Fuel Saving by Gradual Climb Procedure

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***Abstract*—This paper proposes a new climb procedure called a gradual climb procedure. When aircraft climb to the cruise altitude or change the flight level, maximum climb thrust is usually applied. However, this maximum cruise thrust leads to a higher fuel consumption compared to the cruise thrust, which results in non fuel optimal higher climb rate. An optimal rate of climb which minimizes the fuel consumption exists, but aircraft usually opt for a faster rate of climb. This paper focuses on the part of the climb phase just prior to the cruise altitude, and clarifies the relationship between fuel consumption and the rate of climb via numerical simulations. The author also proposes a practical fuel-saving climb procedure considering actual air traffic control constraints and pilot operation. The expected fuel saving is of the order of 50 lb, which corresponds to 0.1 % of total cruise fuel consumption. However, the proposed procedure will be applicable to almost all aircraft and flights worldwide, so the cumulative effect will be significant. In addition, the negative effects to air traffic control and a pilot are minor, so the proposed gradual climb procedure can be applied in the near future operation.**

Keywords-component; trajectory optimization, climb trajectory, step-up climb, top of climb

NOMENCLATURES

x : longitudinal distance [m]

z : altitude [m]

v : true air speed [m/s]

γ : path angle [rad]

m : weight [kg]

T : thrust [N]

M : Mach number

L : lift [N]

D : drag [N]

g : acceleration of the gravity [m/s^2]

v_z : target rate of climb (vertical speed) [m/s]

I. INTRODUCTION

Fuel saving is a keyword in air traffic management, and there are many researches which consider the issue from various aspects. In the ATM field, most researches relate to multiple aircraft to minimize fuel consumption, because aircraft can rarely follow their optimal trajectory due to the aircraft interference of others. For example, optimal operation of multiple aircraft are considered in arrival manger (AMAN) [1][2] and airport surface management[3][4].

On the other hand, CCO (Continuous Climb Operation) and CDO (Continuous Descent Operation) have been proposed for a single flight operation[5][6]. According to these concepts, an aircraft should fly near its optimal profile and stay high as long as possible, minimizing level segment during descent or ascent. However, these operations are also affected by airspace congestion, and are therefore often applied in non-congested times only. The potential benefit of CDO and CCO is significant [7][8][9], so various researches have been conducted on the operation of CDO/CCO in congested airspace [10][11].

The problem of single aircraft trajectory optimization is an old one mainly studied in 1970-80s. Its objective function is set to minimize fuel consumption or flight time, or a mixture of those. The trajectory optimization was described as an optimal control problem, and was solved by maximum principle theoretically [12][13][14]. The computational burden is quite little and easy to implement onboard, so recent aircraft calculate their climb/descent profile using this theory. There are also some recent extended studies, one of which proposes an improvement in calculation with limited information[15].

As for a single trajectory optimization of previous studies, the whole optimal trajectory is categorized into three phases; climb phase, cruise phase, and descent phase. However, the transition between two phases cannot be easily optimized theoretically by the maximum principle. Since the impact of the trajectory optimization is not so big in the transition phase, this transition phase has been overlooked by other researchers and practitioners. However, if the fuel saving is possible and not negligible, it is worth optimizing the climb profile during this transition phase. There are some recent studies calculating the optimal trajectory and obtain the possible fuel saving[16]. However, such a pure "optimal" trajectory often cannot be flown by current aircraft and is therefore difficult to implement the near future.

This paper proposes a new practical climb procedure to reduce fuel burn by changing the climb profile. In the current climb operations, maximum climb thrust (MCT) is usually used (de-rated thrust is also used actually, which will be explained later). The past works[12][13][14][15] also assume the MCT during climb phase. The MCT climb is simple and currently widely in use, but MCT climb is not optimal in terms of fuel consumption. Therefore, this paper clarifies the following points: 1) possible fuel saving by changing the climb profile, 2) proposition of an optimized climb profile for practical use.

This paper is organized as follows: Sec II provides an overview of a typical operation of two climb procedures (climb operation to top of climb (TOC) and step-up climb operation). Sec III explains the simulation model and the optimization method applied in this research. Sec IV shows simulation results for both step-up climb operation and climb to TOC operation under the current operation and the proposed operation in various conditions. Sec V summarizes this paper.

II. Research Scope and Climb Operations

A. *Research scope*

First of all, the scope of this paper is clarified. As described in the Introduction, this paper proposes a new procedure for the transition phase between climb and cruise, i.e. climb phase near TOC. On the other hand, the "optimal" trajectory cannot be flown by the current aircraft due to ATC constraints and aircraft capability. This paper also considers these aspects, and proposes a practical way to fly on a sub-optimal trajectory providing a better fuel burn performance than the current one. Regarding the optimization, due to difficulties associated with accurate theoretical description of the transition state, here a numerical analysis is applied.

