Open Flight Trajectories for Reproducible ANS Performance Review

Enrico Spinielli, Rainer Koelle
EUROCONTROL
Performance Review Unit
Brussels, Belgium
enrico.spinielli@eurocontrol.int
rainer.koelle@eurocontrol.int

Ken Barker, Nicholas Korbey
Via Technology
Poole, UK
ken.barker@via-technology.aero
nick.korbey@via-technology.aero

***Abstract*—Air Navigation Service[1] (ANS) performance review needs to be based on an open set of flown flight trajectories and computed according to published, accepted standards and methodology. This paper describes the set-up of such a trajectory repository for the European region, the associated cloud based trajectory processing infrastructure and trajectory production algorithms. It also demonstrates the application for an initial use case within the terminal area. With a view to enhance transparency of performance results and to increase the level of reproducibility, we have developed a system to combine different data sources in order to produce a freely available set of flown flight trajectories for the assessment of European ANS performance. The capability is demonstrated on the basis of a use case analysis of holding patterns at a major European airport. Stakeholders are capable of reproducing the results by inspecting the underlying data processing and sample data. The presented cloud based infrastructure and processing setup form the nucleus of a reproducible and open data based approach to ANS performance in Europe. Both will be iteratively further developed to achieve the overarching goal of reproducible ANS Performance Review in Europe.**

I. Introduction

The steady growth of air transportation and how to accommodate it represents a challenge for policy making, operations, and research. The International Civil Aviation Organization (ICAO) promotes a performance based approach to address this challenge [1]. However access to data, reproducibility of performance results, and availability of relevant algorithms is still limited.

Throughout the recent years the concept of open data and the validation of operational performance results has been identified as a key stepping stone towards increasing transparency and reproducibility [2]–[4]. The prospect of this approach is that other researchers and also decision-makers can readily validate the assumptions or data processing steps, verify results by replicating the data processing, or apply the processes to other data, and subsequently build on earlier results.

With the advent of community-based data collection and sharing across the air transportation value chain (aircraft databases, aeronautical information, surveillance data), there is a need to ensure the validity of such sources by combining open data with data curated by (inter)governmental organisations. The latter is a key driver for the on-going developments by the Performance Review Unit (PRU) of EUROCONTROL aiming at the establishment of a pan-European open flown flight trajectory repository for the analysis and monitoring of the operational performance of the European Air Navigation System.

This paper proposes the establishment of an open repository of flown flight trajectories for ANS Performance review and monitoring within the European region. The aim is to establish an openly available data repository and provide access to the underlying data processing to achieve the overarching goal of transparent and reproducible ANS performance review in Europe. In particular the contributions of this paper are as follows:

- description of the open repository of flown trajectories for the European region;
- development of the data merging and trajectory production algorithms and the set-up of the relevant cloud infrastructure; and
- initial use-case analysis for the application of Reference Trajectories for operational performance related measures in the terminal area.

II. Background

The evaluation of ANS performance is not a fundamentally new topic. In Europe, EUROCONTROL initiated an independent performance review system[2], governed by the Performance Review Commission[3] (PRC), in 1997 [5]. The PRC is supported by the PRU which is responsible for the

[1] **A**ir **N**avigation **S**ervice (ANS) refers to the services provided in order to ensure the safety, regularity and efficiency of air navigation and the appropriate functioning of the air navigation system.

[2] Performance review is carried out for EUROCONTROL's 41 Member States.

[3] The **P**erformance **R**eview **C**ommission (PRC) was established in 1998 by EUROCONTROL's Permanent Commission. It provides objective information and independent advice to EUROCONTROL's governing bodies on European **A**ir **T**raffic **M**anagement (ATM) performance, based on extensive research, data analysis and consultation with stakeholders. Its purpose is "to ensure the effective management of the European air traffic management System through a strong, transparent and independent performance review," as stated in Article 1 of the PRC Terms of Reference and Rules of Procedure.

day-to-day activities of the PRC work programme, including the regular preparation of performance data products. Next to the yearly performance review reports, the PRU publishes performance related data on a monthly basis on its online Data Portal [6]. This monthly performance related data represents an initial step towards providing performance data as open data.

At a global level ICAO promotes a performance based approach via an integrated framework under the ICAO's Global Air Navigation Plan (GANP) [7], [8]. This framework is predominantly built on the performance measures used in Europe (c.f. above, EUROCONTROL Performance Review System) and in the United States of America (measures regularly reported by the Federal Aviation Authority (FAA)). The current ICAO framework proposes 19 operational Key Performance Indicators (KPI). This year ICAO Air Navigation Conference (autumn 2018) has endorsed the development of further guidance material for the proposed KPI's, including the further development of additional performance measures for the on-going ANS transformation to meet the growing demand for air transportation. Such developments will benefit from the availability of open data and associated reproducible data processing.

