Wind-Based Robust Trajectory Optimization using Meteorological Ensemble Probabilistic Forecasts

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***Abstract*—A major challenge for Trajectory-Based Operations is the existence of significant uncertainties in the models and systems required for trajectory prediction. In particular, weather uncertainty has been acknowledged as one of the most (if not the most) relevant ones. In the present paper we present preliminary results on robust trajectory planning at the pre-tactical level. The main goal is to plan trajectories that are efficient, yet predictable. State-of-the-art forecasts from Ensemble Prediction Systems are used as input data for the wind field, which we assume to be the unique source of uncertainty. We develop an ad-hoc optimal control methodology to solve trajectory planning problems considering uncertainty in wind fields. A set of Pareto-optimal trajectories is obtained for different preferences between predictability and average efficiency; in particular, we present and discuss results for the minimum average fuel trajectory and the most predictable trajectory, including the trade-off between fuel consumption and time dispersion. We show how uncertainty can be quantified and reduced by proposing alternative trajectories.**

I. Introduction TBO-Met

A. Motivation

In the future ATM system, the trajectory becomes the fundamental element of a new set of operating procedures collectively referred to as Trajectory-Based Operations (TBO). By replacing the current airspace-based ATM system, this trajectory-centric paradigm will be able to accommodate airspace users' requests to a greater extent (SESAR Consortium, 2007). The Business Trajectory constitutes a fundamental element of the TBO concept; it is the trajectory that will best meet airline business interests and will evolve out of a collaborative and layered planning process.

A major challenge for Trajectory-Based Operations is the existence of significant uncertainties in the models and systems required for trajectory prediction. Understanding and managing the impact of these uncertainties is necessary in order to increase the predictability of the ATM system whenever possible and desirable. In turn, predictability and robustness improvements in trajectories will produce gains in the high level goals (capacity, efficiency, safety, and environmental impact) pursued within a modernized ATM system. Some examples of relevant uncertainty sources are: 1) meteorological uncertainty; 2) uncertainty in the aircraft performance model [1]; 3) uncertainty in initial mass[2] and other parameters and 4) uncertainty in the aircraft intent [3]. In this paper, the focus is on the former, i.e., meteorological uncertainty; while we won't consider these additional uncertainty sources, we'll note that out methodology could be extended to include them.

Weather uncertainty is one of the most important sources of uncertainty that affect the ATM system. Indeed, the recently granted SESAR ER TBO-Met Project[1] focuses on the analysis of meteorological uncertainty coming from the following two sources: 1) wind, and 2) convective regions.

The analysis of the effects of meteorological uncertainty in TBO is an extraordinarily broad problem. The TBO-Met Project focuses on two particular problems, both at the pre-tactical and tactical levels: 1) Trajectory planning; and 2) Prediction of sector demand.

B. Scope of the paper

In the present paper we present preliminary results on robust trajectory planning at pre-tactical level (in this context, around 3 hours before departure). A methodology for robust route optimization is presented that can serve as a stepping stone towards robust 4D flight planning. To that end, we make use of Ensemble Prediction Systems and optimal control techniques. Wind is considered as the unique source of uncertainty. Figure I-A sketches the intended methodology for the Trajectory planning problem in TBO-Met Project. Recall that in this paper we focus on the pre-tactical level, i.e., the left hand side of the figure, not considering convective phenomena.

II. State of the Art

A. Ensemble Prediction Systems and ATM

Numerical Weather Prediction (NWP) centers developed Ensemble Prediction Systems (EPS) in order to provide probabilistic meteorological forecasts in addition to deterministic predictions. They seek to provide an estimation of the uncertainty that is inherent to the NWP process [4], a task that cannot be achieved with deterministic forecasting. In an EPS, several runs of the NWP model are launched with different characteristics in order to produce a set of (typically) 10 to 50 different forecasts or "members" of the ensemble. There are

[1]TBO-MET project (https://tbomet-h2020.com/) has received funding from the SESAR JU under grant agreement No 699294 under European Union's Horizon 2020 research and innovation programme. Consortium members are UNIVERSITY OF SEVILLE (Coordinator), AEMET (Agencia Española de Meteorología), METEOSOLUTIONS GmbH, PARIS-LODRON-UNIVERSITAT SALZBURG, and UNIVERSIDAD CARLOS III DE MADRID