During the climb, MCT (the maximum thrust) is widely used in the current operation. Therefore, the baseline scenario is assumed that the aircraft climbs to a cruise altitude with MCT. During the cruise, it is well-known that the optimal cruise altitude gradually increases as the aircraft weight decreases[13]. However, the rate of climb (ROC) becomes very small to track the optimal trajectory, so the aircraft cannot fly on the optimal trajectory due to ATC requirements. Instead, the aircraft often applies a step climb. Although the optimal altitude changes gradually, ATC usually assigns altitude to each aircraft every 1000 ft, so the step climb allows the aircraft to fly on sub-optimal altitude by changing the altitude by 1000 ft. The optimal timing of the step climb exists, and some researchers consider the optimal step climb points including wind effects[17][18], which are also a well-discussed problem. However, ROC during step climb has not been discussed by other researchers. This step climb can also be handled as the transition from climb to cruise, and there is room for improvement by changing the climb profile during step climb. Therefore, this paper considers two cases: climb profile near TOC and climb profile during step climb. The flight trajectories at low altitude climb and the cruise phase flight are assumed to be the same as the current operation, and the proposed climb profile does not affect other phases.

B. *Current aircraft capability and possible sub-optimal climb profile*

This subsection considers the aircraft operational aspects. The current aircraft may not be able to follow the optimal flight profile, because the aircraft should select a certain FMC (Flight Management Computer) mode. FMC can provide the target path and speed profile by considering various aircraft information such as route structure and aircraft weight. The target path and speed profile is calculated by the maximum principle explained in the previous section. Since the lateral route is usually decided by a flight chart, longitudinal and vertical motion are optimized by FMC. Here, speed and ROC are controlled by two control devices (pitch angle and engine thrust). Each control variable (speed and ROC) can be controlled by either control device, so there are two options. The first option is to keep the target speed by controlling the pitch angle and setting the thrust constant (usually MCT during climb). ROC is automatically determined by the thrust setting. The second option is to keep the target ROC by controlling the pitch angle and using the engine thrust to control speed. The first option is usually used during climb operation as VNAV SPD mode (Boeing) or flight level change (FLCH/LVL CHG) mode. During the climb, the thrust is set to MCT, so only the target speed profile is required. The second option can be flown using VNAV PATH mode (Boeing) or vertical speed (V/S) mode. Since both speed and ROC are controlled, both the target speed profile and the target ROC profile are required. Both options are summarized in Table 1.

TABLE I. Two FMC mode options during climb.

	Option 1	Option 2
Pitch angle	Speed control	ROC control
Thrust	Set to MCT	Speed control
Target speed profile	Required (calculated in FMC)	Required (calculated in FMC)
Target ROC profile	Not required	Required

In reality, Option 2 is rarely used during the climb, so only the speed profile is given during this stage. In order to change the climb profile, it is recommended to use Option 2 but the target ROC should be given. If the pilot wants to use Option 2 during the climb, it can only be done with a manual target ROC setting.

C. *Current climb operation*

This research focuses on two different climb operations: the first operation is the climb operation to TOC, and the second operation is a step-up climb operation.

First, the climb operation to TOC is considered. When an aircraft climbs to TOC, it usually uses VNAV SPD mode

(Boeing), i.e. Option 1. FMC calculates the optimal speed during climb. During the climb, the thrust is set to MCT, so the aircraft climbs at the maximum possible ROC while maintaining its target speed.

To extend the engine cycle, the derated thrust setting is often used. The derated thrust sets the climb thrust smaller than the MCT by up to 30 %, which depends on the aircraft type. During the ascent, if the ROC is set smaller than the optimal one, the aircraft consumes additional fuel. However, the engine life span can be extended, and such a benefit is usually larger than the additional fuel consumption. However, the rate of derated thrust becomes smaller with higher altitude, and most aircraft do not activate the derated thrust above 30000 ft altitude, which means that MCT is applied when climbing above 30000 ft. This research considers the flight above 30000 ft only, so the derated thrust is not considered.

As describe above, the aircraft uses the MCT (or derated thrust) to reach TOC, but the engine is designed to be most efficient at a certain cruising thrust, not at MCT. When an aircraft climbs, thrust larger than cruising thrust is used, but the fuel consumption can be theoretically reduced by setting the thrust near the cruising thrust during the climb, so ROC decreases. On the other hand, there is an optimal cruising altitude once the aircraft type and weight are determined. Too small a ROC means that the aircraft has to fly on the non-optimal altitude, which consumes additional fuel. Therefore, in regard of these two factors, it is safe to say that there exists a fuel-optimal ROC for each altitude.