Within this context, the guiding principles of PRU's approach towards performance review are impartiality and transparency. Both entail that stakeholders, for example political decision-makers, operational planners and researchers, are able to consult, critique, and validate results and algorithms. The ultimate goal (or *dream*) is to enable stakeholders to reproduce the numerical performance results via the agreed methodology (and associated algorithm). This goal is categorically different from today's practice which limits the action space of stakeholders to the published results.

Figure 1 contrasts these different approaches as ends of the *reproducibility spectrum*. As depicted in Figure 1 the aim is to move as far as possible from "Marketing" (i.e. *spit the numbers and big claims* – today's predominant paradigm) to "Science" (*be my guest, reproduce my claims* – a more transparent approach).

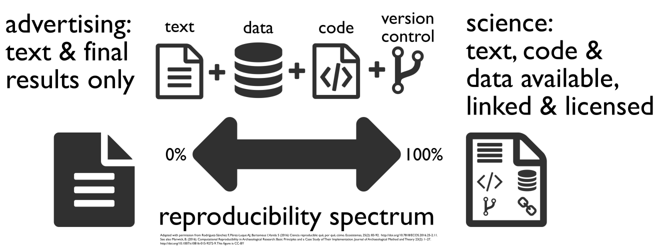

Figure 1. Reproducible research, Fig. 2 in [9].

Conceptually, the origins of the idea of reproducible research date back to the publication of first scientific results and claims. Throughout the recent years, the proliferation of communication has made it difficult to allow stakeholders to follow the work done or assess the appropriateness of the results as insights to the underlying data and how the data was processed is not readily accessible [10].

The central idea behind reproducible research is that (scientific) claims and results are published with the associated data analyses, software code, and input data. This will allow others to validate and verify the results or build on such findings [11].

Reproducibility works towards achieving both impartiality and transparency by fostering:

- Collaboration, i.e. scrutiny, feedback and engagement. Stakeholders have a platform to critique and validate the performance results, confirm the implementation of the algorithm and assess data preparatory transformations, and provide input for further developments or propose alternative data analyses.
- Trust. This is a fundamental property of good stakeholder relations. Even when the results are not favoring a specific stakeholder, the way they are produced is understood and repeatable.

In order to align with its principles, the PRU:

- develops its methodologies in collaboration with its stakeholders and domain experts
- makes the methodologies and results (Performance Indicators and studies) publicly available
- is open to feedback and revision

Pursuing full reproducibility means that the input data (i.e. flown trajectories), the methodology (i.e. documentation *and* source code for Performance Indicators (PI) calculation) and the computed metrics have to be made openly available for scrutiny.

This paper proposes a first step in the direction of sharing input data and addresses the establishment of an open dataset of flown trajectories, a.k.a. Reference Trajectory (RT) data set, which will:

- foster an open and collaborative approach to performance review for its Member States and stakeholders,
- facilitate the production of studies in collaboration with International Partners (e.g. Brasil, Singapore, China, Japan)
- define a foundation for comparative studies between different world regions (e.g. EU-USA).

It is the popular availability of ADS-B[4] position reports that overcomes the past obstacles and resistances towards sharing trajectories openly and massively. Even arguments such as privacy and confidentiality from governmental agencies or reluctance to disclose massive amounts of trajectory data are

[4]**A**utomatic **D**ependent **S**urveillance **– B**roadcast (ADS–B) is a surveillance technology in which an aircraft determines its position via satellite navigation and periodically broadcasts it [12].

being subsumed by crowdsourced efforts, for example by OpenSky Network [13], that make ADS-B data and derived artefacts openly available in a timely manner and at higher and higher quality.

III. Platform Architecture and Implementation

The RT production system runs various processes in multiple copies of a Docker[5] [14] image for the different stages of data processing taking positional and environment data as input and delivering RT and other data as output.

