Aircraft Atypical Approach Detection using Functional Principal Component Analysis

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***Abstract*—Airports Terminal Maneuvering Areas (TMA) and Control Traffic Regions (CTR) are characterized by a dense air traffic flow with high complexity. In nominal operations, approach flight path safety management consists in procedures which guide the aircraft to intercept the final approach axis, and the runway slope with an expected configuration in order to land. Some abnormal flights are observed and considered as Non Compliant when the intermediate and the final leg intercepting conditions do not comply with the prescription of the operational documentation. This kind of trajectories generates difficulties for both crew and Air Traffic Control (ATC) and may induce undesirable events such as Non Stabilized Approaches or ultimate events like Control Flight Into Terrain (CFIT), in the worst cases. There is a real need for atypical flights detection tools in order to improve safety. In this paper, a post-operational detection method based on functional principal component analysis and unsupervised learning will be presented and compared to geometric features.**

***Index Terms*—Flight Path Safety Management, Atypical Flight Event, Non-Compliant Approach, Functional Principal Component Analysis, Unsupervised Learning, Anomaly Detection**

I. Introduction

A. Operational Motivations

Approach and landing accidents (i.e. accidents that occur during initial approach, intermediate approach and landing) represent every year 50% of total hull losses and 55% of fatalities. Moreover, a great majority of accidents presents significant atypical events from nominal approaches such as atypical speed or atypical altitude [1], [2]. In addition, Airports Terminal Maneuvering Areas (TMA) and Control Traffic Regions (CTR) are characterized by a dense air traffic flow with high complexity. This complexity will surely increase since IATA forecast a growth of air passenger worldwide from around 4 billion today, up to 7.8 billion in 2036 [3]. Consequently, there is a crucial need for aircraft atypical approach detection.

To respond to the International Civil Aviation Organization (ICAO) safety requirement, the French Civil Aviation Authority has launched since 2006 a national safety program, which for the time being, is divided into two State Safety Program (SSP) published for the period 2009-2013 [4] and 2013-2018 [5]. A SSP for the period 2018-2023 is currently being published. The risk portfolio [6] distinguishes undesirable events such as Non Stabilized Aproaches (NSA), from ultimate events such as Control Flights Into Terrain (CFIT) or mid-air collisions. Undesirable events may lead to final events and therefore jeopardize safety or reduce airfield capacity. Their identification and detection is an important issue.

In nominal operations, flight path safety management consists in procedures which guide the aircraft to intercept the final approach axis, and the runway slope with an expected configuration in order to land. A particular undesirable event called Non Compliant Approach (NCA) was defined in the second version of the 2008-2013 safety program risk portfolio [6]. An approach is considered not compliant when the intermediate and the final leg intercepting conditions do not comply with the prescription of the operational documentation. It may occur during radar vectoring or not, and for visual or instrument approaches. A NCA is a potential precursor of NSA [7]. A stabilized approach is one in which the pilot establishes and maintains a constant angle glide-path, an approach speed and an aircraft configuration towards a predetermined point on the landing runway.

Geometrical criteria with horizontal and lateral margins from the nominal path were defined to distinguish a compliant from a non-compliant approach. In particular, interception chevrons were created. They define a 45° maximum angle of procedure radial interception. This angle may be reduced to 30° in specific situations such as dependent parallel runways. Besides, a flight is expected to attend a 30 second level-off flight during the intermediate leg before descending on runway slope in order to reduce speed and to configure properly for landing. Figure 1 illustrates those criteria.

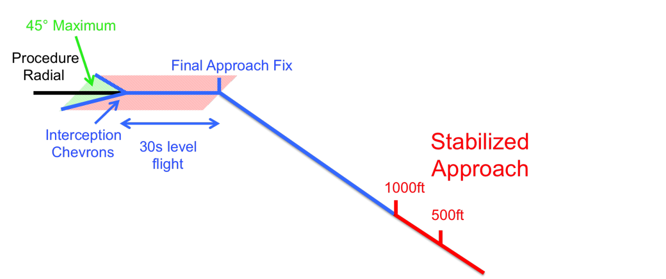

Figure 1. Description of Compliant Approach Criteria and illus-tration of Stabilized Approach

For example, NCAs were found in different accidents such as the Air Nostrum flight 8313 on July, 30th 2011, where the aircraft suffered structural damages following hard landing at Barcelona Airport [8]. Peeks of descent rate above 3000ft/min were recorded and the aircraft flew over threshold at 315ft, where nominal Reference Datum Height (RDH), i.e. the nominal height above threshold on-glide is around 50ft. Another example is the crash of Asiana Airline flight 214 of July, 6th 2014 at San Francisco Airport, which counted 3 fatalities and 185 injuries [9]. The airplane was recorded with a very low speed in final approach, and finally stalled before crashing.