Second, the step-up climb is considered. The optimal flight altitude depends on the aircraft weight. With thime the aircraft becomes lighter due to the burnt fuel and so the optimal flight altitude becomes higher as the cruise proceeds. At a certain timing, the pilot can place a request to ATC to fly on a higher altitude. If clearance is obtained, the pilot usually sets the designated altitude in MCP and pushes the altitude nob, then the aircraft starts climbing with MCT. For reasons analogous to the ones stated earlier in the climb to TOC explanations, there should be an optimal ROC.

D. Possible sub-optimal climb profile

To account for the implementation of the proposed climb procedure in the real world, the operational aspects should be considered. From ATC perspective, the aircraft should fly on a certain cruise altitude and should not apply too low ROC. From aircraft control perspective, the target ROC should be set, i.e. the climb by Option 2.

First, the comparison of the various climb profiles is shown in Fig. 1 under a certain environment (a certain aircraft type and a certain weight). This is just an example, and the calculation method will be shown in the next section. If MCT is applied, the aircraft can climb with the highest possible ROC. On the other hand, the optimal trajectory shows a fast climb at the beginning, but as the ROC gradually reduces, a very low ROC (optimal cruise ROC) is observed. However, as mentioned before, such a ROC is not acceptable from ATC perspective. The proposed climb profile is within the allowable window defined by these two profiles. First the aircraft climbs with MCT, but at a certain point (33000 ft in this example) small ROC (1000 ft/min in this example) is set. After reaching the cruise altitude, the aircraft maintains this altitude. By setting a small ROC at 33000 ft, TOC is shifted by about 10 NM compared to MCT climb. The proposed profile is possible for the current aircraft, and will be acceptable for ATC. (The details of the possible negative impacts will be discussed in Sec. IV (D).) The questions are how much fuel can be saved by the proposed procedure, and what ROC should be set during the climb. These questions are investigated in the following sections.

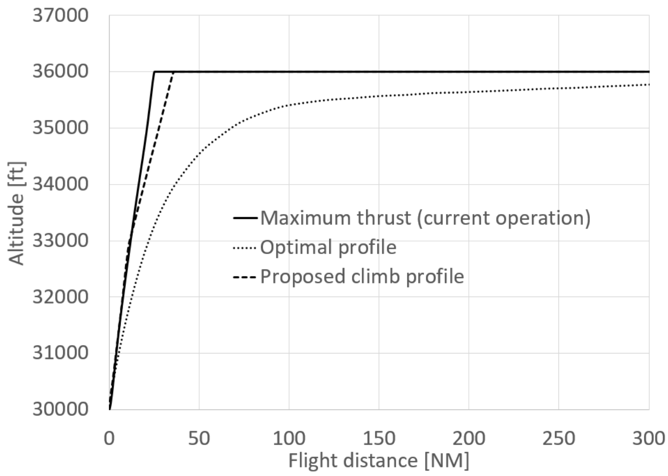

Figure 1. Comparison of climb profiles.

III. Simulation and Optimization

A. Simulation model

To investigate the effect of ROC on fuel consumption, numerical simulations are conducted. Here, the details of the simulation model are described. It is assumed that an aircraft flies on a straight track, and only the longitudinal and vertical dynamics are considered. The aircraft dynamics are given by the following point mass model with five state variables $\mathbf{x} = [x, z, v, \gamma, m]^T$ and two control variables $\mathbf{u} = [\dot{\gamma}, T_{ratio}]^T$. Here, null wind is assumed.

$$\dot{x} = v \cos \gamma \tag{1}$$

$$\dot{z} = v \sin \gamma \tag{2}$$

$$\dot{v} = \frac{T - D}{m} - g \sin \gamma \tag{3}$$

$$\dot{\gamma} = \frac{L}{mv} - \frac{g \cos \gamma}{v} \tag{4}$$

$$\dot{m} = f(M, T) \tag{5}$$

$$T = T_{ratio}(T_{\max} - T_{\min}) + T_{\min} \quad T_{ratio} = [0 \quad 1] \tag{6}$$

where $f(M, T)$ is the fuel consumption. Since $\dot{\gamma}$ is a control parameter, L is automatically determined. D, $T_{\max}$, $T_{\min}$, and $f(M, T)$ are calculated by the Base of Aircraft Data (BADA) 4 model[19]. Here, two aircraft types (B777-300 and A330-300) are used in the simulation. The operational constraints are also set based on BADA model.

B. Optimization method

To determine the optimal flight path, a trajectory optimization problem is formulated. Here, two operations are assumed: 1) climb with MCT, 2) climb at constant ROC. To account for both operations, a three-stage optimization problem is formulated. For the step-up climb operation, the first stage is for a level flight before the step-up climb, and the second stage is for a climbing operation, and third stage is for a level flight after the step-up climb. The following initial and terminal conditions and constraints are set in each stage.