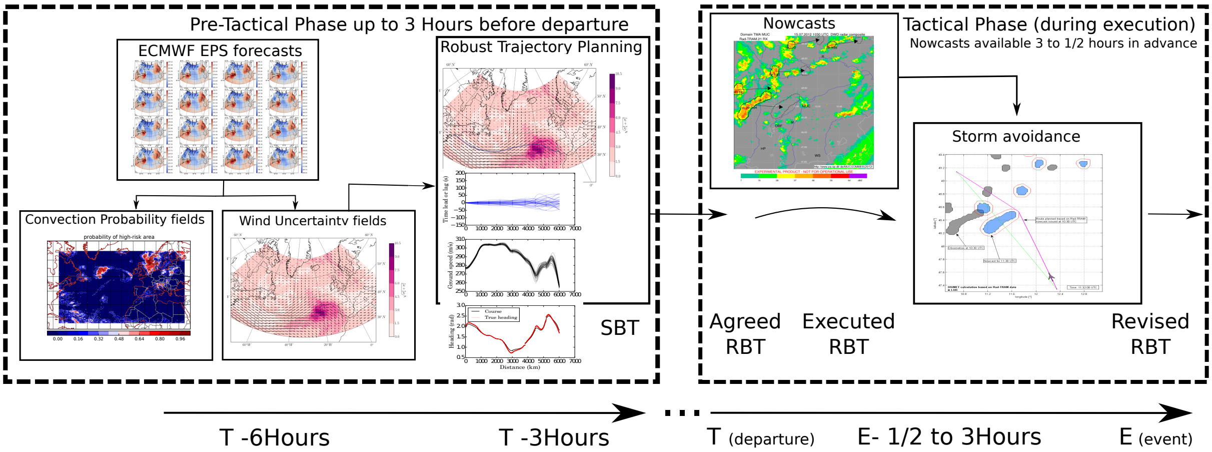


Figure 1. TBO-Met Trajectory Planning Methodology for both pre-tactical and tactical levels. Recall that the present paper focuses on the pre-tactical level.

several techniques in use to produce different simulations, such as strategic perturbations of the initial conditions or different parametrizations of physical processes [5] [6]; each NWP center employs a different combination of them. We refer to [7] for a review of the status of NWP as well as the relevance of EPS in a wider meteorological context.

The ATM research community has recently started to use EPS in order to study the predictability of flight plans and the sensitivity to weather prediction uncertainty. The main research effort in this direction has been undertaken within IMET, a SESAR WP-E project. It seeked to develop a ”probabilistic trajectory prediction” (PTP) system, where a deterministic TP is run once for each member in order to produce a trajectory ensemble. Preliminary results of this project were presented in [8] and a follow-up publication [9] showed how the information obtained with this approach could be used to improve decision-making at the pre-tactical level. Outside IMET, we computed optimal trajectories for each member in [10] in order to study the impact of uncertainty on trajectory optimization instead of trajectory prediction. Finally, the work presented in [11] is an analysis of the impact of uncertainty in average wind on final fuel consumption.

B. *Wind-optimal trajectories*

The calculation of wind-optimal trajectories is a problem that has been approached in the literature in several ways. In [12], a technique called neighbouring optimal control based on analytic optimal control is used; this procedure is based on a linearization of a perturbation around a nominal trajectory. In [13], it is compared to a method that relies on the interpolation of precomputed extremals. Other works based on analytical optimal control include [14], based on the numerical integration of the analytical solution of the problem that uses shooting to find the initial heading. The work in [15] belongs to this tradition, introducing some numerical techniques to solve the wind-optimal routing problem. A variant of dynamic programming called ordered upwind is shown to be effective in [16] [17].

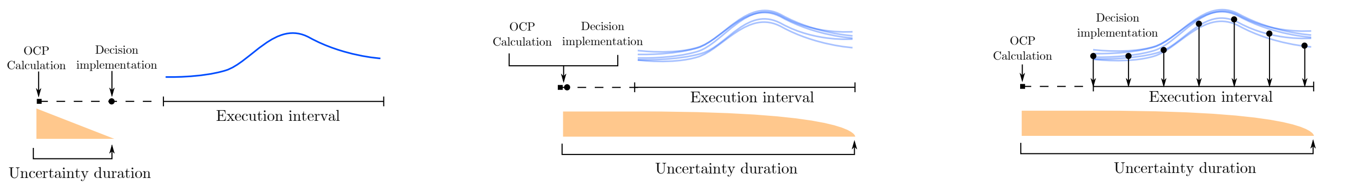
We also want to highlight approaches based on numerical direct methods[18] such as [19] or [10], as we will employ them too. When compared with the previously mentioned works, the main advantage of direct methods is the ability to solve more complex problems, as the former usually rely on assumptions such as constant airspeed or altitude that direct methods do not require. Therefore, direct methods can, in principle, be applied to more general problems. In exchange, direct methods generally have higher computational costs.

