffers at the end of each time window.

1) Time Window Constraints for Solutions with Generous Buffers: To assure feasible solutions, we can generally claim 150 extra seconds as buffers in every time window that contains a Heavy aircraft and 125 seconds otherwise (150 and 125 seconds are the maximal values in our separation time matrix, see Table I), except for the last time window of the considered time horizon. Then, for all $j \in W \setminus \{m\}$ both constraints (4) and (5) can be formulated as one:

$$75 \sum_{i \in L_j} x_{ij} + 75 \sum_{i \in M_j} x_{ij} + 100 \sum_{i \in H_j} x_{ij} \leq s - 50 \quad (6)$$

Obviously, (4) turns into (6) by simply adding those 150 seconds on the left hand side. And further, $x_{ij} = 0$ for all $i \in H_j$ in (6) yields (5) with additional 125 seconds on the left hand side. Of course, this approach only provides a heuristic procedure for solving the problem because those buffers will be unnecessarily large for some time windows.

2) Time Window Constraints (and Additional Variables) for Optimal Solutions with Individual Buffers: In the following we describe a way to model distance requirements at the window boundaries precisely (dependent on corresponding aircraft types). For this purpose, we have to consider the arrivals that are planned at those boundaries. We accomplish this by introducing additional variables. These new variables assure suitable minimum separation times at the end of each time window: We define z_j^{HH} with

- $z_j^{HH} = 1$ if we have a Heavy at the end of time window j and a Heavy at the beginning of the subsequent one (assuming a sequence from Light to Heavy) and
- $z_j^{HH} = 0$ otherwise,

and variables z_j^{HM}, z_j^{HL}, z_j^{ML} analogously. Note that for a certain time window j, at most one of these four variables can be 1. If $z_j^{HL} = 1$ for instance, we need to assure 150 extra seconds of separation time at the end of time window j (according to Table I). If all four variables are 0, we need to assure 75 extra seconds at the end of j (and we don't need to differentiate if we separate Medium and Heavy or Medium and Medium etc.). Adding the different cases for the required extra seconds, the inequality (4) splits into the following inequalities for each $j \in W \setminus \{m\}$:

$$75 \sum_{i \in L_j} x_{ij} + 75 \sum_{i \in M_j} x_{ij} + 100 \sum_{i \in H_j} x_{ij} + 100 z_j^{HH} \leq s + 100 \quad (7)$$

$$75 \sum_{i \in L_j} x_{ij} + 75 \sum_{i \in M_j} x_{ij} + 100 \sum_{i \in H_j} x_{ij} + 125 z_j^{HM} \leq s + 100 \quad (8)$$

$$75 \sum_{i \in L_j} x_{ij} + 75 \sum_{i \in M_j} x_{ij} + 100 \sum_{i \in H_j} x_{ij} + 150 z_j^{HL} \leq s + 100 \quad (9)$$

Note that we have such inequalities for each time window except for the last one again, because we always consider the end of the windows. For the last time window $j = m$ we still have the separation constraint (4). Also note that if, e.g., $z_j^{HH} = 1$ in (7), then (8) and (9) become redundant for that time window j, since $z_j^{HM} = z_j^{HL} = 0$.

Inequality (5) can be extended analogously.

To determine the new z-variables within the model, we use another group of binary variables which state whether there are Heavies, Mediums or Lights, respectively, in a given time window j or not. Then, we introduce additional constraints to model the relations between those new variables and the z-variables. For instance, $z_j^{HH} = 1$ if and only if we have the following situation:

- there is at least one Heavy in time window j and at least one Heavy in $j + 1$,
- there are no Lights and Mediums in time window $j + 1$ (those would precede the Heavies in the assumed sequence).

IV. Computational Results

In a computational study, we tested both models introduced in Section III, the one that yields globally optimum solutions with individual time buffers at the end of each time window (in the following referred to as *optimal MIP*) and the one that uses general buffers that might be unnecessarily large (*generous MIP*). We considered runtime, solvability, quality and impact of disturbances. In order to investigate the impact of disturbances, we studied the following situation: Assume an optimal assignment has been determined for the nominal realization of the uncertain parameters, i.e. for nominal ET's and LT's. Uncertainties have been ignored in the model. Now some disturbances on the ET's occur (and therefore on the dependent LT's as well). A natural question is how good the former optimum solution is for the disturbed situation.