B. Previous Related Works in ATC

Compliance criteria were applied to flight operations to give a state of the current situation at Paris Charles-De-Gaulle (CDG) Airport between February and August 2014. The module NCA of the French Civil Aviation Authority tool called ELVIRA [10] was used. This module is a post-operational analysis tool that studies radar trajectories and describes their compliance. Over this period, 22% of flights were detected as non-compliant with approximately 2% being significant [11]. It implies that the definition of compliance could be improved since a large majority of detected non-compliant flights, do not present significative safety issues. Too many false non-compliant alarms may occur, which is troublesome for Air Traffic Control (ATC) operations. Besides, there is a real lack of energetic features. This study led to the identification of different contributing factors and bias for NCAs such as extra energy owing to overspeed or downwind in final approach, or the influence of the QNH during the operations. Specific atypical situations called Glide Interception From Above (GIFA) were pointed out. These situations are particularly critical owing to the potential difficulties to manage the aircraft energy and because aircraft are nether designed nor certified to intercept glide slope from above [12], [13].

To improve the safety and decrease the number of GIFA an online detection tool was set up at CDG Airport and used by ATCs on real time. It consists in four 3D-volumes using the Area Proximity Warning (APW) described in Figure 2. The first three volumes are warning volumes, the ATCs advise pilots that they are too high on glide. The final volume is a decision volume, where the ATC and pilots must take the decision to continue or to interrupt the approach.

The results of the experiments are positive since GIFAs are detected and an appropriate response is now taken. Approximately 5 flights over 700 per day rise an alarm and in about half cases, ATCs suggest a recovery slope as recommended [13].

Our work consists in enhancing compliance criteria and developing methodologies to detect NCA and atypical flights post-operatively and online. This paper presents new criteria that extend those defined in the NCA module of ELVIRA and defines off-line methodologies based on the specific total energy of the aircraft.

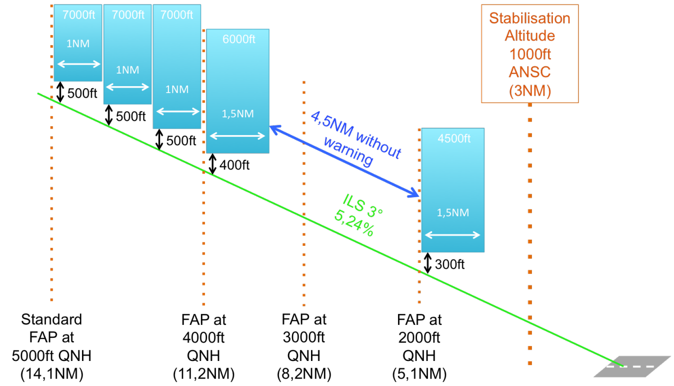

Figure 2. Illustration of the Area Proximity Warning set up at CDG

C. Functional Data Analysis Approach

Functional Principal Component Analysis (FPCA), is a powerful mathematical tool from Functional Data Analysis (FDA). FDA consists in studying a sample of random functions generated from an underlying random variable [14]. They deeply evolved during the 2000s with Ramsay et Silverman [14]–[16]. Other theoretical and applied aspects like regression or clustering were published by Ferraty and Vieu [17], [18].

The applications of FDA are numerous. In [19] Ullah et al. state many applications and underline its multidisciplinary purpose. In particular, FDA is used in various research fields such as medecine, biomedical, biology, finance, demography. In aeronautics, FDA is also widespread. Gregorutti [20] uses data from flight recorder to develop a prediction tool for long and hard landings. The tool mixes FDA and wavelets decomposition with machine learning like random forests. During his PhD a software was developed and is now commercialised by a company called SafetyLine. Suyundikov [21] presented a multivariate functional data clustering from trajectories using FPCA in Sobolev spaces. Hurter et al. [22] developed a bundling algorithm for radar trajectory visualization based on a smoothing splines decomposition and FPCA. Tastambekov [23] developed an aircraft trajectory predictor based on local functional regression with wavelet decompostion and a k-mean clustering algorithm. Nicol [24] applied FPCA to study the underlying mode and pattern of variation of aircraft trajectories. Barreyre et al. presented a novel outlier detection tool in functional data [25], and a statistical outlier detection [26] for space telemetries, based on wavelet decomposition and principal component analysis. Finally, Yan et al. [27] proposed to apply FPCA to a sliding window for dynamic prediction of longitudinal biomarker data, in order to enhance performance robustness.

In this paper we will propose a method to detect atypical approaches by applying recursively on a sliding window the following process. First, reduce the dimensions using a FPCA on total energy trajectories. Then, apply a hierarchical clustering and finally give a compliance score from outlier detection.

Our paper is divided in four parts. In the first part, mathematical backgrounds around functional data analysis and data clustering will be presented. In the second part, we will present different features that extend the actual geometric criteria to detect non-compliant approaches. Then, our atypical approach detection method will be presented. Finally, the method will be illustrated on real data and specific operational situations.