In this work, we will employ direct methods in order to solve a problem which is similar, in formulation, to the analytic and DP-based methods with the addition of uncertainty; thus, while the problem we solve is not as general as the one presented in [19] or [10], it can be extended to a comparable problem (and, indeed, we are working on this task already).

C. *Robust and stochastic optimal control*

Robust and stochastic optimal control is a field that is not as methodologically consolidated as deterministic optimal control or stochastic dynamic programming, so there are a number of approaches being employed in the literature that are sometimes referred by similar or identical names but correspond, in fact, to different problem formulations where uncertainty appears in different forms and thus correspond to different practical problems. We will classify them in three categories, referring to :

1) The “Uncertainty Quantification (UQ) + Optimal Control” approach: combines a non-intrusive UQ methodology with a deterministic OCP solver. The deterministic problem is solved for different values (as determined by the UQ rule) of the uncertain parameters and the solution is statistically characterized using the UQ rule. If the uncertain variables are realized before implementation of the solution, the optimal solution can be quickly computed by interpolation or analogous methods. However, if they are not known at execution, the UQ+OCP approach doesn’t provide an obvious decision rule since the “most likely to be the deterministic-optimal” trajectory is not, in general, the optimal trajectory



(a) Uncertainty Quantification OCP (b) Robust OCP (c) Stochastic OCP

Figure 2. Robust and stochastic optimal control methodologies.

under uncertainty. Examples of successful application of this methodology in aviation and ATM include optimal routing in an environment with uncertain threats [20] and 4D conflict-free trajectory optimization [21] [22]. See Figure 2 for a scheme of this methodology.

2) The "robust" (or "tychastic", in the terminology of [23]) approach: optimizes all of the possible trajectories simultaneously by augmenting the state space of the dynamical system with the states of different possible trajectories, corresponding to different values of the uncertain parameters and a single control law (usually, an open-loop sequence of controls). See Figure 2 for a diagram. Some examples of aerospace applications are [24] (based on the method of [25]) and [26].

3) The "stochastic differential equations" (SDE) approach: models uncertainty as a dynamic stochastic process instead of a "static" probabilistic set of parameters. Therefore, the dynamical system is described by a system of stochastic differential equations[27] instead of controlled ordinary differential equations or differential-algebraic equations. The solution features a closed feedback law $u = u(t, X_t)$ as a representation of the controls. The simplified linear-quadratic version of this problem has been extensively explored in the literature (like its deterministic version), but practical methods for the nonlinear version have not been explored in depth yet (see [28] and references therein for examples). Examples within the field of aviation rely instead on discretization to Markov Decision Processes (see [29] [30]). This methodology is described in Figure 2

We have chosen to develop a methodology that is similar in spirit to the "robust approach". The reasons that justify our methodological choice are the following: 1) we cannot assume that meteorological uncertainty has been fully realized at the departure time, thus making the "UQ+OCP" approach not convenient; 2) Ensemble Prediction Systems provide a model of the uncertainty that fits the formulation of the "robust" problem better than the "SDE" one; 3) from an operational perspective, obtaining a fixed guidance law (as we will do with our variant of the "robust formulation") corresponds with current operating procedures to a greater extent than a feedback policy of the form that is usually sought in a "SDE-based" formulation.

III. METHODOLOGY

The class of dynamical systems that we will consider is what [23] call a *tychastic* dynamical system. We denote the state vector by $\mathbf{x} \in \mathbb{R}^n$, the control vector by $\mathbf{u} \in \mathbb{R}^m$, $t \in \mathbb{R}$ is the independent variable (usually time) and the uncertain parameters are a continuous *constant* random variable $\xi : \Omega \to \mathbb{R}^q$. The dynamics of the system are given by the function $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R}^q \times \mathbb{R} \to \mathbb{R}^m$, such that:

$$\frac{d}{dt}\mathbf{x}(\omega, t) = f(\mathbf{x}(\omega, t), \mathbf{u}(\omega, t), \xi(\omega), t) \tag{1}$$

where $\omega \in \Omega$ is the sample point on the underlying abstract probability space. Thus, for each possible realization of the random variable $\xi(\omega)$, the trajectory will follow the deterministic differential equation (III-B)[2]. To emphasize the dependence of the trajectories on the random variables, we will use the notation $\mathbf{x}(\omega, t)$ and $\mathbf{u}(\omega, t)$

In order to fully determine the trajectory, we'll need a control or guidance law in addition to the realization of the uncertain parameters ξ. We will discuss this topic in Section III-C; consider, meanwhile, a general control law $\mathbf{u}(\omega, t) = \mathbf{u}_L(t, \mathbf{x}(\omega, t))$