A. Setting

The results were obtained by the integer programming solver *Gurobi* (version 5.6). For the experiments we used a laptop with Intel i7 CPU, 4 cores (2.70 GHz) and 8 GB RAM. We considered time windows of 10 minutes (600 seconds) and investigated three different instance sizes which concerns numbers of aircraft with corresponding time horizons:

- 100 aircraft on 2,5 hours (15 time windows),
- 200 aircraft on 5 hours (30 time windows),
- 400 aircraft on 10 hours (60 time windows).

The distribution of the weight classes was always 90% Medium, 5% Heavy and 5% Light according to the actual data from a large German airport. The ST_W's for all aircraft are chosen randomly, i.e. uniformly distributed. The ET_W's are assumed to be the predecessors of the ST_W's. According to the latest time calculation formula (1), we assumed LT_W to be two hours (12 time windows) after ET_W. For simplification, we set maxLT = LT.

In our setting, the ET_W and the LT_W are the only uncertain parameters. The uncertainties are selected at random. For proof of concept, we used a Gaussian distribution in this preliminary study. In further investigations, we will test our optimization approaches with more realistic uncertainties. As we explain in Section V, this will lead to asymmetric Gamma-distributions. We assumed that the expected value (EV) for shifting the ET_W is 1 with standard deviation $\sigma = 1.5$.

For each of the three test cases, we generated 5 random instances. In Table II we see the averaged results.

B. Results

The first observation considering Table II is that most runtimes are very low. Further, they increase with increasing number of aircraft, because our allocation graph grows bigger (which yields more variables and constraints in our model). Only in case of 400 aircraft we frequently reach the determined time limit of 15 minutes. However, those instances that could be solved to optimality here were also solved in less than two minutes.

Mathematically, the optimal MIP is more challenging than the generous MIP due to an increasing number of variables and constraints including non-linearities. This is reflected in longer runtimes. The generous MIP actually turns out to be extremely fast on all feasible instances. In fact, runtimes for all those instances were less than 1 second. However, also the optimal MIP can be solved rather fast in most cases.

As expected, the optimal MIP performs much better regarding objective values and numbers of delayed aircraft than the generous MIP. This is because the buffers in our time windows for the generous MIP can be unnecessarily large.

The parameter *Infeasible Assignments* shows whether the optimal solution of the corresponding approach is still feasible after the disturbances occurred. It describes the percentage of aircraft that have been assigned to time windows to which they cannot be assigned in the disturbed situation. Thus, it can be seen as probability for an aircraft to be assigned to an infeasible time window. This situation is very unsatisfying for a practitioner. Namely, the solution that has been computed earlier becomes useless because it is not feasible anymore. Then, usually replanning has to be performed, often in real-time. The goal of this study is to understand how sensitive the computed solutions are to disturbances. As we see in Table II, the optimal MIP assigned $23.8-27.2\%$ of the aircraft to time windows that became infeasible after disturbances occurred. $11-21.9\%$ were assigned to "wrong" time windows by the generous MIP. This experimental result shows that it is crucial to enrich the optimization approaches by protection against uncertainties, such that less replanning is necessary. Thus, in the future we will incorporate disturbances directly in the model.

We did the above tests also for time windows of 15 minutes and 5 minutes. In general, we observed the following trends: Runtimes increased with decreasing window sizes because of a bigger allocation graph. Further, the smaller the windows the more delayed aircraft were obtained, due to measurement accuracy. Also the objective value increased with decreasing window sizes for the same reason.

So far, in this paper we have described mathematical approaches for optimizing runway utilization in the pre-tactical planning phase. Further we have tested the impact of (Gaussian distributed) disturbances on our computed solutions. In the following section, we now analyze real-world disturbances from our database from a large German airport. Finally, we describe a simulation environment to test our current and future approaches with those realistic disturbances.