II. Mathematical Backgrounds

A. Functional Data and Functional Principal Component Analysis

In FDA, data are functions, and methoods focus on the statistical analysis of a sample of curves. In practice, we observed a discretization of these functions on a grid of time locations. For this reason, the first step of FDA consist in recovering the functional nature of curve data from discretised curve data by using a decomposition on a functional basis. There are different basis function systems in the literature like polynomial basis, Fourier basis, wavelet basis or smoothing spline basis [16]. Then, most of the multivariate statistical method may be extended to the functional setting, such as the Principal Component Analysis (PCA), in order to identify the most important source of variation in the sample of curves and cut down the complexity of the data.

In our context, approach trajectories are functional in nature, mapping a time interval to a state space $\mathbb{R}^d$. In this paper, such curve data are discretely recorded by radar every 4 seconds. Trajectories are observed on a time interval $[0, T_i]$, which can be different for each trajectory.

PCA is a powerful statistical method that summarizes a significant amount of data information by creating new variables as the linear combination of existent variables. It is an orthogonal projection that concentrates the majority of the variance in the first components. It enables simpler representation and analysis of complex or even large dimension variables. In practice, data are projected over the eigen basis of the covariance matrix of the observations ordered by decreasing eigen values.

PCA was extended to the functional case called FPCA by Deville [28] and Dauxois [29], [30]. When data are functions sampled from an underlying stochastic process, FPCA enables dimensionality reduction by estimating a truncated Karhunen-Loève decomposition. Therefore, when the principal components are determined, the trajectories can be represented by their decomposition coefficients on the principal component basis and considered as a small dimension vector.

In this paper, we will not use a grid of time location but a grid of curvilinear distance to the runway threshold location. The curvilinear distance is the distance flown by the aircraft along the trajectory. Indeed, to enable consistent comparisons between flights in the approach phase, flights must be compared regarding their curvilinear distance to the threshold rather than flight time since aircrafts do not operate at same speeds. Besides, in FPCA the entire interval is usually studied. However, in our methodology we propose to recursively apply the whole process on smaller intervals with a sliding window in order to give a local non-compliance score.

B. Introduction to Data Clustering

Data clustering, consists in grouping similar data samples together into subsets. The inputs are unlabeled data, and the idea is to find underlying information to classify those data [31].

A way to perform a data clustering is to solve an optimization problem that minimises the intra-class variance and maximises the inter-class variance over the possible clusters. In our paper, a clustering algorithm called Hierarchical Density-Based Spatial Clustering of Applications with Noise (HDBSCAN) will be used. This algorithm extends the DBSCAN clustering method [32] by converting it into a hierarchical clustering algorithm. It finally extracts a flat clustering based on the stability of clusters [33]. The algorithm is divided into five steps. It first transforms the space according to the density. Secondly, it builds the minimum spanning tree of the distance weighted graph. Thirdly, a cluster hierarchy of the connected components is constructed. Then, the cluster hierarchy based on minimum cluster size is condensed. Finally, it extracts the stable clusters from the condensed tree.

An extension to data clustering is called anomaly or outliers detection. After the clustering process, it is possible to consider as outlier the elements that fall outside the clusters, i.e. the elements that are far from any cluster. An algorithm called Global-Local Outlier Score from Hierarchies (GLOSH) gives a score between 0 and 1 for outliers [34]. It compares the density of a point to the density of any points in the associated current and child cluster. Points with substantially lower density than the cluster density are likely to be considered outliers.

In our method, hierarchical clustering and outlier scoring will be applied to the decomposition coefficients on the principal component basis of the FPCA to compute the local compliance coefficient.

III. Compliance Criteria Extension

In this section we introduce the extension and the new compliance features we have created. On all the figures, colors are defined as the following. Green is compliant, orange is a warning, red is critical, blue corresponds to the study interval and gray means that it does not belong to the study interval. The current situation and the conformity limits for trajectories from ELVIRA NCA module [35] are the baseline for our features. Indeed, the chevrons, the 30 seconds level-off flight and the glide path define geometric limits.

In order to make the notion of compliance more restrictive, reducing the number of false alarms, we now consider two limits. A warning limit and a critical limit. For both altitude and lateral feature, the limits defined in the ELVIRA module become the warning limit. Besides, we introduce the critical limit for the altitude feature as the low altitude of the GIFA's 3D volumes. For the horizontal feature, the critical limit, is defined as twice the warning limit. Both limits are represented in Figure 3

The false alarms are possibly due to the lack of features like energetic features for example. In the following, we present complementary features. In operations, an aircraft is