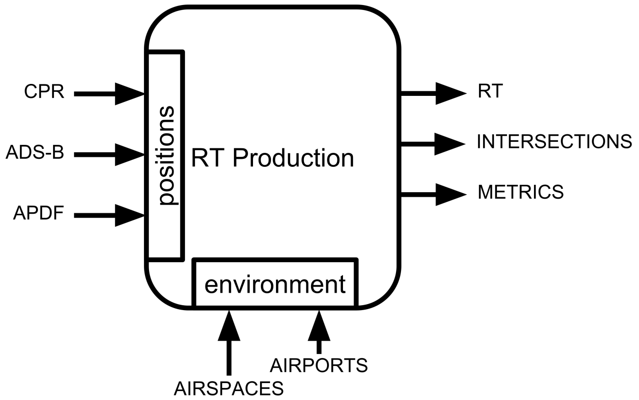

Figure 2. Data input, output for the RT system.

The position (P), environment (E) and output (O) data are, Figure 2:

- (P) ADS-B position reports, currently from FlightRadar24 live feed over Europe. The subset of the API fields fed to the input is `START_TIME`, `ADEP`[6], `ADES`[7], `CALLSIGN`, `FLIGHT`, aircraft `REG`istration, aircraft `MODEL`, aircraft `ADDRESS` (the ICAO 24-bit address that uniquely identifies an airframe [15]) for flight, and `LAT`itude, `LON`gitude, `TRACK_GND`, `ALT`itude, `SPEED`, `SQUAWK`, `RADAR_ID`, `EVENT_TIME`, `ON_GROUND`, `VERT_SPEED` for position.
- (P) CPR[8] from Network Manager which provides longitude/latitude/timestamp, callsign, ADEP/ADES.
- (P) APDF[9]: airport movement data such as flight, stand, runway and various timestamps. For future work when apron positions will be more widespreadly available.
- (E) Airspaces such as Elementary Sectors (ES) from the Network Manager (NM) or Flight Information Regions (FIR) from Aeronautical Information Publication (AIP) or user defined volumes (in the form of polygon, lower altitude, upper altitude.)
- (E) Airports, i.e. airport reference point (ARP[10]).

RT production system outputs are:

- (O) Reference Trajectories, the subject of the paper.
- (O) Airspace intersections.
- (O) Metrics about the RT production, e.g. error metrics, position, time and altitude accuracy metrics.

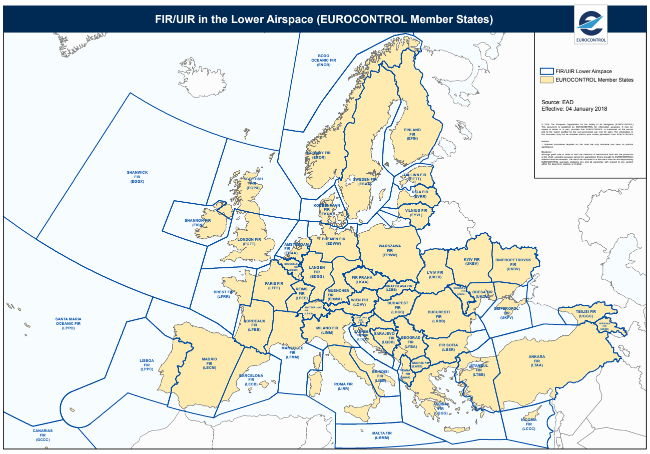

Figure 3. EUROCONTROL Member States.

The RT dataset is produced by running a mix of Python and open source Spherical Vector Geometry C++ code [17] using Earth Centered, Earth Fixed (ECEF) coordinates [18]–[21]. This approach has been taken to avoid map projection distortion that would otherwise inevitably arise given that the EUROCONTROL Member States encompass the whole of Europe and beyond, spanning an area over 50 degrees of longitude and 40 degrees of latitude, Figure 3.

The organisation of the different stages of processing is that of a typical UNIX pipeline [22] whereby files of data are processed by scripts/programs that then produce output files of data. Further down the chain these can be the input for the subsequent stage, cf. Figure 4.

There are two high level modules:

1. Trajectory Module.
2. Trajectory Assessment Module.

The first refines and merges data from a variety of different sources, converting the data into common formats, cleaning and merging it with the aim of producing a complete gate-to-gate trajectory for every flight.

The second determines sections of trajectories belonging to airspaces of interest (4D intersections for Elementary Sectors,

[5] Docker provides the ability to package and run an application in a loosely isolated environment called a container. The isolation and security allow you to run many containers simultaneously on a given host.

[6] **A**erodrome of **DEP**arture

[7] **A**erodrome of **DES**tination

[8] **C**orrelated **P**osition **R**eport (CPR) is a radar position report from Air Traffic Control which contains information about the flight it is associated to.

[9] **A**ir**p**ort Operator **D**ata **F**low provides departure and arrival data on a per airport basis [16].