V. Statistical Analysis and Modeling of Arrival Delay Data

A. Stochastic Delay Modeling

Understanding and modeling the statistics, dynamics, and propagation of air-traffic arrival and departure delays is a prerequisite of any attempt to optimize the punctuality of schedules and airport capacity, and minimizing necessary buffer times for required robustness of performance (e.g. Wong and Tsai [19], Tu et al. [18]). In support of the development of

TABLE II
EXPERIMENTAL RESULTS

Number of Aircraft	Approach	Runtime (sec)	Solved (%)[a]	Objective Value	Delayed Aircraft (%)	Infeasible Assignments (%)
100	optimal MIP	0.43	100	48.00	33.60	23.80
	generous MIP	0.05	100	137.20	62.00	17.40
200	optimal MIP	2.41	100	53.40	25.80	25.50
	generous MIP	0.07	100	150.00	51.10	21.90
400	optimal MIP	170.86	60	149.33	34.83	27.17
	generous MIP	0.16	100	2355.60	80.85	11.00

[a]Amount of instances that are solved optimal within the time limit of 15 min. If < 100, averages are taken over the instances that could be solved to optimality only.

new optimization algorithms we investigate simple stochastic arrival and departure delay models derived from empirical delay data from a large German airport. In general, histograms of delay data exhibit a pronounced non-symmetry (e.g. [18]). Wu [20], [21] used the two-parametric Beta-probability density function (PDF) for modeling arrival time statistics (with actual arrival times ATA, = actual in-block time AIBT, in what follows). The ATA-PDF is also valid for delays (= ATA−STA) which differs only by a translation by the deterministic scheduled arrival time STA. The argument in favor of the Beta-PDF as compared, e.g., to the log-normal PDF was its analytical form and easier tractability in calculations. The two parameters were quantified by fitting to 1999-flight data from an European airline with sample sizes of 90 flights each for three different cases of operation: domestic flights, exhibiting a quasi normal pattern (Beta$(18, 20)$), short haul international flights exhibiting a right tailed PDF (Beta$(4, 14)$), and long haul (inter-continental) flights which showed a long right tail (Beta$(2, 13)$).

Because the Beta-PDF is strictly limited to the open $(0, 1)$ interval and because standard models of inter-arrival statistics usually are based on the Poisson process (exponential inter-arrival time distribution), we prefer the two-parametric Gamma-PDF as fitting model for empirical delay-histograms. It appears more appropriate for comparing the empirical delay statistics because it extends to $+\infty$ which captures the empirical data after shifting the values to positivity by subtracting the largest negative delay (minimum earliness). The family of Gamma models includes the Poisson process and it has been extensively investigated recently (e.g. Dodson and Scharcanski [10]). As will be shown below, the analysis of measured arrival and departure delay histograms together with Monte Carlo computer experiments indicate Gamma models to be a suitable approach for modeling the stochastic part of the delay dynamics. Following the definition in [10], the Gamma density represents a generalization of the Poisson model of inter-arrival time (t) density with mean inter-arrival time τ and variance $\sigma^2 = \frac{\tau^2}{\alpha}$:

$$f(t; \tau, \alpha) = \left(\frac{\alpha}{\tau}\right)^{\alpha} \cdot \frac{t^{(\alpha-1)}}{\Gamma(\alpha)} \cdot e^{-\frac{\alpha t}{\tau}} \qquad (10)$$

with shape parameter α and scaling parameter β (defined via $\tau = \alpha \cdot \beta$ yielding the variance $\sigma^2 = \alpha \cdot \beta^2$) as parameters for maximum likelihood fitting of empirical delay histograms. For $\alpha = 1$, (10) reduces to the Poisson case of maximum randomness, i.e. exponential t-distribution ([10]). For $\alpha < 1$, (10) models a process with larger variance than the random process due to clustering, i.e. non-independent clustered events. For integer $\alpha = 1, 2, 3, \ldots$, (10) models a process that is Poisson with intermediate events removed to leave only every α-th, which has a smoothing effect for $\alpha > 1$. Citing Dodson and Scharcanski [10]: "...Gamma distributions can model a range of stochastic processes corresponding to non-independent clustered events, smoothed events, and the random (Poisson) case."

B. Statistical Analysis of Empirical Arrival Delay Data

We want to investigate the question in how far arrival and departure delays can be modelled as random events with random deviations from scheduled times. It is expected that any realistic model has to be a combination of deterministic and random components (Abdel-Aty et al. [1], Tu et al. [18]). This is due to the deterministic character of the flight plan and the possible influence of delay of an aircraft a_{i-1} on delay of its successor a_i, as well as influence of origin airport departure delay on destination airport arrival delay, and arrival delay on departure delay from the destination airport (turnaround delay). As proposed in [1], we analyze daily delays observed within the time series of all flights during full days of operation, as well as delay data from a selection of single flights over a couple of months (with ≥ 150 monitored arrival or departure times).