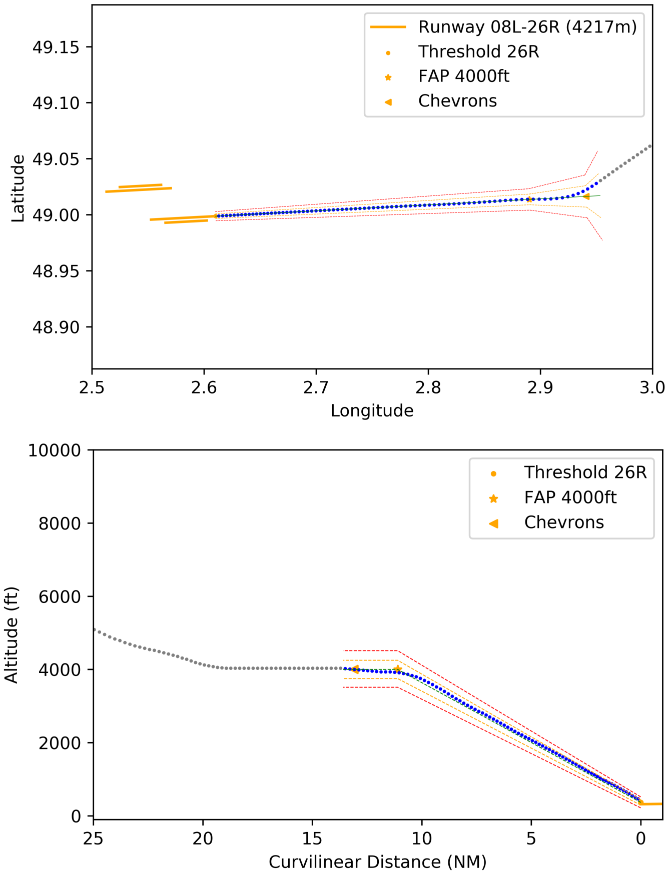

Figure 3. Actual compliance criteria of NCA Module: horizontal limit (top) and vertical limit (bottom)

supposed to attend a level-off flight before intercepting glide slope. Consequently, we introduce a new feature Glide Angle (GA), which corresponds to the slope to join the touchdown point from current position. Considering the earth as a sphere implies an altitude correction to compute the feature. This is illustrated in figure 4. To give an example, with 11NM distance from the runway threshold, there is an altitude difference of 107ft. The feature is illustrated for a trajectory in figure 5.

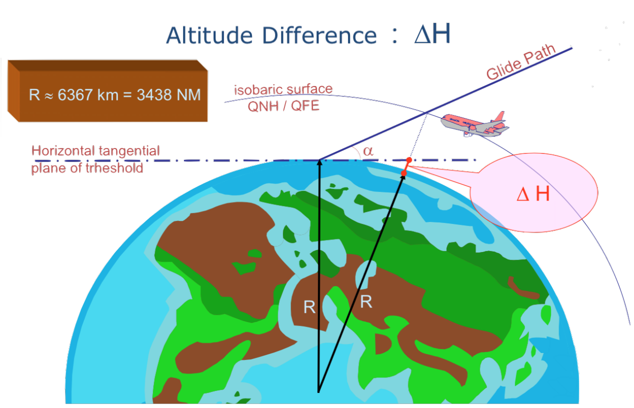

Figure 4. Illustration of the altitude correction [36]

To define a warning limit and a critical limit, we computed the slope angle of intercepting Glide Slope at 4350ft and 4700ft instead of 4000ft at FAP. The warning limit (resp. critical limit) for the Glide Angle feature is consequently set to 0.4° (resp. 0.7°) up to the published glide angle and increases linearly to 0.9° (resp 1.2°) approaching the touchdown point to consider the latent fluctuation of the tangent function used to compute the feature nearby the touchdown point.

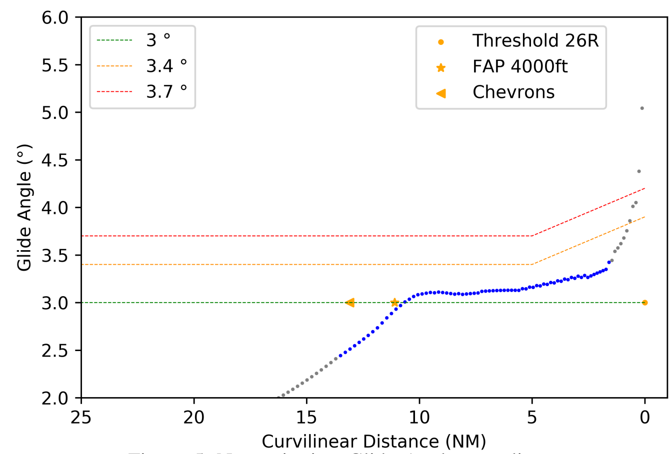

Figure 5. New criteria : Glide Angle compliance

Finally, we introduced two other features, the Ground Speed (GS) and the Vertical Speed (VS) to complete the energy analysis of the trajectories. The nominal, warning and critical limits are based on operational on-glide deceleration issues. The nominal operational on-glide deceleration in the literature is between 10 and 20 kts/NM [37]. Consequently, we consider nominal a constant ground speed deceleration of 15 kts/NM from the Chevrons to the stabilization height at 1000ft with the aircraft computed approach speed v_{app} (average speed over the last 3NM). For the warning limit, we consider a 17.5 kts/NM on-glide deceleration from Chevrons to $v_{app} + 15$kts at 1000 ft and for the critical limit a 20 kts/NM deceleration from Chevrons to v_{app}+30kts at 1000 ft. Those limits are illustrated in Figure 6.