[10] **A**irport **R**eference **P**oint (ARP) is a point on the airport designated as the official airport location.

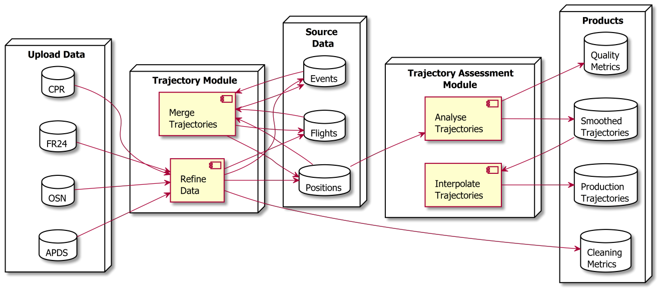

Figure 4. High level architecture for Reference Trajectory pipeline.

cylinders around airports, user defined airspaces).

A. *Trajectory Module*

The Trajectory Module performs the following tasks:

- Trajectory Cleaning
- Trajectory Smoothing
- Trajectory Interpolation

Trajectory Cleaning

The purpose of trajectory cleaning is to identify and remove erroneous positions from the data sources.

Erroneous horizontal positions are identified by calculating ground speeds between adjacent positions: erroneous positions have very high speeds, while low speeds are likely to be caused by missing positions.

Erroneous vertical positions are identified by comparing changes in vertical attitude (climbing, cruising or descending) between adjacent positions: a change in attitude from climbing to descending (or vice versa) is likely to be erroneous, especially when accompanied by a change in Secondary surveillance radar (SSR) code.

Other erroneous positions include duplicate positions, and different aircraft addresses. Note different aircraft addresses are only found in radar surveillance data since, since ADS-B uses the aircraft address to identify individual trajectories.

Trajectory Smoothing

Trajectories are smoothed by considering positions in: horizontal, temporal and vertical dimensions.

- Horizontal Path
  The Trajectory Smoothing algorithm assumes that horizontally, aircraft either fly straight (i.e. along route legs or on headings), or that they perform turns (either between route legs or onto headings). Manoeuvres such as holds, circuits, etc. are simply combinations of straight legs and turns.
  The horizontal path flown by an aircraft is derived from its time ordered positions and defined by a pair of sequences:
  - path waypoints
  - and turn initiation distances at the path waypoints.

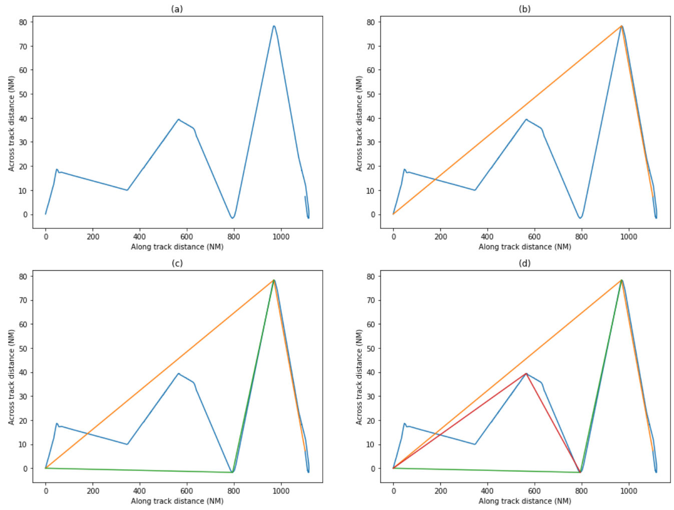

Figure 5. Across track distance (exaggerated) compared to baseline for a trajectory (a). Recursive calculation of widest positions, (b), (c) and (d) . Widest points indicate turns, either at the ends of straight legs or within holds or circuits. (b) shows lines (in orange) drawn from the first and last points of the baseline to the furthest point from the baseline. (c) shows lines (in green) drawn from the first orange line to the furthest point from the first orange line. (d) shows lines (in red) drawn from the first green line to the furthest point from the first green line, and so on. . .

The first stage in deriving a horizontal path is to find positions where the aircraft turns from one straight leg to another. They are found as follows:

1. Create a baseline (Great Circle arc) from the first point to the furthest point,
2. Calculate the widest point from the baseline; if it is further than a given tolerance then divide the baseline in two at the widest point to create two new baselines to and from the widest point.
3. Repeat above until all the widest points are within a given tolerance.

Figure 5 shows graphically how the straight portions of a trajectory are derived from along track and across track distances relative to a baseline Great Circle arc.