Figure 2 shows examples of arrival delay statistics $f(\text{ATA}(=\text{AIBT}) - \text{STA})$ for a single full day of traffic and arrival delays for a single flight periodically repeated over half a year. The χ^2-acceptance tests of the Γ-fits to the empirical delay histograms differs significantly between single days as well as between single flights. This is no surprise, of course, due to the neglection of any deterministic effect (correlations between flight arrival times or delays depending on traffic density, ATC-sequencing, departure delays, previous flight delay, etc.). Despite the rough matching between histograms and Γ-fits, the Γ-hypothesis for the single-day case of Figure 2a) and

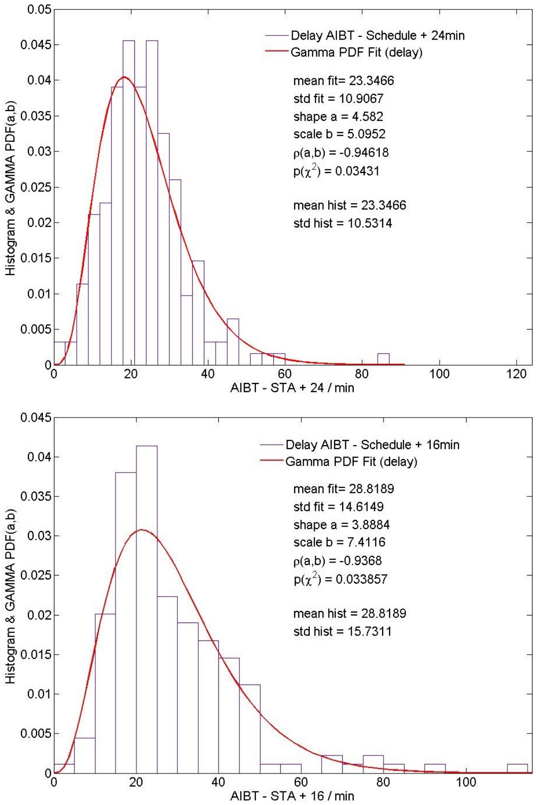

Fig. 2. Examples of empirical arrival-delay histograms (AIBT−STA shifted into $\mathbb{R}^+$ by adding |min. earliness|) with Γ-density max. likelihood fits. a) (top) Full-day (17 hours) traffic with 205 evaluated arrivals (ATA−STA+24 min). b) (bottom) One of 33 single flights measured over 6 months (July-December 2013), 179 arrivals (ATA−STA+16 min).

for the single flight example of Figure 2b) are both rejected at the $p = 5\%$ level. This is basically due to the deviations around STA which shows the necessity for including correlations into a complete model of the delay statistics ([1], [18]).

C. Simulation of Arrival Delays with Baseline Algorithm

With random disturbance of start times we expect our simulation to yield realistic delay distributions, i.e. Γ-distributions. For testing, preliminary simulations using a heuristic scheduling algorithm (Take Select with parameters 8 and 2 (Helmke [11]) for complete flights with repeated optimization during each simulation time step were performed. For this purpose a simple arrival time-interval model is used as shown in Figure 3.

The arrival time-interval between latest and earliest time (LT−ET) converges linearly to the target time as optimization result with regard to minimizing deviation from schedule STA (according to (1) for the pre-tactical phase, $>$ 20 min before ET), with a constant phase after entering the TMA, and a very small time interval leaving path stretching area, i.e. turning to final approach segment. An iteration of optimized a_i-sequences is calculated for each simulation time step. As an initial test, disturbance effects introduced at start time were analyzed for a single day of traffic with the same

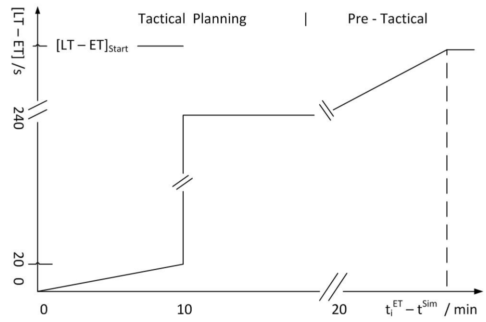

Fig. 3. Simplified model for arrival time-interval during a simulated flight between origin and destination airport. Abscissa: remaining flight duration ET$-t^{\mathrm{Sim}}$. Ordinate: arrival time-interval LT−ET.