1st stage constraints

$x_0^1 = 0$

$z_0^1 =$ initial altitude (z_0)

$v_0^1 =$ initial TAS calculated from initial Mach

$\gamma_0^1 = 0$

$m_0^1 =$ initial weight

$z_f^1 = z_0$

$\dot{\gamma}^1 = 0$

2nd stage constraints

$z_0^2 = z_0$

$z_f^2 =$ final altitude (z_f)

$$\left.\begin{aligned} t_f^2 - t_0^2 &= \frac{z_f - z_0}{v_z} + \alpha \\ \dot{z} &\le v_z \end{aligned}\right\} \ldots\text{(a)}$$

$T_{ratio}^2 = 1 \ldots$(b)

Mach number is between cruise Mach number – 0.005 and cruise Mach number + 0.005.

3rd stage constraints

$z_0^3 = z_f$

$\dot{\gamma}^3 = 0$

$x_f^3 =$ final distance

$z_f^3 = z_f$

$v_f^3 =$ final TAS calculated from final Mach

where the superscript indicates the stage number, the subscript 0 indicates the initial condition in each stage, and the subscript f indicates the final condition in each stage. Either constraint (a) or (b) in 2nd stage are used depending on the problem, i.e. climb with MCT or constant ROC. The constraints (a) are used for 2) climb with constant ROC, and the constraint (b) is used for 1) climb with MCT. α is a parameter to account for the aircraft movement. If α is set to 0, the aircraft has to climb at the maximum ROC (v_z) during the 2nd stage. However, at the beginning and the end of the stage, the ROC should be 0, so the solution becomes infeasible. Therefore, α should be as low as possible if the solution is feasible, and here it is set by trial and error.

The objective function is usually set to include both the fuel consumption and flight time, and its weight is given as a cost index (CI). Since CI is given in the unit of 100 lb/hour (Boeing), the following objective function to be minimized is set.

$$J = \frac{100}{3600} CI \cdot t_f^3 + 0.453592 \int_0^{t_f^3} -\dot{m} dt \tag{6}$$

The unit of J is lb. 1 s flight time corresponds to 0.0278 CI lb. Therefore, if CI is set to 100, 1 s flight corresponds to 2.78 lb fuel consumption. To solve the optimization problem, the pseudospectral discretization method is applied. Using this method, the continuous trajectory optimization problem can be formulated as a nonlinear programming (NLP) problem. In this study, IPOPT is used as a NLP solver, and PSOpt (optimal control solver software) is used for software implementation [20]. The nodes in each stage are also set by trial and error, with 10-30 nodes being set in each stage.

IV. SIMULATION RESULTS

A. Optimal flight profile of the step-up climb procedure

To account for a step-up climb procedure, three initial and terminal conditions are assumed as shown in Table 2. In all scenarios, the aircraft changes altitude by 2000 ft. Once the initial weight is determined, the optimal vertical flight profile is determined, and the appropriate initial and terminal altitudes are set in each scenario.

Figs. 2 and 3 show the optimal flight profile in scenario 1 and 3. (Neither constraints (a) nor (b) are applied.) Both figures show a similar trend; the mach number is almost constant during the simulation period, and the aircraft gradually climbs from the initial altitude to the terminal altitude. The same trend is observed in scenario 2 as well. Since the optimal altitude gradually changes theoretically, these results show that the calculation is feasible. In all cases, the ROC during the climb is about 10 ft/min. This ROC is too low and this operation is never performed in practice. Instead, step climb is often used. During a step climb, MCT is usually applied. Therefore, the optimal step climb profile is obtained via optimization calculation. In the step climb calculation, the constraint (b) is applied. Fig. 4 shows the optimal step climb flight profile with MCT in scenario 1. The altitude change happens around 1000 NM point, which agree with the optimal flight profile shown in Fig. 2, because the aircraft passes around 1000 NM point at 37000 ft. Since MCT is applied, about 1500 ft/min ROC is observed. The interesting point is the difference of the objective function. The difference of the objective function between the optimal profile (Fig. 2) and the optimal step climb profile with MCT (Fig. 4) is 89.3 lb. The objective function consists of fuel consumption and flight time, but the difference of flight time is less than 1 s and therefore negligible. The difference of the objective function is almost equal to the difference of the fuel consumption. The same trend is observed in all scenarios (1-3).

TABLE II. INITIAL AND TERMINAL CONDITIONS FOR STEP-UP CLIMB PROCEDURE.