Regarding the vertical speed, while on-glide, the aircraft vertical speed is directly linked to the ground speed. Besides, we consider that before the FAP, the vertical speed is not supposed to change since the aircraft has to operate a level-off flight. We finally consider as warning (resp. critical) a vertical speed 50% (resp. 100%) greater than the nominal vertical speed after the stabilization height.

IV. Atypical Flights Detection Methods

In this paper, the data we used is composed of 20746 landing radar records at CDG Airport during December 2011. The radar records are composed with the longitude, the latitude, the altitude, the ground speed, the time, the vertical speed, the heading, and the aircraft type. Radar information are recorded every 4 seconds.

A. *Energy Motivations*

The real problem for aircraft to land is an excess of energy. Excess of energy corresponds to situations where an aircraft is

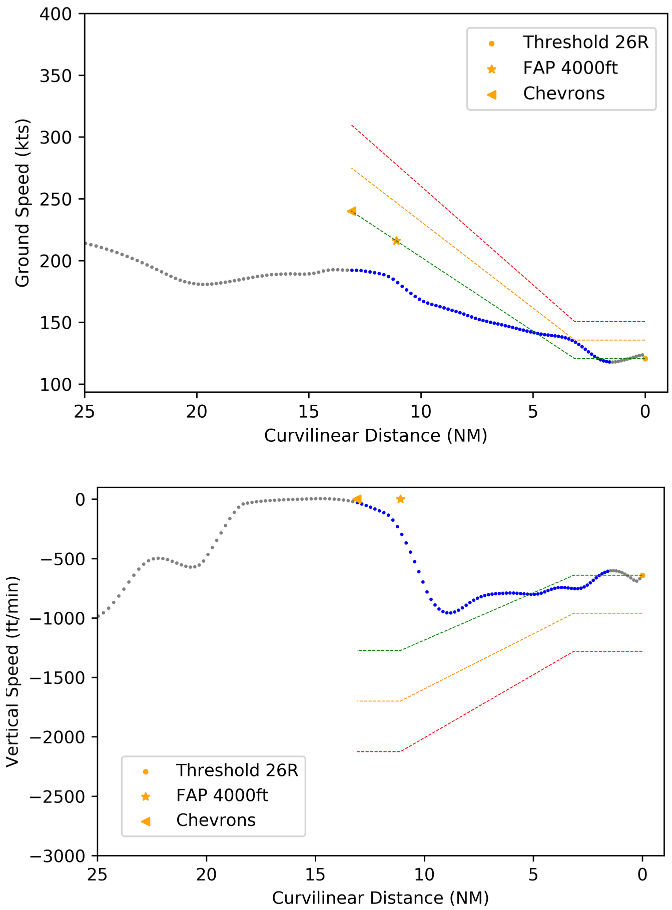

Figure 6: New criteria : Ground Speed (top) and Vertical Speed (bottom) function of the curvilinear distance

for example too high on glide owing to GIFA resulting in high potential energy, or with an overspeed owing to down wind in final approach or late power reduction resulting in high kinetic energy. By using the total energy we have a tool that is able to detect both cases of non-compliance.

The main idea is to use the total energy of the aircraft in the runway coordinate system. Radar records do not contain aircraft mass. Since, our study only concerns last phases of flight b efore l anding, w e a ssume t hat t he m ass i s constant. Therefore, we compute the total specific e nergy E_T(energy per unit mass) as:

$$E_T = E_c + E_p; \; E_c = \frac{1}{2} \cdot (G_s^2 + V_z^2); \; E_p = g \cdot h$$

Where Ep is the specific potential energy, Ec the specific kinetic energy, G_s is the ground speed, V_z the vertical speed, h the height and g the gravity constant.

Our input is the total energy function of the curvilinear distance.

B. Algorithm

Our method consists of applying the following process on a sliding window (defined by its width ν and its shift δ) recursively. First, apply a smoothing spline decomposition and a FPCA over the pieces of trajectories. Then, project over the k first principal components as representation of each piece of trajectory. Finally, apply a clustering to detect outliers far from every cluster. The HDBSCAN algorithm was used to perform the clustering. The Global-Local Outlier Score from Hierarchies (GLOSH) is used to give an outlier scoring. The value given is between 0 for a nominal sample and 1 for an outlier.

With this algorithm, each shift of the sliding window is attributed a coefficient. To represent and give a smooth representation of the coefficient, we use an averaging process to give the discrete score. The local compliance coefficient at a fix point for example 10NM is computed by averaging all the sliding window shift coefficients containing the 10NM point.

Finally, the detection phase is done by computing the length of the maximum interval for which compliance coefficients are over a threshold τ. If the maximum interval length is greater than a reference length λ, the trajectory is considered as atypical.