Derive Straight Legs

The widest points are likely to be within turns between straight legs, not along the straight legs themselves.

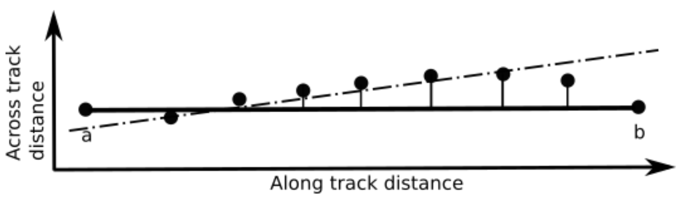

Figure 6. Path Leg Line Fitting.

A line of best fit is calculated from the along track and across track distances of the points between the widest points, see Figure 6.

A turn between straight legs can be defined by a turn radius (r) and a turn initiation distance (d), as depicted in Figure 7).

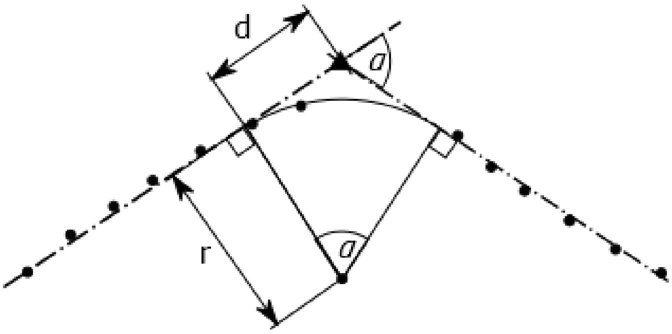

Figure 7. Calculate Turns.

A turn is the arc of a circle between inbound and outbound straight legs. Finding a circle that is tangent to both the inbound and outbound legs while passing through a turn point is a Problem of Apollonius, specifically: one point and two lines [23].

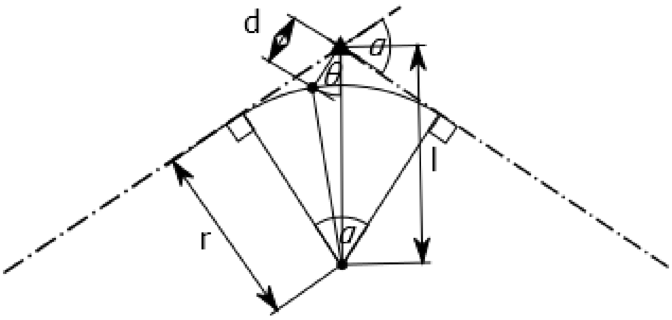

Figure 8. Calculate Turn Radius.

The turn radius is calculated from the distance (d) of the closest point to the intersection and its angle (θ) from the bisector of the turn legs using the cosine rule, Equation 1, and solving the quadratic, see Figure 8 and Equation 3

$$r^2 = d^2 + l^2 - 2dl\cos(\theta) \tag{1}$$

$$\sin^2(\alpha/2) + \cos^2(\alpha/2) = l \tag{2}$$

resolving Equation 1 for r we obtain:

$$r = d\cos(\alpha/2)\frac{\cos(\theta) \pm \sqrt{\cos^2(\theta) - \sin^2(\alpha/2)}}{\sin^2(\alpha/2)} \tag{3}$$

Calculate Path Distance

The path distance is the distance flown by an aircraft around derived waypoints, i.e. it is the distance between waypoints minus the turn initiation distances plus the turn arc lengths at each turn, see Figure 9.

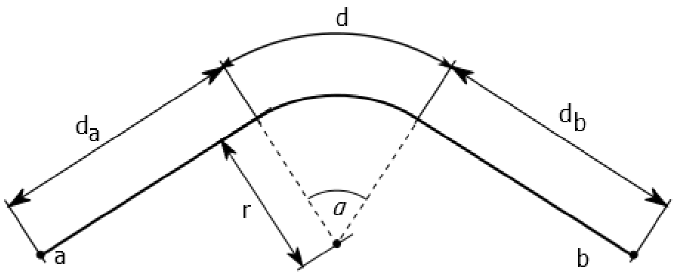

Figure 9. Path Distance.

The path distance of an input point is the distance along the closest adjacent straight leg or turn, see Figure 10.

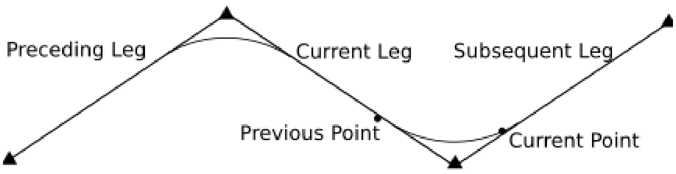

Figure 10. Find Closest Leg from the Current Leg.