Scenario	1	2	3
Initial weight [lb]	540,000	610,000	440,000
Initial altitude [ft]	36,000	33,000	37,000
Terminal altitude [ft]	38,000	35,000	39,000
Initial/Terminal Mach	0.83	0.83	0.80
Flight distance [NM]	2,000	2,000	2,000
Aircraft type	B773	B773	A333
CI	100	100	100

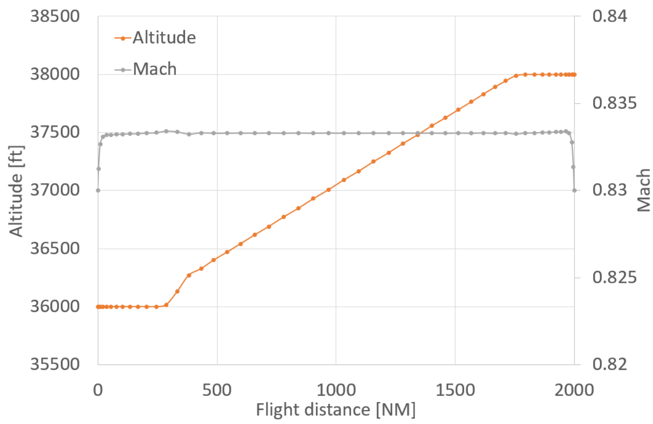

Figure 2. Optimal flight profile in scenario 1. (J = 103,530.5lb, fuel used = 61,687.2lb, flight time = 15,063.6s)

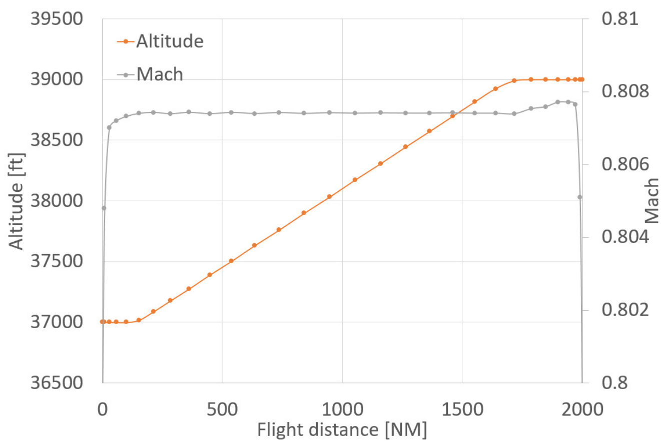

Figure 3. Optimal flight profile in scenario 3 (J = 94,523.3lb, fuel used = 51,334.6lb, flight time = 15,547.9s)

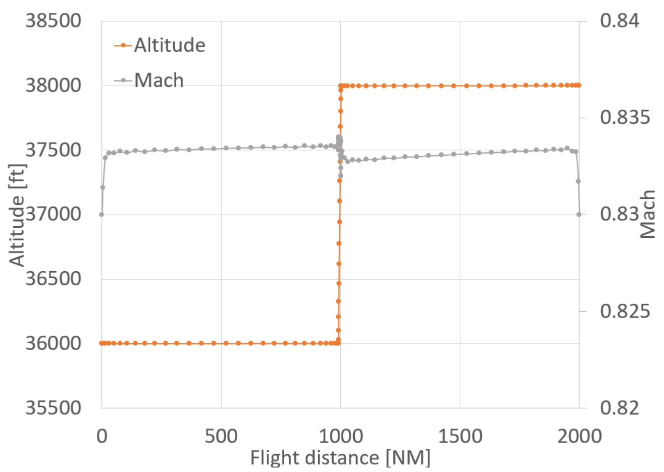

Figure 4. Flight profile in scenario 1 with the maximum climb thrust. (J = 103,618.0lb, fuel used = 61778.4lb, flight time = 15,062.2s)

TABLE III. FUEL CONSUMPTION IN EACH ROC AND EACH SCENARIO.

ROC [ft/min]	Objective function [lb]		
	Scenario 1	Scenario 2	Scenario 3
Optimal	103530.5	111172.3	94523.3
50	103563.1	111208.4	94553.2
500	103590.5	111235.9	94580.3
1000	103604.5	111248.4	94593.0
MCT	103618.0	111265.0	94594.0

However, as mentioned before, climb at 10 ft/min is not realistic, so higher ROC is required in practice. Even basic FMCs provide a climb function at a fixed ROC (V/S mode) with a minimum value of 50 ft/min, and it can be set every 50 ft/min up to 1000 ft/min. This time, 50 ft/min, 500 ft/min, 1000 ft/min are chosen for the calculation. The values of objective function are summarized in each scenario and ROC, and shown in Table 3. The climb profile in each ROC in scenario 1 is shown in Fig. 5.