C. Why a sliding window is used ?

The first question is, why do we apply a sliding window and not using the whole trajectory as usually done in FPCA ? To answer this question we applied the process on the whole trajectory and discuss the pros and cons in the following.

We used radar records at CDG Airport during December 2011. We focus on the last 15 nautical miles (18NM to 3NM from the threshold) before stabilization of A320 landing on runway 26L. First, using the whole trajectory implies only to use the threshold τ to seperate nominal from atypical trajectories. In this illustration, we fixed τ such that we detect the most distant percentile of trajectories.

The detected atypical flights are analysed using the geometric limits defined in Section III. Over the 20 flights detected, there are 7 flights that were too high and did not respect level flight, 9 flights with an overspeed. Besides, we notice a huge Glide Interception From Above shown in Figure 7. This method also detects interesting cases. First, a flight with very strong speed reduction at 9NM and with a big lateral deviation. Second, a flight that intercepts the glide at 3000ft instead of 4000ft. Then, a flight with a low speed at 5NM which speed up back to the approach speed. Finally, a landing after a go around, which therefore started with a low speed and had a small overspeed at the beginning of the glide slope.

Using the whole interval presents different limitations. First, τ is a parameter that was fix based on the percentile the most distant and is dataset dependent. Besides, we considered as atypical, outliers with an important score. The distribution of the scores obtained with our process is shown on top of Figure 8. Score near 1 are in red, and score near 0 are in green. If we compare this distribution with the labels obtained by the energetic features that we develop previously, it shows that the outlier coefficient is not always appropriated since the warning and critical trajectories seem to be located on the right side of the distribution. A possible alternative could be to use a supervised learning model, using the resulting features as

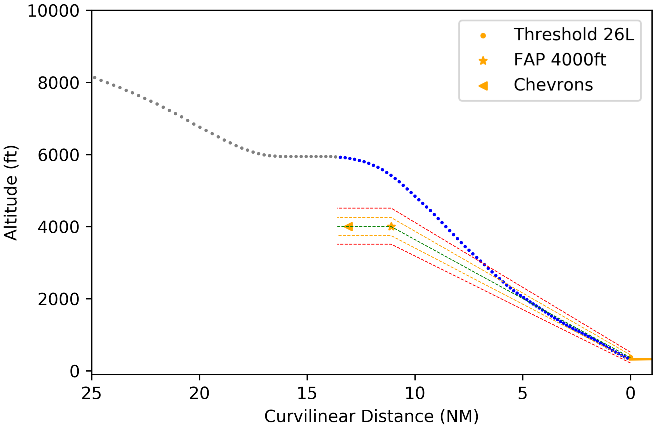

Figure 7. A GIFA detected as outlier by the FPCA detection method

labels. Then, to consider the outlier score to be a function of the probability given by the supervised learning method to be in the class. An illustration is shown on bottom of Figure 8.

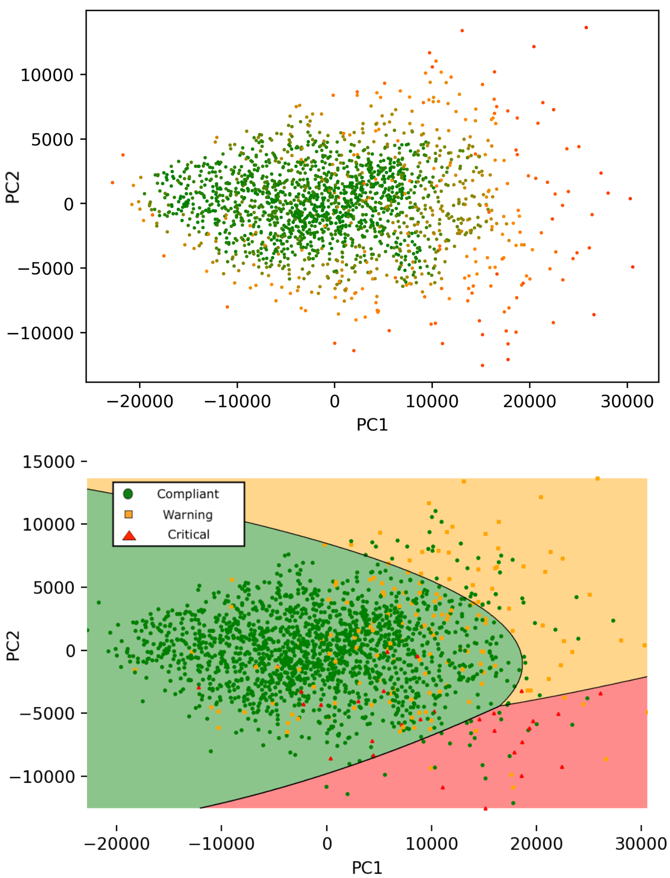

Figure 8. Representation of the outlier coefficient (top) and of the supervised classification with a simple learning model (bottom)

Besides, when the principal component analysis is made over the whole trajectory, local events could have been hidden by the process. All these reasons motivate the sliding window FPCA. In our context, it will give an outlier score for each interval, which can be interpreted as a local compliance score.