STA's as those in Figure 2a), resulting in a delay statistic after the final optimization at arrival time (ATA−STA). This is motivated by the fact that departure delays at the origin airport have the dominating effect on the arrival delays at the destination (Performance Review Commission [15], 2013). An example histogram is shown in Figure 4 of a single flight a_i out of the total number of $n = 210$ arrivals. The statistics is obtained from a Monte-Carlo simulation with Gaussian N(18.2 min, 11.8 min) density for random disturbances of departure time. Runtime of a single MC-run was 25 s yielding about 1.5 hrs for the complete statistics of $n = 200$. The difference between maximum likelihood fit and histogram appears significantly better than that one of the empirical delay data (Figure 2b)) which can certainly be attributed to the neglected deterministic contributions (e.g., daily and seasonal periodicities) in the latter case.

Figure 4 depicts the modification of the departure delay-pdf by the scheduling algorithm. The correlation coefficient $r(\alpha, \beta)$ of the Γ-coefficients appears counterintuitive when comparing the good fit of the simulated data with low $\alpha - \beta$ (anti-)correlation value $r = -0.88$, with the worse fit of the empirical data, however higher (absolute) value of $r = -0.94$. The reason for this effect is the independence of mean τ and coefficient of variation $\frac{\sigma}{\tau}$ of a random sample with common PDF being equivalent to the PDF being a Γ-density (Dodson and Scharcanski [10]). The χ^2-test in this case supports the Γ-hypothesis. However, specific experiments are required, e.g. with different objective functions, in order to investigate the reason for the Γ-PDF to provide a good model for the delay statistics.

As a next step, we will develop optimization methods that cope with uncertainty, and test them within the simulation environment instead of using the Take Select algorithm. These new methods will contain techniques from robust optimization and stochastic optimization. The abovementioned results of the empirical data analysis and baseline simulations indicate the two-parametric Γ-PDF to be a reasonable approach for modeling the random disturbances for the validation of our future approaches. For improving the empirical data modeling we have to include the correlations and periodicities of the

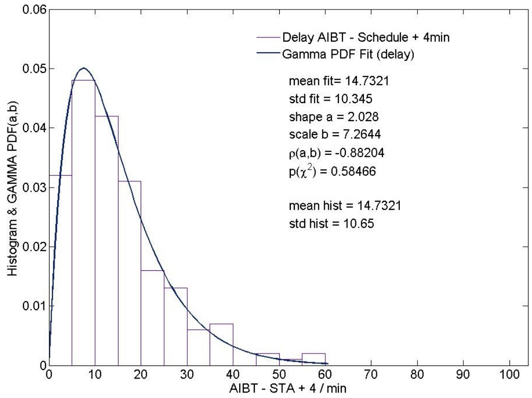

Fig. 4. Example Monte-Carlo simulation of arrival delays with 200 repeated runs, with random variations of start time (departure delays) drawn from $\mathrm{N}(18.2, 11.8)$-Gaussian PDF and using a standard optimizer (Take-Select82). Histogram depicts a single flight out of a full day scheduling-sequence of simulated ATA$-$STA$+4$ min with the same 210 flights as those of Figure 2. Γ-PDF fit with χ^2-test acceptance ($p = 0.58$) at 5% rejection level.

delay time-series. For the planned validation of the stochastic optimizer algorithms additional disturbances will be added during flight time.

VI. Conclusion and Future Work

We have developed two mathematical optimization models for the pre-tactical optimization of assigning time windows for runway utilization. In these models, several aircraft can be assigned to the same time window which reduces the complexity of the problem. The first model can be solved very fast in practice for determining good pre-tactical solutions with generous time buffers at the window boundaries. The second model extends upon the first and yields globally optimum solutions by modeling occasions at the boundaries more precisely.

We tested the impact of disturbances on both models. The results strengthened our intention to enrich the models by protection against uncertainties. We performed a statistical analysis of real-world data from a large German airport and described a simulation environment to test current and future optimization approaches.

As a next step, we will analyze the nominal models, presented in this paper, in more detail using realistic data and test it against standard algorithms (e.g. FCFS) within our simulation. Furthermore, we will incorporate uncertainty into our optimization models. Therefore, we will adjust our nominal models using techniques from robust and stochastic optimization.

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