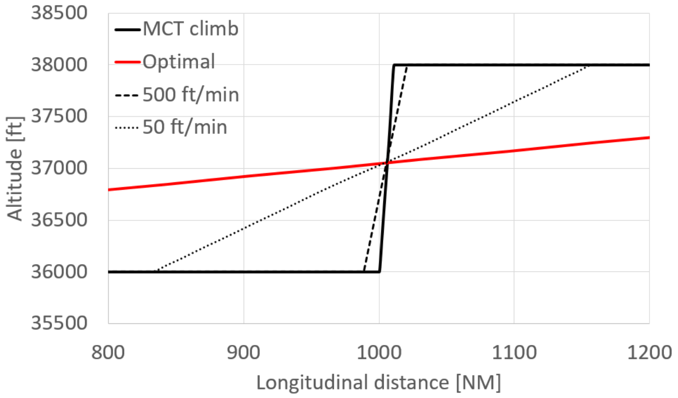

Figure 5. Climb profile between various climb patterns.

As for the climb profile as shown in Fig. 5, MCT climb achieves the highest ROC while the optimal climb profile shows the smallest ROC. All climb profiles cross at a specific point, which indicates the feasibility of the calculation. Note that the climb profiles between 500 ft/min climb and MCT climb do not differ greatly. MCT climb completes the climb for about 10 NM and 80 s, while 500 ft/min climb completes the climb for 30 NM and 240 s.

As seen in Table 3, less fuel is used with slower ROC, and the overall trend is similar for all scenarios. By using 50 ft/min climb, 40-60 lb fuel can be saved compared to MCT step climb. However, if 50 ft/min climb is applied, it takes 40 minutes to complete 2000 ft altitude change, and all altitude ranges (in this case three flight levels) have to be blocked. Such multiple altitude blocking is possible in the current ATC operation, but of course it reduces the capacity of airspace. However, it might be worth trying 50 ft/min climb if the airspace is not crowded. On the other hand, 13-30 lb fuel can be saved with 500 ft/min step climb compared to MCT step climb. According to ATC controllers, 500 ft/min climb is not slow, because the climb performance of some aircraft is less than 1000 ft/min even with MCT. 500 ft/min climb might be more realistic for implementation.

The possible fuel saving can differ depending on the flight conditions, such as cruise mach number, aircraft type, wind condition, weight, CI, and so on. On the other hand, the possible fuel saving is 15 – 50 lb which corresponds to about 0.1 % of total fuel consumption for 2000 NM flight. Although it might seem quite little and even negligible, the proposed gradual climb procedure is applicable to almost all aircraft flying worldwide. Its cumulative effect will be significant. Under specific conditions, gradual climb might save no fuel at all, so the calculation in various conditions will be a subject of future work.

B. *Optimal flight profile for climb to TOC operation*

Next, the climb to TOC operation is considered. Here, two initial conditions (Scenario 4 and 5) are considered based on scenario 1 as shown in Table 4. Since it is already known that the optimal altitude under the initial weight in scenario 1 is a bit less than 36000 ft, the optimal profile is calculated up to 36000 ft under the same initial weight. For the comparison purpose, in scenario 5, it is assumed that the aircraft is not cleared to fly at optimal altitude (36000 ft) due to airspace congestion and is allowed to fly at 34000 ft.

TABLE IV. INITIAL AND TERMINAL CONDITIONS FORCLIMB TO TOC PROCEDURE.

	Scenario 4	Scenario 5
Initial weight [lb]	540,000	540,000
Initial altitude [ft]	30,000	30,000
Terminal altitude [ft]	36,000	34,000
Initial climb angle [deg]	1.0	1.0
Initial/Terminal Mach	0.83	0.83
Flight distance [NM]	500	500
Aircraft Type	B773	B773
CI	100	100

Fig. 6 shows the optimal profile and the profile with MCT in scenario 4. When MCT is used, the aircraft climbs to 36000 ft with the highest possible ROC and flies level. On the other hand, in the optimal profile, the aircraft climbs fast at the beginning, but the ROC gets lower with climb. After passing 35000 ft, the ROC becomes small. Fig. 7 shows the relationship between altitude and the ROC under MCT and optimal profile. The initial climb angle of 1.0 deg corresponds to about 800 ft/min ROC. With MCT, about 2000 ft/min ROC is achieved during the entire altitude range. However, in the optimal profile, the optimal ROC decreases with altitude almost linearly. If a small ROC is applied near TOC, the fuel burn can be reduced.