D. Hyperparameters and inherent effects

We must point out the inherent effects of the sliding window width and the coefficient computation. First, the sliding window width must be selected properly. If it is too short, we will only detect noise, and if it is too large, we will not detect local events. Then, the computation uses an average over all the intervals containing the point. This implies a smoothness of the compliance coefficient but also a decrease of the maximum value and a possible delay to change from atypical to nominal and inversely. It implies possible problems to generalize in a real-time situation. Finally, the detection rule also implies hyperparameters that must be fixed properly.

We underline that our method is made with different hyperparameters. Those parameters and their influence will be studied in future works. The motivation of this paper is to present a methodology and some first results.

V. Case Study Analyses

In this subsection, we present the results we obtained for a fix configuration of our algorithm over specific situations. We selected a 2NM sliding window, which corresponds to a flight of around 30s, and a shift of 0.2NM (around the radar refresh time). The compliance coefficient of a point is obtained by averaging all the coefficients over the sliding windows that contain the point. The threshold τ was fixed at 0.6 and the reference length λ to 2NM, which corresponds to the sliding window width. For the FPCA, we used the 3 first principal components coefficients. Finally, we used 10 minimum samples per cluster in HDBSCAN.

a) Continuous Descent Approach (CDA): CDAs are situations where an aircraft operates a continuous descent and therefore does not attend the level-off flight. The geometric limits will always notify the situation with an altitude deviation warning since the flight overpass the altitude limits designed for the level-off flight. Nevertheless, it does not present any safety issue since it is a known procedure. The only possible issue with CDA is over-energy owing to over speed. We wanted to analyse how our methods deal with this kind of situations.

We consider 30 flights that intercepted the glide slope at an altitude above the published interception altitude, and then proceeded a CDA. For all the flights that presented a nominal speed none is considered as atypical. It means that our method does not detect abnormal energetic behaviour. Nevertheless, for those with a high ground speed like the flight illustrated in Figure 9, which has a ground speed around 250kts at FAP, the sliding window detect an overenergy. The energy was finally dissipated but before the FAP it shows that it was a potential dangerous situation.

Consequently, our methods seem to be relevant to study CDA. Indeed, only approach with high speed are considered as atypical.

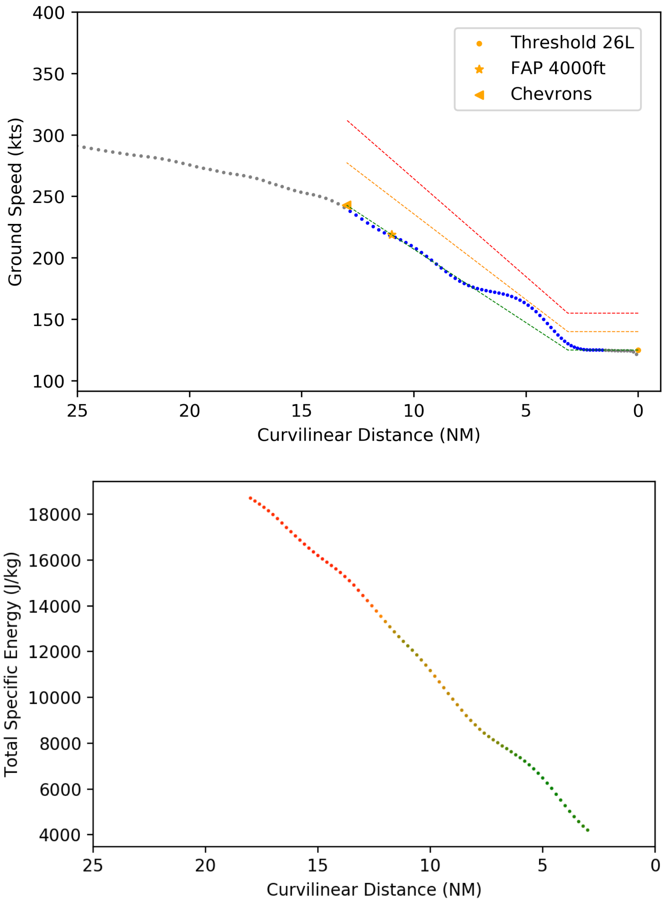

Figure 9. Ground speed (Top) and Sliding Window Energy Com-pliance Score (Bottom) of a Continuous Descent presenting an high ground speed around FAP