A. Stochastic quadrature rules

The first component of this methodology is a stochastic quadrature rule: a finite set of quadrature points $\{\xi_k\}$, $k \in \{1, \ldots, N\}$ and weights $\{w_k\}$, $k \in \{1, \ldots, N\}$, such that we can build an approximation to the stochastic integral $I = \int_\Omega g(\xi(\omega))d\omega$ with the sum:

$$Qg = \sum_{k=1}^{N} w_k g(\xi_k)$$

where $g(\xi)$ is an arbitrary function. Basic statistical quantities, such as averages and variances, can be obtained with this integral by the corresponding function choices. There are a number of approaches with different approximation techniques that can provide a stochastic quadrature rule:

Monte Carlo methods: $w_k = N^{-1}$ and ξ_k are randomly sampled from the probability distribution. Under mild assumptions on g, the approximation error of the integral converges at an $\mathcal{O}(1/\sqrt{N})$ rate. This is usually too slow for our purposes as we would like to use as few points as possible in order to reduce the size of the problem; it is, however, independent of the dimension of ξ.

[2] Note that, despite the similarity in notation, this is not a stochastic differential equation because the random parameters are not random processes, i.e., are constant.

Quasi-Monte Carlo methods: replace random sampling by deterministic low-discrepancy sequences that sample the outcome space in a more even manner [31]. For certain problems, a rate of convergence of $\mathcal{O}\left(1/N\right)$ (faster than Monte Carlo) is observed; several explanations have been advanced in the literature [32][33][34].

Cubature techniques: are high-dimensional analogues of regular one-dimensional quadrature rules that look for exact approximation of certain classes of functions. A compilation of cubature rules can be found in [35] and [36]. In [23], a "Hyper-Pseudospectral" cubature is developed specifically for robust optimal control problems.

Generalized Polynomial Chaos (gPC) methods: rely on the expansion of the random inputs and outputs on an orthogonal polynomial basis, thus allowing for recovery of some statistical quantities directly from the expansion coefficients [37] [38]. The *stochastic collocation* of gPC variant is the form we're interested in, as it characterizes the solution as an interpolant at the set of nodes $\{\xi_k\}$. It is efficient for problems with low-dimensional random variables [39], but the number of required nodes grows quickly with the dimension even when using higher-efficiency sparse grids [40]

In this work, we don't need a stochastic quadrature rule because the uncertainty information is already presented in discrete scenarios (that we weigh equally) from EPS forecasts; however, integrating other sources of uncertainty in future work may require the usage of a stochastic quadrature rule.

B. The trajectory ensemble

Given a quadrature rule and a given number of samples N, we define the *trajectory ensemble* associated to a control law $\mathbf{u}_L$ and a stopping criterion s as the set of trajectories $\{(t_{f,k}, \mathbf{x}_k, \mathbf{u}_k)\}$ with $k \in \{1, \dots, N\}$ such that the trajectory k is generated by the control and stopping rules with $\xi = \xi_k$ and the stopping criterion is met at $t = t_{f,k}$, i.e.

$$\frac{d}{dt}\mathbf{x}_k(t) = f(\mathbf{x}_k(t), \mathbf{u}_L(t, \mathbf{x}_k(t)), \xi_k(\omega), t)$$
$$s(t, \mathbf{x}_k(t)) < 0, \; \forall t < t_f$$
$$s(t_{f,k}, \mathbf{x}_k(t_{f,k})) = 0$$

We consider a virtual dynamical system whose state vector contains the state vectors of all the trajectories in the trajectory ensemble, which evolve each according to the dynamics in each scenario (i.e. for each value ξ_k of ξ). Using this trajectory ensemble, the robust optimal control problem can be reformulated as a large deterministic OCP, where the N trajectories are considered simultaneously.

C. The "state-tracking" ROCP

In previous literature employing this approach (see [23], [24], [25] or [26]), the control law is considered as only dependant on time $\mathbf{u}(\omega, t) = \mathbf{u}_L(t)$, thus leading to an "open-loop" control scheme. This "open-loop" formulation is, however, not a practical scheme for general optimal control problems. In some problems, the dynamic system could be unstable and the trajectories would diverge towards undesirable regions of the state space; in other (as the one we face in commercial aircraft trajectory optimization), we need to apply final conditions and/or have a unique path for some of the states.