Input positions are sorted in path distance (followed by time) order to derive the time and altitude profiles.

- Vertical Profile
  A plot of altitude versus path distance (a vertical profile), should form a smooth curve during climbs and descents. Between climbs and descents, aircraft fly at a single (cleared) altitude, which is just represented by the altitude and start/finish points, see Figure 11.

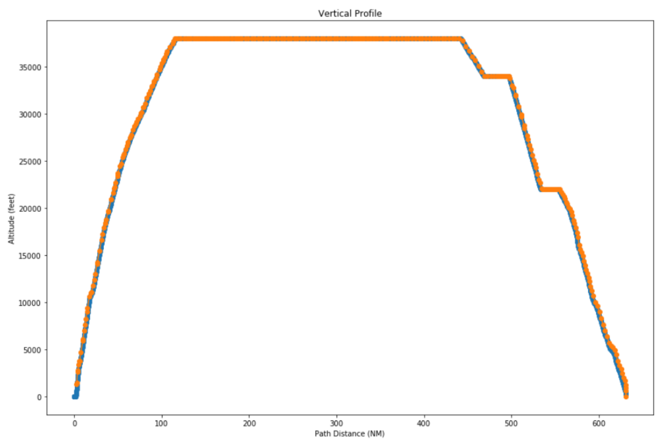

Figure 11. A Vertical Profile.

- Time Profile
  A plot of time versus trajectory distance should also form a smooth curve. However, position times have a relatively

low precision and significant errors have been observed in ADS-B times.

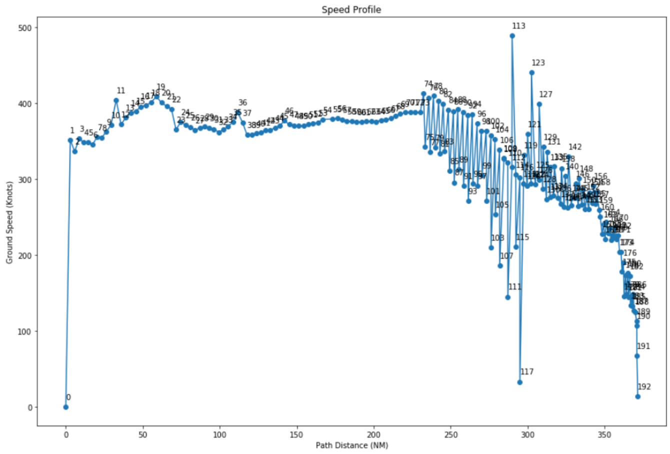

Figure 12. A Raw Speed Profile.

The inaccuracy of position times can be observed in plots of ground speed calculated from position path distances and times in Figure 12.

Ground speeds are smoothed by passing through moving median and moving average filters, e.g. see Figure 13.

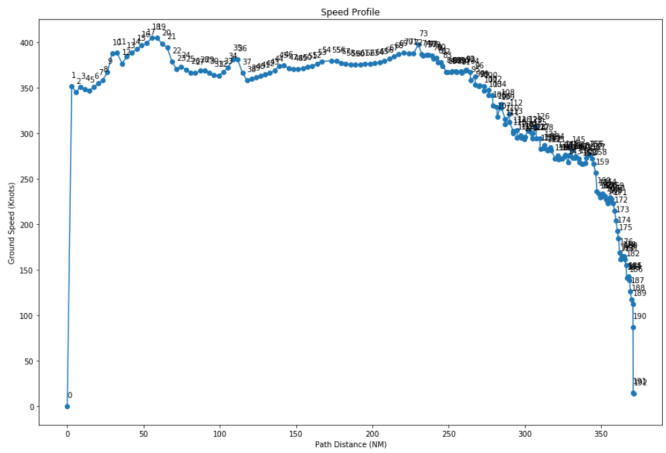

Figure 13. A Smoothed Speed Profile.

The time profile is then calculated from the smoothed ground speeds and path distances.

Trajectory Interpolation

Synthetic reference trajectories are composed of positions at regular time intervals. The user can specify different time intervals for straight and turning trajectory sections.

Synthetic reference trajectories are constructed as follows:

1. Path distances to the starts and ends of turns are calculated from the Horizontal Path;
2. Position times are calculated from the Time Profile and specified time intervals, using the path distances to find straight and turning sections;
3. Position path distances are calculated by interpolating the Time Profile at the position times;
4. Position: latitudes, longitudes, altitudes, vertical speeds, ground speeds and ground tracks are calculated by interpolating the Horizontal Path, Vertical Profile and Time Profile at the position path distances.