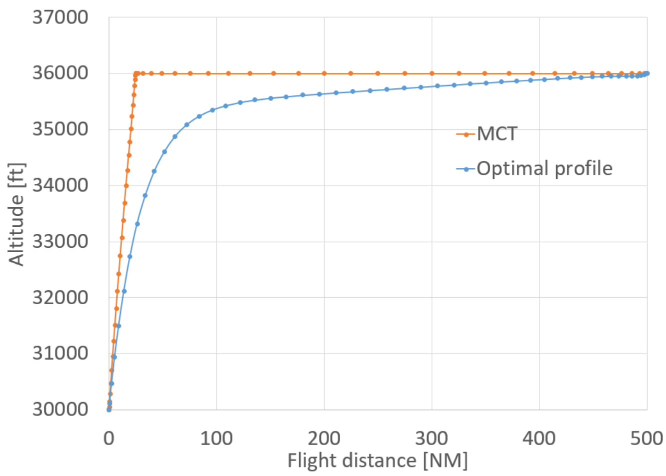

Figure 6. Optimal flight profile and the flight profile with MCT in scenario 4.

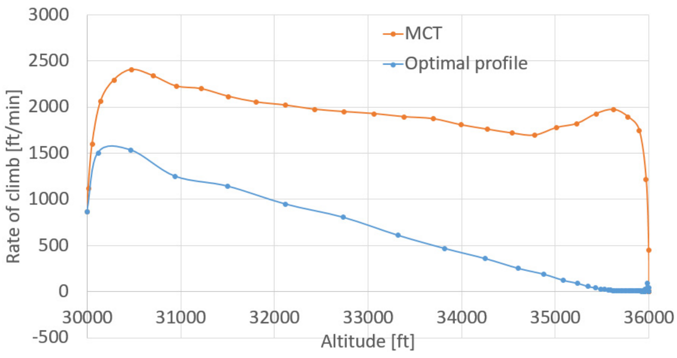

Figure 7. Relationship between ROC and altitude in optimal flight profile in scenario 4.

In order to save fuel to reach TOC in real world implementation, the following is a possible operation: climb to a certain altitude (defined as transfer altitude) with MCT and maintain a certain ROC between the transfer altitude and the cruise altitude. Fig. 1 (in Sec II) shows an example of flight trajectory when transfer altitude is 33000 ft and ROC is 1000 ft/min as well as the MCT climb profile and optimal climb profile. As shown in the figure, the aircraft starts climbing at 30000 ft and climbs to 33000 ft with MCT (about 2000 ft/min). After passing 33000 ft, the ROC is changed to 1000 ft/min and climbs to 36000 ft (TOC).

Since the proposed climb procedure requires two parameters (transfer altitude and ROC), Fig. 8 shows the calculated fuel saving by the proposed method compared to MCT climb in Scenario 4. First, the difference of fuel consumption between MCT and optimal climb is 64 lb, which is the maximum possible fuel saving. If 500 ft/min ROC is applied, the best transfer altitude is 32000 ft, and 42 lb fuel is saved compared to MCT climb. As for 1000 ft/min ROC case, 41 lb fuel saving is achieved when the transfer altitude is set to 30,000 ft. If an appropriate transfer altitude is chosen, the selection of ROC does not cause a big difference in this case.

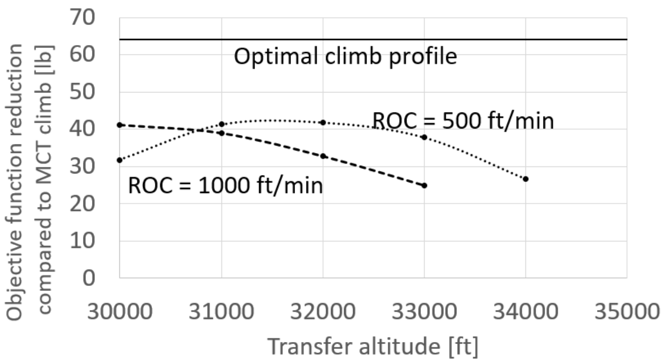

Figure 8. Objective function reduction by the proposed climb with various parameters in Scenario 4.

Next, the same calculation is conducted for Scenario 5. The optimal profile is calculated and shown in Fig. 9. At the beginning, the optimal profile between Scenario 4 and 5 are the same, but the optimal trajectory of Scenario 5 smoothly goes below the optimal trajectory of Scenario 4. In Scenario 5, the aircraft has to fly below the optimal cruise altitude (around 35500 ft), so the optimal trajectory is to cruise the highest cleared altitude. In scenario 5, the fuel saving of the proposed procedure is calculated with various parameters as shown in Fig. 10. Compared to Fig. 8 (Scenario 4), the maximum altitude is constrained to 34000 ft, so the possible fuel saving by the optimal climb profile is reduced to 34 lb. However, using the proposed climb procedure, about 28 lb fuel saving is possible by choosing appropriate ROC and the transfer altitude. Note that 1000 ft/min ROC is overall better than 500 ft/min ROC in Scenario 5, while it is opposite in Scenario 4. This is due to the difference of the optimal profile between Scenario 4 and 5, but we have to find the best transfer altitude and ROC easily for the real implementation. These will be functions of the various factors such as aircraft weight, cleared altitude, wind and temperature. Further details will be examined and analyzed in a future work.