Instead of looking for an optimal *control*, then, we will look for an optimal *guidance*; we designate some of the states as "tracked" states and we replace the unique controls $\mathbf{u}_L(t)$ that are applied identically in all scenarios by scenario-specific controls $\mathbf{u}_k(t)$ that ensure that the tracked states follow a unique trajectory for all likely values of the random variables (as long as it is feasible within the dynamics and constraints of the problem). In a real-world implementation, where the realized uncertainty would generally be a mix of the discrete scenarios that we are considering, we assume that the controls can be computed by existing controllers in order to track the calculated trajectory. In our context, the controls can be computed by the autopilot in order for the aircraft to follow a route at the calculated airspeeds and altitudes.

Let $\{i_1, \dots, i_q\}$ be the indexes of the states we are interested in tracking (e.g. if we are tracking x_2 and x_5, $i_1 = 2$ and $i_2 = 5$). Let e_i be the column vector that has a 1 at the position i; we define the matrix $E \in \mathbb{R}^{q \times n}$ as

$$E = \begin{bmatrix} e_{i_1}^T \\ \vdots \\ e_{i_q}^T \end{bmatrix}$$

We define the problem as:

$$\min J = \mathbb{E}\left[\phi(\mathbf{x_f}) + \int_{t_0}^{t_f} \mathcal{L}(\mathbf{x}(\omega, t), u(\omega, t), t)dt\right]$$

subject to the differential equations (III-B), the state-tracking condition:

$$E(\mathbf{x}(\omega_1, t) - \mathbf{x}(\omega_2, t)) = 0, \; \forall t, \; \forall \; \omega_1, \omega_2 \in \Omega$$

the stopping rule $s(t, \mathbf{x}(t)) = t - t_f$ and the boundary conditions:

$$\mathbf{x}(\omega, t_0) = \mathbf{x}_0$$

$$\mathbb{E}\left[\psi(\mathbf{x}(\omega, t_f))\right] = 0$$

where ψ is the function that represents the final conditions. As emphasized earlier, the controls are no longer unique as in the open-loop problem; they depend on the realization of $\xi(\omega)$. Here, the final conditions that depend only on the tracked states and the final time can be imposed exactly and not only in average. The corresponding discretization is

$$\min J = \sum_{k=1}^{N} w_k \left[\phi(\mathbf{x}_k(t_f)) + \int_{t_0}^{t_f} \mathcal{L}(\mathbf{x}_k(t), \mathbf{u}_k(t), t)dt\right]$$

subject to:

$$\dot{\mathbf{x}}_k = f(\mathbf{x}_k(t), \mathbf{u}_k(t), \xi_k, t),\ k \in \{1, \dots, N\}$$
$$\mathbf{x}_k(t_0) = \mathbf{x}_0,\ k \in \{1, \dots, N\}$$
$$E(\mathbf{x}_k(t) - \mathbf{x}_1(t)) = 0,\ \forall k \in \{2, \dots, N\}$$
$$\sum_{k=1}^{N} w_k \psi(\mathbf{x}_k(t_f)) = 0$$

IV. Application

We consider an aircraft that flies at constant airspeed v_{TAS} and altitude h[3] following a horizontal route in a static wind field in an ellipsoidal Earth. We consider the standard 3-DoF point-mass model of aircraft used widely in ATM studies [41] simplified according to our constant airspeed and altitude assumptions.

We cannot apply the methodology described in Section III directly to the standard dynamical system that arises from this model. We would like to track the position variables, latitude and longitude, in order to generate a unique flight plan; however, under such a scheme, the trajectory in each scenario would have to adjust the airspeed in order to absorb the entirety of the uncertainty in wind speed. This is neither convenient not practical, as it would force the aircraft to choose inefficient airspeeds for some of the scenarios and limit the average airspeed to an undesirably low value in order to have margin for the most unfavourable scenarios.

Instead, a conventional flight plan specifies a route and any velocity or airspeed changes[4] are associated to specific waypoints, not points in time. Therefore, we will reformulate the dynamical system so that the independent variable is not time, but distance flown along the trajectory, which we will denote by $r \in [0, r_f]$. We will parametrize the route by r: denoting latitude by ϕ and longitude by λ, we define a route as a smooth mapping $r \mapsto (\phi(r), \lambda(r))$ that satisfies condition 2, needed to ensure that r does indeed correspond to undistorted (ground) distance flown along the route:

$$\left((R_N + h)\frac{d\phi}{dr}\right)^2 + \left((R_M + h)\cos\phi\frac{d\lambda}{dr}\right)^2 = 1 \qquad (2)$$

where the radii of curvature of ellipsoid meridian and prime vertical are denoted by R_M and R_N respectively. We can now track the position with respect to distance r instead of time t.