Note: ground tracks are interpolated along straight sections, since the straight sections are Great Circle arcs and ground track usually varies along a Great Circle, unless the Great Circle is the Equator or a meridian.

IV. Results and Discussion

A repository with an initial set of reference trajectories is being set up in order to share, collaborate and further study the quality and usefulness of the data set.

The finer granularity of the RT data set compared to the FTFM[11] or CTFM[12] trajectories from Demand Data Repository [25] allows for a different approach to performance measurements. For the terminal area, for example, this means better identification, classification and measurement of the holding patterns and the possibility to analyse their impact upon efficiency performance indicators such as Arrival Sequencing and Metering[13] (ASMA) additional time, Vertical Flight Efficiency[14] (VFE) or CO_2 emissions.

Arrivals to Heathrow airport have been selected as a case study for use of the RT dataset for the identification, classification and calculation of performance attributes linked to holding patterns.

Heathrow Airport has designed four holding stacks for flights arriving from the four quadrants, cfr. Figure 14.

They are named BNN, BIG, LAM and OCL after the villages/toponyms adjacent to the relevant VOR-DME listed in Table I.

Figure 15, a 2D plot of flight BAW34N on 2017-08-01, clearly shows a holding around OCL as captured by RT. CTFM does not capture it and misses the actual runway too by referring to the ARP (black dot in the middle of the northern runway).

[11]The **F**iled **T**actical **F**light **M**odel (FTFM) or Model 1 is a flight trajectory constructed (by the ETFMS system of NM) from the last filed flight plan.

[12]The **C**urrent **T**actical **F**light **M**odel (CTFM) or Model 3 is a flight trajectory constructed (by the ETFMS system of NM) to tactically represent a flight being flown. It refines the previous Tactical Flight Models when CPRs show a significant deviation (1 min in time, more than 400 feet in en-route phase, more than 1000 feet in climb/descent phase or more than 10 NM laterally) and/or upon message updates from ATC (DCT, level requests, FPL update), see 14.3.1 [24].

[13]The Additional ASMA time is the difference between the actual ASMA transit time and the unimpeded ASMA time calculated for non-congested conditions. See http://ansperformance.eu/references/definition/additional_asma_time.html for further details.

[14]http://ansperformance.eu/references/methodology/cd_vertical_flight_efficiency_pi.html

TABLE I
EGLL ARP, RUNWAYS AND HOLDING STACKS

Longitude	Latitude	ID	Name
-0.4613333	51.47750	EGLL	EGLL ARP
-0.4850000	51.47750	09L	Runway 09L
-0.4333333	51.47767	27R	Runway 27R
-0.4823333	51.46483	09R	Runway 09R
-0.4340000	51.46500	27L	Runway 27L
-0.5497222	51.72611	BNN	Bovingdom
0.0348111	51.33087	BIG	Biggin
0.1516667	51.64611	LAM	Lambourne
-0.4472222	51.30500	OCL	Ockham

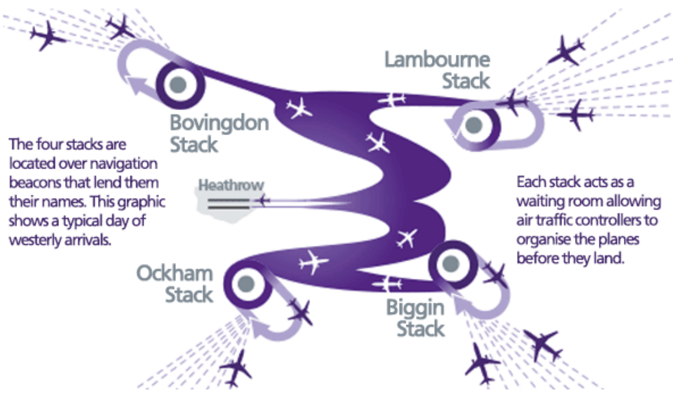

Figure 14. Holding stacks at Heathrow Airport [26].

The classification of an arrival as subject to holding is currently carried out using the following features:

- quadrant at 50 NM
- number of minima/maxima of distance from airport's ARP
- level flight segments, and
- flown distance in the holding stack boxes (the polygons have been manually defined)

Figure 16 shows the current approach with respect to flight BAW887 on Aug, 1 2018.