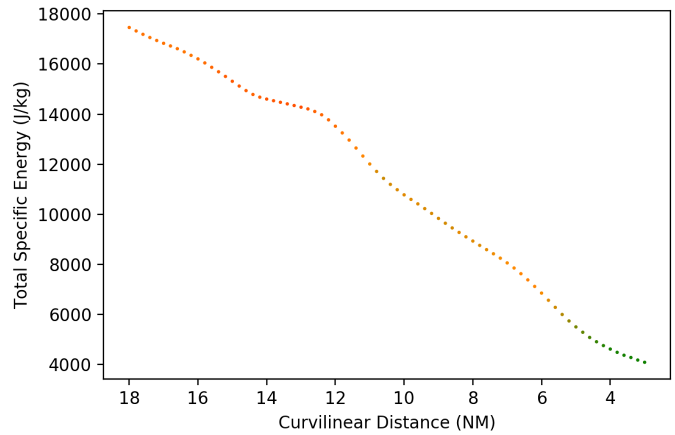

Figure 10. Sliding Window Energy Compliance Score over a GIFA

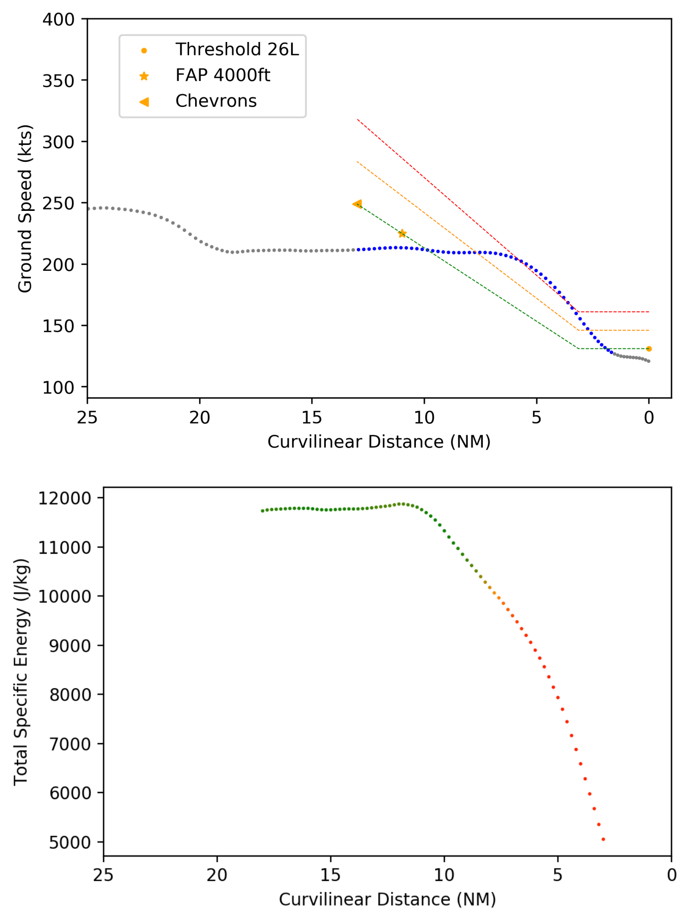

Figure 11. Ground speed (Top) and Sliding Window Energy Compliance Score (Bottom) of a Ground Speed Warning flight

b) Glide Interception From Above: Other interesting events to analyse are GIFAs. In the dataset, there are 6 cases of GIFAs. The result obtained for the GIFA represented in Figure 7 is shown in Figure 10. The sliding window method is really efficient since the atypical behaviour is well localized before 6NM. The results are similar for the 6 cases.

Our method is efficient to detect important GIFAs. We need to underline an aspect of our method. Small GIFAs like bumpy profiles (flights which attend a level-off flight on glide to decelerate for example), which means a potential energy excess, might be counterbalanced by a low ground speed. Nevertheless, this is coherent since the non-compliance induced by the excess of altitude is averaged by the low speed in the energetic point of view.

c) Ground Speed Warning: We now focus on flights which had a ground speed warning with the geometric features. A typical example is shown in Figure 11. The aircraft maintained a ground speed of 210 kts untill 6NM and finally reduced speed joining approach speed apparently after stabilization. For all these situations, the sliding window presents a large Non Compliance area for the last nautical miles and flight are detected as atypical.

d) Nominal Flight: Finally, we want to make sure nominal flights without any non-compliant aspects are considered as nominal. Over 1270 nominal flights, only 10 are considered as non compliant by the algorithm. Over the 10 flights, 7 flights presented a low ground speed on final approach possibly owing to the wind, and 3 presented a high speed and altitude before FAP and therefore a very high total energy.

VI. Conclusions

In this paper we presented an atypical flight detection method based on FPCA to enhance safety in flights approach and landing. The results of our method were compared to geometric features built with operational limits and analysed on typical flight approach patterns. Our method detects and localizes properly different type of abnormal energy situations. Nevertheless, we want to underline some limitations. An inappropriate sliding window size may induce changes in the results. Besides, local events such as brutal changes in the trajectory behaviour may be detected without being relevant safety issues.

Future works will focus on analysing our algorithm hyper-parameters behaviour. Then, on developing a complete post-operational analysis tool based on our methodologies. Besides, we want to use those methods to develop a real-time detection tool using the result of our algorithms to label the data. In addition we are currently working on a novel data generation method in order to enhance our atypical flights database.

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