For a given route and wind field, we can define a function $t(r)$ which represents the time at which the aircraft flies through a position r in the route. Relying now on an EPS composed by N members, we will consider N scenarios and N trajectories such that each trajectory corresponds to the forecasted wind field for each member. We define $t_i(r)$ as the time at which the aircraft flies through the position r in the route if the wind field corresponds to the forecast for member number i. We will consider each member as equally likely and define average in this sense (as an empirical average).

We now look to find routes that minimize a weighted sum of average flight time and flight time dispersion. By changing the relative weight of parameter p, we can obtain routes that are more efficient on average or routes that are more predictable. We will denote this parameter by p, where $p = 0$ means that we look for maximum average efficiency and higher values of p put more weight on dispersion, which we will define as the difference between the earliest and the latest arrival time. As we're flying at constant airspeed and environmental conditions (except for wind), fuel burn is only dependent on flight time.

We proceed to model this problem under the framework described in Section III. For computational efficiency, we don't include copies of the latitude and longitude states for each scenario, as they're all equated by the state-tracking constraints. We will introduce the course χ_G and the member-specific headings χ_i and groundspeeds $v_{G,i}$ in order to build the system of differential-algebraic equations given by equations 4 and 5 that describes the dynamics of the problem. We complete the formulation of the optimal control problem by adding the cost functional 3 and the boundary conditions 6 - 9 (note that $w_{x,i}$ denotes the Eastbound component of the wind for member i while $w_{y,i}$ denotes the Northbound component).

Optimal Control Problem 1: minimize

$$\min J = \frac{1}{N}\sum_{i=1}^{N} t_i(r_f) + p \cdot (t_{f,\max} - t_{f,\min}) \qquad (3)$$

subject to the dynamical constraints:

$$\frac{d}{dr}\begin{bmatrix}\phi \\ \lambda \\ t_1 \\ \vdots \\ t_N\end{bmatrix} = \begin{bmatrix}\dfrac{\cos(\chi_G)}{R_N + h} \\ \dfrac{\sin(\chi_G)}{(R_M + h)\cos\phi} \\ 1/v_{\text{GS},1} \\ \vdots \\ 1/v_{\text{GS},N}\end{bmatrix} \qquad (4)$$

$$\begin{bmatrix} v_{\text{GS},1}\cos(\chi_G) \\ \vdots \\ v_{\text{GS},N}\cos(\chi_G) \\ v_{\text{GS},1}\sin(\chi_G) \\ \vdots \\ v_{\text{GS},N}\sin(\chi_G)\end{bmatrix} = \begin{bmatrix} v_{\text{TAS}}\cos(\chi_i) + w_{y,1}(\phi,\lambda) \\ \vdots \\ v_{\text{TAS}}\cos(\chi_i) + w_{y,N}(\phi,\lambda) \\ v_{\text{TAS}}\sin(\chi_i) + w_{x,1}(\phi,\lambda) \\ \vdots \\ v_{\text{TAS}}\sin(\chi_i) + w_{x,N}(\phi,\lambda)\end{bmatrix} \qquad (5)$$

and the boundary conditions:

$$(\phi(0), \lambda(0)) = (\phi_0, \lambda_0) \qquad (6)$$
$$(\phi(r_f), \lambda(r_f)) = (\phi_f, \lambda_f) \qquad (7)$$
$$t_i(0) = 0\ \forall i \in \{1, \dots, N\} \qquad (8)$$
$$t_{f,\min} \le t_i(r_f) \le t_{f,\max}\ \forall i \in \{1, \dots, N\} \qquad (9)$$

[3]While, as we noted in the introduction, these are restrictive assumptions for a problem solved with direct methods, they are comparable to most of the published routing algorithms and not indispensable for our methodology; we only choose them for simplicity

[4]Again, the presented formulation does not feature airspeed or altitude changes but it will include them in the future