The result of the classification is quite encouraging and will enable the estimation of the time, fuel and environmental impact of holding with respect to flights not subject to them.

The very low costs associated with data storage and processing resources needed for the production of one month of RT data on Google Cloud Platform (8.67 USD) is encouraging too because it allows for further upscaling of the platform.

V. Future work

The RT data set is in its early stages of development and PRU is willing to engage with the aviation community at large to improve it. Some areas under consideration are:

- Calibration with aircraft logs in order to assess adherence with *ground truth*. PRU is currently in contact with some airlines in order to obtain a set of logs.

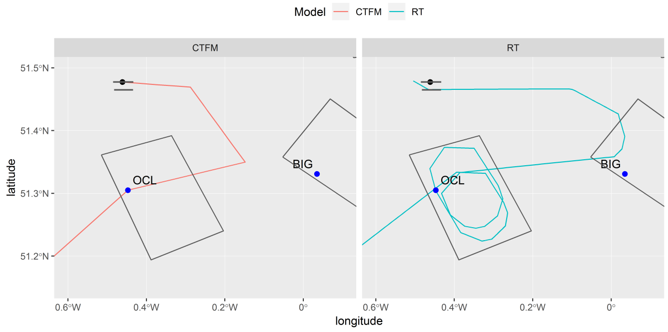

Figure 15. Comparison of CTFM and RT trajectories: CTFM connects to the ARP rather than the actual landing runway and does not capture the holding pattern.

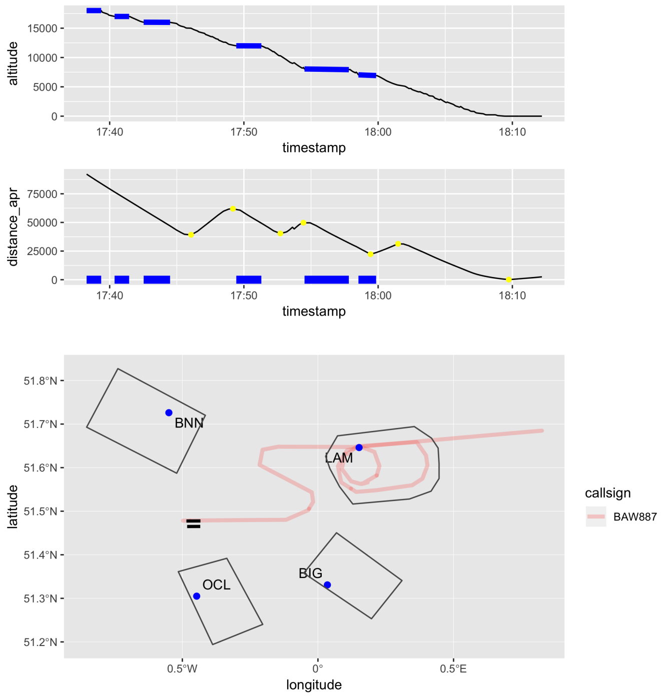

Figure 16. Holding stacks and level portions during descent at Heathrow Airport.

- Smoothing algorithms. Some of the current trajectories still show some spiky portions that could benefit from different/better algorithms to eliminate them.

Currently PRU is limiting the availability of RT to a few days in Aug 2017. This is mainly due to the still pending calibration activities outlined above, but the ultimate goal is to start providing RT data as from 2017 onwards. Thus PRU will in the near future further increase the offer of data (larger and larger temporal span), documentation and performance indicators based on this data set which is available at [27].

It is nevertheless important to clarify that the input data, i.e. FR24, CPRs or APDF in Figure 2, won't be made fully available but will eventually be released (if agreed by the owners) as a limited sample in order for interested parties to validate/reproduce the RT production steps.

In terms of usage of the RT dataset, PRU continues with the holding patterns case study and plans to eventually extend it to other major airports.

PRU is also committed to compare the horizontal flight efficiency and traffic complexity PI's calculated with current set of trajectories and the RT data set.

Further future work includes: apron modelling with stand positions, taxiways layouts, runway details and layout (from AIPs) in order to measure apron movements once ground positions eventually become available.

VI. Conclusions

This paper reflects on the establishment of an open flight RT dataset for operational ANS performance within the European region and addresses the need for reproducibility of performance analyses. Based on the development of such a capability, this paper provides an initial evaluation and use case application of the RT to determine operational performance in the terminal area of a European airport.

With a view to reproducibility, this paper is built on an openly accessible repository of trajectory data for the use-case analyses, the associated data processing, and the paper production including associated data analytical visualizations. The code and data can be found via a GitHub repository [27].

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