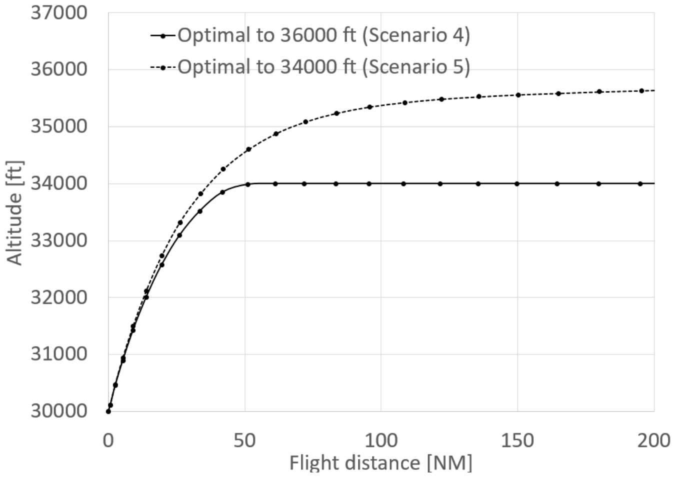

Figure 9. Optimal trajectory in Scenario 4 and 5.

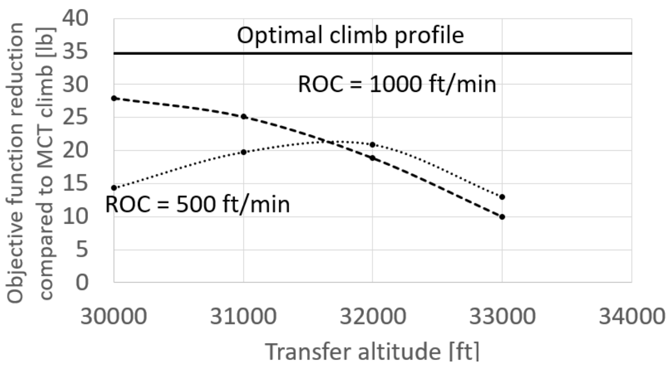

Figure 10. Objective function reduction by the proposed climb with various parameters in Scenario 5.

C. Possible negative effects and feedbacks from pilots and air traffic controllers

The introduction of the proposed gradual climb procedure might cause negative impacts to the aircraft operation. Therefore, the author discussed the proposed procedure with several pilots and air traffic controllers and obtained their valuable feedback.

According to the pilots, gradual climb operation should save fuel to some extent. The optimal ROC varies with the conditions, but it is preferable to have a simple rule, such as 500 ft/min for step-up climb, or 1000 ft/min for the last 2000 ft prior to TOC. In addition, TCAS monitors climb or descent when the climb/descent rate is 500 ft/min or larger, so if the ROC is too small, other aircraft might think that the aircraft is not climbing/descending. Therefore, the ROC for 500 ft/min or greater is recommended for situational awareness. To conduct the proposed gradual climb procedure, the pilot should select V/S mode by pushing the V/S button and set an appropriate ROC. After reaching the cruise altitude, the aircraft automatically starts cruise flight and V/S mode is automatically changed. Therefore, the impact of the gradual climb procedure to the pilot workload will be limited.

As for ATC perspective, it takes longer time to reach the cruise altitude by using a gradual climb procedure. However, during the normal climb, ATC does not assign ROC of the aircraft and does not know how long it will take to reach the cruise altitude. Therefore, the aircraft are sufficiently separated horizontally from each other during climb, so no safety issue will be occurred by a gradual climb procedure. As for the ATC efficiency, ATC does not feel that 500 ft/min climb rate is slow. Since sufficient horizontal separation is set, no impact will be given to another aircraft. Also, even if the pilot applies gradual climb procedure, no report to ATC is required. However, if the aircraft conducts a 50 ft/min climb, multiple flight levels should be blocked, which might affect other flights in the vicinity.

According to these comments, the negative impacts will be almost negligible by operating gradual climb procedure. Even if the fuel saving per flight by gradual climb is not big, little negative effect is expected, so it is worth performing the gradual climb procedure.

V. Conclusions and Future Works

This research considered a practical way to implement gradual climb which has theoretically been known to save fuel. Potential fuel saving was calculated considering the current ATC and pilot operation. It would be impossible to fly on the “optimal profile”, but this research showed that a sub-optimal profile such as a fixed ROC could achieve fuel saving. The possible fuel saving per flight is not significant, the order of 10-100 lb. However, the proposed gradual climb procedure is applicable for all commercial aircraft flying worldwide, and so the cumulative effect will be significant. Clarification of the conditions under which the proposed operation can be applied and implemented in practice will be a subject to future work.

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