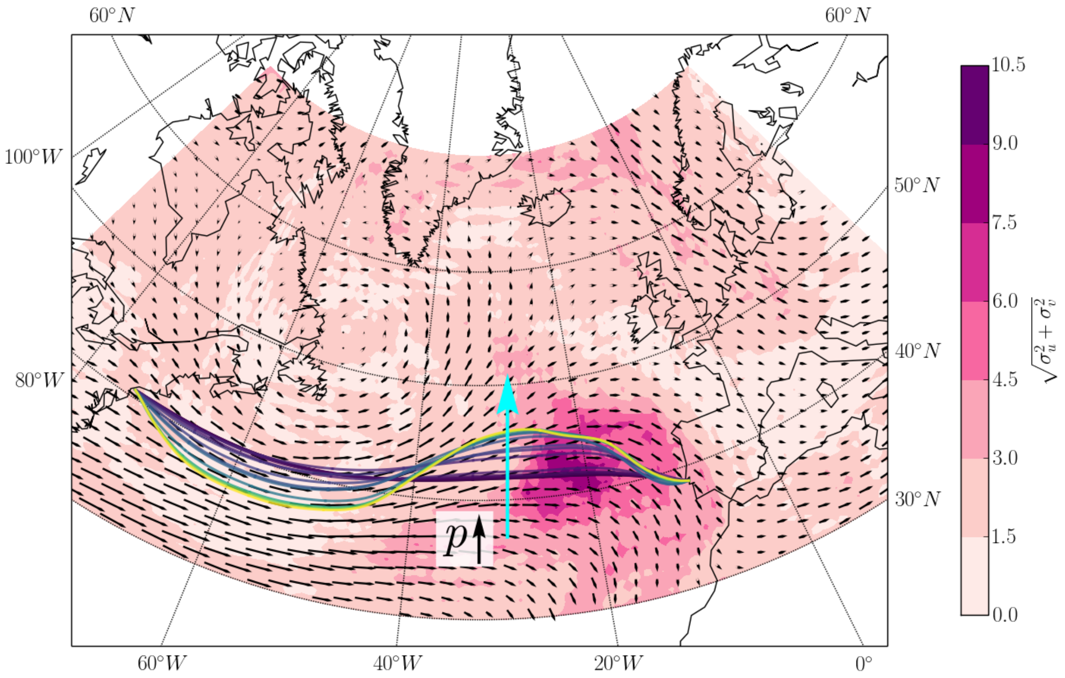


Figure 3. Optimal trajectories from NY to Lisbon, for values of p from 0 to 50. Higher brightness in the trajectory color indicates higher values of p. We also color regions of higher uncertainty, which we have defined as $\sqrt{\sigma_u^2+\sigma_v^2}$, with σ_u being the standard deviation of the u component of wind across different members and σ_v analogous for the v-component.

In order to solve this problem, we rely on an initialization and wind approximation procedure described in [10]. We solve them with direct methods, discretizing the trajectory with a trapezoidal scheme and then solving the resulting nonlinear optimization problem with NLP software (see, for example, [18])

V. Case Study

A. Description and Statement

We consider an BADA3 A330 Aircraft model flying from the vertical of New York to the vertical of Lisbon at flight level FL380 and at Mach 0.82. Initial mass has been considered to be 200 tons. We use a forecast for a pressure of 200 hPa 6 hours in advance for the 20th of January, 2016 from the PEARP ensemble. This is an ensemble elaborated by Météo France with 35 members that is hosted among others in the TIGGE dataset [42] by the European Center for Medium-Range Weather Forecasts (ECMWF)[5]. We rely on the Pyomo library as NLP interface [43] and IPOPT [44] as NLP solver.

B. Results and discussion

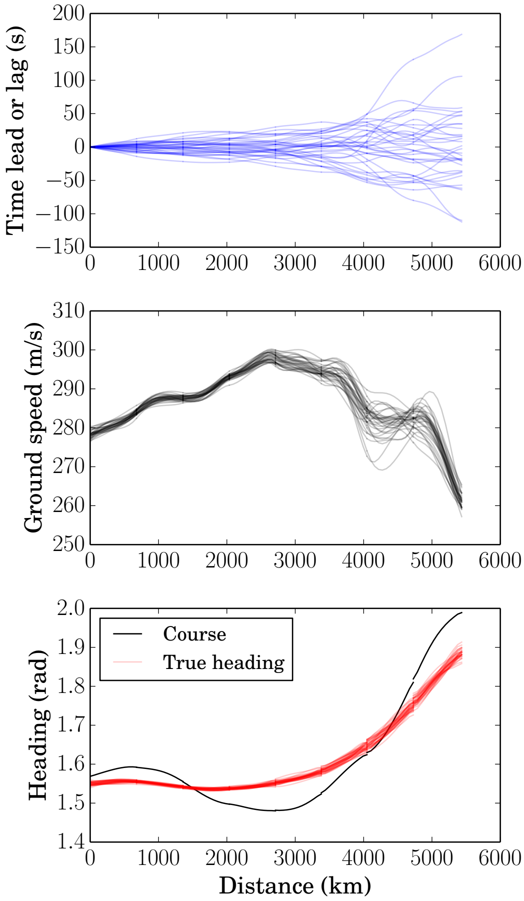
Figure 3 displays the geographical routes for different values of p. It can be seen that routes computed with higher p tend to avoid the high uncertainty zone in the Atlantic in order to increase predictability, at the cost of taking a more indirect route that is longer on average.

[5]http://apps.ecmwf.int/datasets/

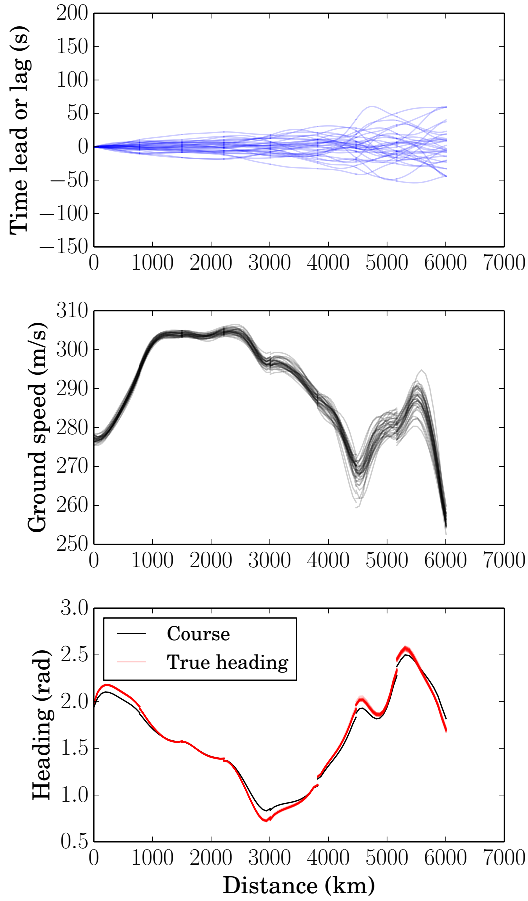
Figure 4.a shows the evolution of the state and control variables along the average-min-fuel-optimal trajectory (corresponding to $p = 0$, the black line in 3). It can be seen that the spread in the ensemble times, ensemble headings, and ensemble ground speeds increases markedly when the aircraft crosses the area of high uncertainty (that can be seen in Figure 3). Figure 4.b shows the evolution of the state and control variables along the average-most predictable trajectory (corresponding to $p = 50$, the yellow line in 3). It can be seen that the spread in times and ground speeds are comparatively lower than in the previous case. In particular, we can see how the ensemble ground speeds and headings present much less dispersion.

Finally, Figure 5 shows the Pareto frontier of the problem, obtained by solving problems with different penalties p (from $p = 0$ to $p = 50$). For the minimum average fuel case ($p = 0$), the time dispersion at the final fix is above 4.5 minutes, whereas for the maximum predictability case ($p = 50$)[6], the time dispersion at the final fix is slightly above 1.5 minutes. In other words, around three minutes reduction in *time uncertainty* could be achieved by flying the most predictable trajectory ($p = 50$). This would be however at roughly 2500 kg of extra fuel burnt. For example, the increase in predictability of about 1.25 minutes would result in 500 kg of fuel consumption. In any case, the Pareto frontier shows different possible solutions with trade-offs dispersion-consumption.

[6]Problems for greater p values have been solved, but the Pareto frontier becomes very flat



(a) min. fuel (p=0)



(b) max. predictability (p=50)

Figure 4. State-space evolution of the variables in the case $p = 0$ and $p = 50$. Time leads and lags are defined with respect to the average trajectory.

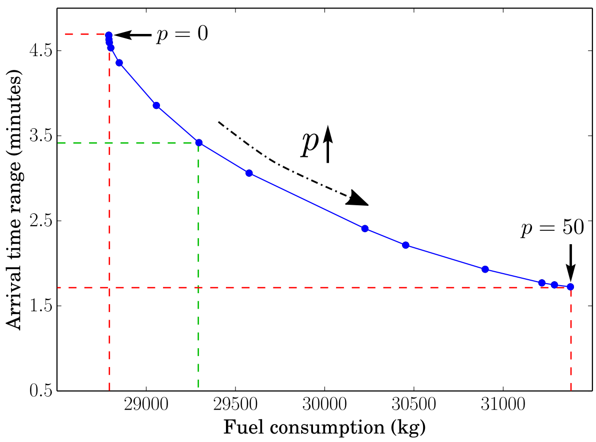


Figure 5. Pareto frontier of the problem.

VI. Conclusions and Future Work

We have introduced a robust optimal control methodology for computing robust optimal routes based on Ensemble Prediction Systems and demonstrated its utility in studying trade-offs between efficient and predictable routes. We can conclude that by using this method, uncertainty (in this case due to wind) can not only be quantified, but also reduced by proposing alternative trajectories. We expect that the methodological choices made will allow us to extend this methodology into a more general and useful robust flight planning framework for balancing average efficiency with predictability. Indeed, we are already working on adding variable airspeed profiles to this methodology, with some promising early results.

VII. Acknowledgments

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